Economic Development in Eastern Germany: History, Expectations, and Public Policy*

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Preliminary Version

Abstract

Motivated by the process of economic development in Eastern Germany since the German reunification we set up a dynamic macroeconomic model of a small open economy where both capital and labor are mobile and there are increasing returns to scale at the aggregate level. The model features multiple equilibria and (local and global) indeterminacy. Expectations matter for resulting equilibrium dynamics, implying that "good or bad moods" are crucial for the process of economic development. We argue that this simple model is in line with the major stylized facts. The model is also instructive when it comes to better understanding the consequences of macroeconomic supply side policy and the pattern of East-West convergence.

Key words: increasing returns to scale; capital mobility; migration; multiple equilibria; indeterminacy; history vs. expectations; economic policy.

JEL classification: E6, H2, O4

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1 Introduction

The process of macroeconomic development in Eastern Germany since the German reunification features a number of interesting characteristics. Motivated by this historical example, we investigate the process of macroeconomic development in small open economies where both capital and labor are mobile and there are increasing returns to scale at the aggregate level. The paper's objective is twofold. First, we aim at a better understanding of the development process in Eastern Germany. Second, we try to better understand the consequences of macroeconomic supply side policy in small open economies.

The process of macroeconomic development in Eastern Germany since 1991 may be sketched by the following list of empirical regularities:

1. Real GDP per capita grew by an average annual rate of 3.6 percent between 1991 and 2007 (Arbeitskreis "Volkswirtschaftliche Gesamtrechnungen der Laender", 2008(a)).

2. The standard deviation of real GDP per capita across Eastern German regions increased between 1992 and 2006 by roughly 80 percent ("Regierungsbezirke", NUTS 2) and by 114 percent ("Kreise", NUTS 3) (Arbeitskreis "Volkswirtschaftliche Gesamtrechnungen der Laender", 2008(b)).[1]

3. At the aggregate level there has been substantial emigration. Between 2000 and 2006 about 70,000 people (0.5 percent of the population) emigrated from Eastern Germany per annum (Burda, 2006, p. 368). However, there are substantial regional differences. Some regions ("Kreise") shrank substantially and, at the same time, there are regions which attracted people to a substantial extent (Statistisches Bundesamt, Genesis database, 2008).

4. At the aggregate level there has been a substantial inflow of private capital. From 1991 to 2004 capital inflows amounted to 80 to 90 billion EUR, or about 20 percent of GDP, each year (Burda, 2006, p. 368).

5. Between 1991 and 2004 Eastern Germany has received massive fiscal transfers of about 80 billion EUR per year (4 percent of Germany's GDP). About 50 percent constitute social assistance (Snower, Merkl, 2006, p. 375).

The observation of regional divergence in per capita income (regularity #2) provides important evidence with regard to the underlying macroeconomic structure. The divergence pattern among

[1] Divergence at the level of Eastern German "Laender" (NUTS 1) is less pronounced. The standard deviation increased between 1991 and 2007 by roughly 29%.

Eastern German regions is therefore considered more deeply. Figure 1 shows the time path of the standard deviation of real GDP per capita between 1991 and 2006 for "Regierungsbezirke" (NUTS 2) and for "Kreise" (NUTS 3). Both graphs clearly indicate regional divergence. Moreover, Figure 2 shows the kernel density estimation (essentially a smoothed histogram) for the regional distribution of GDP per capita (at the level of "Kreise") in 1996 and 2006.[2] These plots confirm regional divergence in per capita income. The regional income distribution in 1996 appears unimodal, whereas in 2006 it appears bimodal. The upper tale became thicker, i.e. there are regions clustering in the upper range of regional income distribution.[3]

Is this empirical pattern compatible with the standard textbook model? Assume that regional output Y is produced with a constant returns to scale technology $Y = K^{\alpha}L^{1-\alpha}$ and both capital K and labor L have an outside option, denoted as $\bar{r}$ and $\bar{w}$ (for simplicity we ignore adjustment costs). It can be easily verified that the equilibrium capital intensity and per capita output are then given by $\tilde{k} = \frac{\alpha\bar{w}}{(1-\alpha)\bar{r}}$ and $\tilde{y} = \tilde{k}^{\alpha}$ (the size of the economy is indeterminate). This model implies convergence of per capita income (outside options $\bar{r}$ and $\bar{w}$ and technologies are the same for each region). The divergence pattern described above is even more remarkable since the degree of factor mobility appears to be quite high. Eastern Germany has unrestricted access to the international capital market and major migration costs associated with cultural and lingual differences do not apply (Hunt, 2006). Moreover, there are substantial productive government expenditures, funded by the central government, which aim at a "harmonization of living conditions" (as prescribed by the German constitution) by uniformly distributed public infrastructure investment.[4] Despite being clearly at odds with observed facts it will turn out, however, that the standard neoclassical approach is a good starting point for a richer and empirically plausible model.

We set up a simple dynamic macroeconomic model that captures the major relevant characteristics: (i) there is a high degree of labor mobility out of and into the domestic market sector (regularity #3);[5] (ii) there is also a high degree of private capital mobility (regularity #4); (iii) productive government expenditures play an important role (regularity #5). The dynamic di-

[2] A complete data set for the 102 East German "Kreise" is available for the first time in 1996. The results for 1994 and 1995 are qualitatively identical (for 1993 no data is provided). For 1992 the data set comprises only 82 "Kreise". The kernel density graph is produced using EViews. As kernel density weighting function the Epanechnikov kernel (default option) is used and the bandwidth parameter (controlling the smoothness of the graph) is data-based.

[3] The following "Kreise" have more than 45 perecnt of average Eastern GDP per capita in 2006: Dresden, Erfurt, Jena, Neubrandenburg, Potsdam, Schwerin, Wismar, Zwickau.

[4] The prescription of "harmonization of living conditions" is codified in Art. 72 GG, § 106 Abs. 3 GG, Art. 20 GG.

[5] It should also be noticed that the unemployment rate increased from 10 percent in 1991 to almost 20 percent in 2004 (Snower, Merkl, 2006, p. 375).

mension is introduced by assuming that labor and capital mobility is associated with adjustment costs. The model features increasing returns to scale (IRS) at the aggregate level and multiple equilibria (ME). There is (local and global) indeterminacy and hence expectations matter for resulting equilibrium dynamics. This aspect is especially interesting since it implies that "good or bad moods" are crucial, first, for the steady state the economy approaches in the long run and, second, for the equilibrium trajectory which leads the economy to the (inferior or superior) steady state. We argue that this model is largely in line with observed empirical facts and delivers a number of non-trivial implications: (i) the success of macroeconomic development is determined by history and expectations; (ii) the relative importance of expectations as major determinant of macroeconomic success depends on public policy; (iii) a strong macroeconomic supply side policy (modeled as high level of productive government expenditures) can foster economic development if optimism prevails. However, the same policy may be detrimental for economic development if moods are predominantly pessimistic.

Two strands of related literature should be mentioned. First, the process of macroeconomic development of Eastern Germany exhibits a number of interesting patterns, which has already attracted the attention of numerous researchers. For instance, Funke and Strulik (2000) and Burda (2006) have investigated the pattern of East-West convergence employing dynamic macroeconomic models. Snower and Merkl (2006) and Uhlig (2006) have studied the sources and consequences of the substantial and persistent increase unemployment figures using labor market models. Second, there is a large number of contributions dealing with IRS and ME in real macroeconomic models (e.g. Murphy et al., 1989; Benhabib and Farmer, 1994). It is well known that IRS may lead to ME. An important question, then, is how the process of equilibrium selection works. A large number of models imply that initial conditions are crucial (Deissenberg et al., 2001). Krugman (1991) has demonstrated that it is, in principle, both history (initial conditions) and expectations (moods) that matter. Our paper contributes also to this strand of the literature since we demonstrate that the relative importance of expectations depends on public policy in a systematic fashion.

The paper is structured as follows. Sections 2 sets up the basic model. Section 3 focuses on an important special case, which allows us to investigate the model more deeply. Section 4 demonstrates the major implications by numerical exploitation. Section 5 concludes.

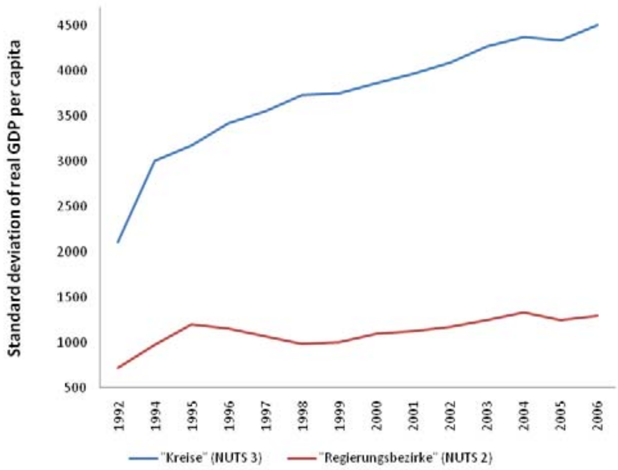

Figure 1: Divergence among East German Regions (data source: Arbeitskreis "Volkswirtschaftliche Gesamtrechnungen der Laender", 2008(b)).

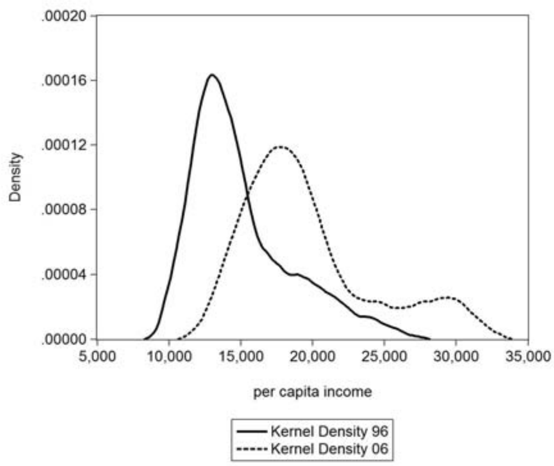

Figure 2: The emergence of twin peaks (data source: Arbeitskreis "Volkswirtschaftliche Gesamtrechnungen der Laender", 2008(b).

2 The model

We consider a dynamic one-sector model of a small open economy, which comprises a number of n regions. Every region is identical except for the initial amount of labor and capital allocated to the region's domestic market sector and the degree of initial optimism or pessimism. For simplicity regions do not interact. Both labor and capital have an outside option. Moving input factors out of or into the domestic market sector causes convex adjustment costs. The model features IRS at the aggregate level due to productive government expenditures à la Barro (1990).

2.1 Production technology and factor prices

Regional output Y_i is produced according to a standard Cobb-Douglas technology, as given by

$$Y_i = G_i^{\beta} K_i^{\alpha} L_i^{1-\alpha}, \tag{1}$$

where G_i denotes regional productive government expenditures (e.g., regional infrastructure investment, commercial zone development), K_i is capital employed in region i, L_i is the amount of labor employed in region i, where $i \in \{1, ..., n\}$ indexes the regions, and $0 < \beta, \alpha < 1$. We assume that regional productive government expenditures are proportional to regional tax receipts, i.e.

$$G_i = q\tau Y_i, \tag{2}$$

where $0 \leq \tau \leq 1$ is the unique tax rate levied on capital and labor income and $q > 0$. We do not impose a balanced budget ($q = 1$) for two reasons: First, there are other uses of tax receipts which are not modeled here (e.g., social transfers) and, second, regional productive government expenditures are typically matched by grants provided by the central government (hence q might be even larger than unity). The crucial assumption here is that there are (regional) productive government expenditures, labeled G_i, which vary positively with regional tax receipts.[6] This might capture productive government expenditures financed exclusively by regional authorities or productive government expenditures cofinanced by the central government through matching grants.[7]

Using $G_i = q\tau Y_i$ the reduced form technology reads

$$Y_i = (q\tau)^{\frac{\beta}{1-\beta}} K_i^{\frac{\alpha}{1-\beta}} L_i^{\frac{1-\alpha}{1-\beta}}. \tag{3}$$

Notice that this model features IRS, since $\frac{1}{1-\beta} > 1$, due to productive government expenditures.[8] Competitive factor prices can be expressed as

$$w_i = (1-\alpha)(q\tau)^{\frac{\beta}{1-\beta}} K_i^{\frac{\alpha}{1-\beta}} L_i^{\frac{\beta-\alpha}{1-\beta}}, \tag{4}$$

$$r_i = \alpha(q\tau)^{\frac{\beta}{1-\beta}} K_i^{\frac{\alpha-1+\beta}{1-\beta}} L_i^{\frac{1-\alpha}{1-\beta}}, \tag{5}$$

where r_i is the rate of return on capital and w_i denotes the wage rate in region i.

[6] In reality regional tax receipts are primarily composed of a share of the income tax, a share of the value added tax, and the business tax.

[7] Public policy instruments which are designed as matching grants comprise sponsorships within the framework of "Gemeinschaftsaufgabe" and "Europäische Strukturförderung" (Bundesministerium für Verkehr, Bau und Stadtentwicklung, 2006)

[8] The specific source for IRS is not crucial, most of the implications hold for other setups as well.

2.2 Workers

In every region there is mass one of identical workers, each of which is endowed with $\bar{L}_i$ hours per period. The representative worker supplies $0 \leq L_i \leq \bar{L}$ units of labor services to the market earning a competitive wage rate w_i per hour. Workers also have an outside option (primarily we think of emigration, but home work, consuming leisure, or unemployment compensation are also plausible examples), which pays a wage rate $\bar{w}$ per hour.[9] The resulting market income is subject to an income tax τ. The problem of the typical worker reads (to simplify the notation, the region index i is suppressed whenever no ambiguity arises)

$$\begin{aligned}
&\max_{\{v_L\}} \int_0^\infty \left[(1-\tau)wL + \bar{w}(\bar{L}-L) - \frac{1}{2\gamma_L}v_L^2\right] e^{-\rho t}dt \\
&s.t.\ \dot{L} = v_L \\
&L(0) = L_0;\ 0 \leq L \leq \bar{L},
\end{aligned} \tag{6}$$

where $\rho > 0$ denotes the time preference rate and $\dot{L} := dL/dt$. Moving labor from the region's domestic market sector to the outside option, or vice versa, causes (symmetric and convex) adjustment costs which reduce current income, as captured by the term $-\frac{1}{2\gamma_L}v_L^2$. The parameter $\gamma_L > 0$ is an inverse measure of the importance of adjustment costs. For $\gamma_L \to 0$ a given reallocation of labor $v_L = \dot{L}$ reduces income substantially and, conversely, for $\gamma_L \to \infty$ adjustment costs become negligible. The associated (current-value) Hamiltonian function is given by

$$H_L = (1-\tau)wL + \bar{w}(\bar{L}-L) - \frac{1}{2\gamma_L}v_L^2 + \lambda_L v_L,$$

where $\lambda_L \lesseqgtr 0$ is the shadow price placed on labor in the domestic market sector. The first-order conditions read

$$\frac{\partial H_L}{\partial v_L} = -\frac{1}{\gamma_L}v_L + \lambda_L = 0 \quad \Longrightarrow \quad v_L = \gamma_L \lambda_L, \tag{7}$$

$$\dot{\lambda}_L = \rho\lambda_L - \frac{\partial H_L}{\partial L} = \rho\lambda_L - [(1-\tau)w - \bar{w}]\,. \tag{8}$$

The efficiency condition (7) says that, in equilibrium, marginal moving costs $\frac{v_L}{\gamma_L}$ must equal the shadow price λ_L. Equation (8) implies that $\lambda_L(0) = \int_0^\infty [(1-\tau)w - \bar{w}]\, e^{-\rho t}dt$, i.e. $\lambda_L(0)$ gives the present value of the difference between earnings in the region's domestic market sector and in the outside option. Since the competitive wage rate w depends on the amount of labor and

[9] Hunt (2006) investigates East-West migration patterns and finds that wages in the source region (Eastern Germany) affect migration much more than unemployment in the source region. This piece of empirical evidence supports the model, which focuses on wage differentials as the prime source for migration.

capital employed in the domestic market sector, $\lambda_L(0)$ captures expectations about future economic development. If the difference between domestic earnings and earnings in the outside option, in present value terms, is positive ($\lambda_L(0) > 0$), then labor flows into the region's domestic market sector ($v_L = \dot{L} > 0$), and vice versa.

2.3 Capital owners

Capital owners are largely modeled symmetrically to workers. There is mass one of identical capital owners. Each capitalist is endowed with $\bar{K}$ units of capital. Capital can be employed in the region's domestic market sector earning a rate of return r. Alternatively, capital can be invested abroad to earn the fixed rate of return $\bar{r} > 0$. The representative capitalist maximizes the present-value of an infinite income stream, i.e. solves the following problem[10]

$$\begin{aligned} &\max_{\{v_K\}} \int_0^\infty \left[(1-\tau) r K + \bar{r}(\bar{K} - K) - \frac{1}{2\gamma_K} v_K^2 \right] e^{-\rho t} dt \\ &s.t.\ \dot{K} = v_K \\ &K(0) = K_0;\ 0 \leq K \leq \bar{K}, \end{aligned} \tag{9}$$

where $\rho > 0$ denotes the time preference rate and $\dot{K} := dK/dt$. Moving capital from the region's domestic market sector to the outside option, or vice versa, causes (symmetric and convex) capital adjustment costs which reduce current income, as captured by the term $-\frac{1}{2\gamma_K} v_K^2$. As before, the parameter $\gamma_K > 0$ is an inverse measure of the importance of adjustment costs. The associated (current-value) Hamiltonian function is given by

$$H_K = (1-\tau) r K + \bar{r}(\bar{K} - K) - \frac{1}{2\gamma_K} v_K^2 + \lambda_K v_K,$$

where $\lambda_K \lesseqgtr 0$ is the shadow price placed on capital in the domestic market sector. The first-order conditions read as follows

$$\frac{\partial H_K}{\partial v_K} = -\frac{1}{\gamma_K} v_K + \lambda_K = 0 \quad \Longrightarrow \quad v_K = \gamma_K \lambda_K, \tag{10}$$

$$\dot{\lambda}_K = \rho \lambda_K - \frac{\partial H_K}{\partial K} = \rho \lambda_K - [(1-\tau) r - \bar{r}]\,. \tag{11}$$

The efficiency condition (10) says that, in equilibrium, marginal moving costs $\frac{v_K}{\gamma_K}$ must equal the shadow price λ_K. Equation (11) indicates that $\lambda_K(0) = \int_0^\infty [(1-\tau) r - \bar{r}]\, e^{-\rho t} dt$, i.e. $\lambda_K(0)$

[10] One could object that this setup is not completely realistic since the typical East German capital owner did not posses substantial wealth allocated outside the economy in 1991. However, this setup is largely equivalent to the case of a typical East German capital owner who has initial wealth $\bar{K}_E$ (invested in domestic economy) and a typical West German capital owner who has initial wealth $\bar{K}_W$ (invested outside East Germany). Opening up the economy then allows cross-border capital investments (see the appendix for details).

gives the present value of the difference between earnings in the domestic market sector and in the outside option. As before, since the competitive rate of return r depends on the amount of capital and labor employed in the domestic market sector, $\lambda_K(0)$ captures expectations about future economic development. Technically, this model is essentially a two-dimensional extension of the Krugman (1991) model.

2.4 Complete dynamic system and steady states

The evolution of the economy within the interior of the state space is governed by the following dynamic system

$$\dot{L} = \gamma_L \lambda_L \tag{12}$$

$$\dot{K} = \gamma_K \lambda_K \tag{13}$$

$$\dot{\lambda}_L = \rho\lambda_L - [(1-\tau)w - \bar{w}] \tag{14}$$

$$\dot{\lambda}_K = \rho\lambda_K - [(1-\tau)r - \bar{r}] \tag{15}$$

$$L(0) = L_0;\ K(0) = K_0$$

with w and r given by (4) and (5). Provided that $\lambda_L(0)$ and $\lambda_K(0)$ are specified, the above system describes a unique trajectory in four-dimensional $(L, K, \lambda_L, \lambda_K)$-space. It should be noticed, however, that $\lambda_L(0)$ and $\lambda_K(0)$ are not uniquely determined. There is rather an infinite number of shadow price combinations $\{\lambda_L(0), \lambda_K(0)\}$ which are admissible as self-fulfilling prophecies (see Section 3.1. below).

A steady state is determined by $\dot{L} = \dot{K} = \dot{\lambda}_L = \dot{\lambda}_K = 0$. We first turn to the interior steady state. From $\dot{L} = \gamma_L\lambda_L$ and $\dot{K} = \gamma_K\lambda_K$ we see that $\dot{L} = \dot{K} = 0$ requires $\lambda_L = \lambda_K = 0$. From (8), (11), $\lambda_L = \lambda_K = 0$ (implying $\dot{\lambda}_L = \dot{\lambda}_K = 0$) one gets $(1-\tau)w = \bar{w}$ and $(1-\tau)r = \bar{r}$. These two equations in L and K characterize the interior steady state in (K, L)-plane.[11] Noting (4) and (5) and solving for L gives

$$L = \left(\frac{\bar{r}}{(1-\tau)\alpha}\right)^{\frac{1-\beta}{1-\alpha}} (q\tau)^{\frac{-\beta}{1-\alpha}} K^{\frac{1-\alpha-\beta}{1-\alpha}} \tag{16}$$

$$L = \left(\frac{\bar{w}}{(1-\tau)(1-\alpha)}\right)^{\frac{1-\beta}{\beta-\alpha}} (q\tau)^{\frac{-\beta}{\beta-\alpha}} K^{\frac{\alpha}{\alpha-\beta}} \tag{17}$$

Since $0 < \frac{1-\alpha-\beta}{1-\alpha} < 1$ the RHS of (16) is increasing and concave in K and since $\frac{\alpha}{\alpha-\beta} > 1$ (assuming $\alpha > \beta$) the RHS of (17) is increasing and convex in K. Hence, there is a *unique interior solution* $\{L^*, K^*\}$ (Figure 3, point A). There are also *two boundary steady states*. The lower

[11] Notice, however, that $(1-\tau)w = \bar{w}$ and $(1-\tau)r = \bar{r}$ are necessary but not sufficient for $\dot{L} = 0$ and $\dot{K} = 0$; sufficient for $\dot{L} = 0$ ($\dot{K} = 0$) is $\lambda_L = 0$ ($\lambda_K = 0$).

(inferior) steady state is $\{L = 0, K = 0\}$ (Figure 3, point C). The upper (superior) steady state reads $\{L = \bar{L}, K = K^{**} \leq \bar{K}\}$ (Figure 3, point B).

As regards the dynamics at the border of the state space, two aspects need to be clarified. First, the economy remains at the boundary once it touches the border of the state space (see the appendix for details). Second, in a world of IRS we need to ensure that factor inflows sooner or later come to a halt. We assume that some political mechanism prevents more than a maximum amount of labor being supplied to the region's domestic market sector. For simplicity, this maximum amount of labor is set equal to $\bar{L}$ (a larger value for this maximum amount of labor wouldn't change the implications). Moreover, we assume that $(1-\tau)r(\bar{K}, \bar{L}) \leq \bar{r}$. This condition guarantees that capital inflows come to a halt. Graphically speaking this condition implies that the RHS of (16) hits the upper L-boundary at some $K \leq \bar{K}$. Assume that the economy hits, say, the upper L-boundary at $t = T$, i.e. $L(T) = \bar{L}$, with $0 < K(T) < \bar{K}$. The dynamics of the economy are then governed by (13) and (15) (noting that $L(T) = \bar{L}$). The shadow price λ_K at $t = T$ jumps in order to satisfy the transversality condition. Capitalists then increase the amount of capital allocated to the domestic market sector until $(1 - \tau)r = \bar{r}$. This movement is sluggish because of convex adjustment costs (for details see the appendix).

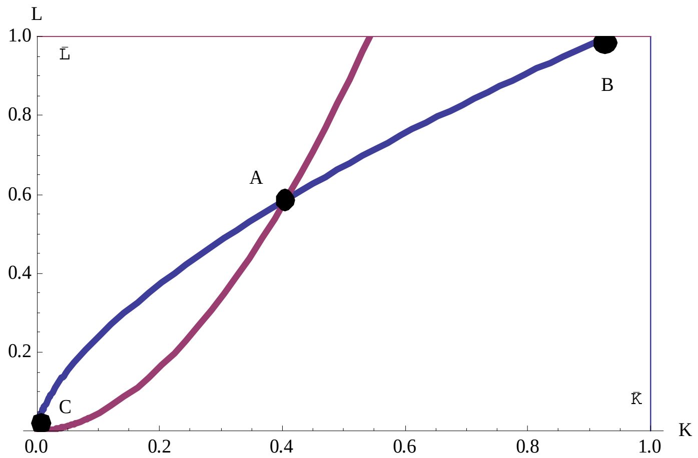

Figure 3: Multiple equilibria.

3 An important special case

We now impose the following parameter restriction $\alpha = \beta = 0.5$. The dynamic system under study then becomes linear and allows for an analytical solution. This special case enables us to analyze the model more deeply and demonstrate its implications. This procedure obviously comes at the cost of imposing an unrealistic parameter restriction since the implied degree of IRS is 2.[12] It must also be assumed that some exogenous (political) mechanism prevents unlimited capital inflows. Despite these shortcomings we think that the analysis of this special case is clearly instructive.[13]

3.1 Equilibrium dynamics

From $(1-\tau)w = \bar{w}$ and $(1-\tau)r = \bar{r}$ together with $w = 0.5q\tau K$ and $r = 0.5q\tau L$, the interior steady state turns out to read $L^* = \frac{2\bar{r}}{(1-\tau)q\tau}$, $K^* = \frac{2\bar{w}}{(1-\tau)q\tau}$, $\lambda_L^* = 0$, $\lambda_K^* = 0$. It can be shown that there are always three eigenvalues with positive real parts (for details see the appendix). The interior steady state is unequivocally unstable. There is a three-dimensional unstable manifold leading away from the interior steady state. Since there are two (predetermined) state variables and two jump variables, there is indeterminacy in the sense of a multiplicity of admissible initial shadow prices $\{\lambda_L(0), \lambda_K(0)\}$ that constitute equilibrium dynamics. Figure 4 shows the (unstable) interior steady state (point A) and several possible equilibrium trajectories leading to one of the boundary steady states (point B or C). As regards the admissible equilibrium trajectories, several remarks are at order:

1. Equilibrium trajectories must approach the border of the state region tangential, i.e. satisfying either $\dot{L}(T) = \lambda_L(T) = 0$ or $\dot{K}(T) = \lambda_K(T) = 0$ ("soft landing"). Once the economy hits the border it does not return into the interior of the state space. Instead it moves along the boundary to one of the border equilibria (for details see the appendix).

2. Starting with initial conditions $L(0) = L_0$ and $K(0) = K_0$ there is an infinite number of shadow price combinations $\{\lambda_L(0), \lambda_K(0)\}$ which are admissible as self-fulfilling prophecies. Hence, there is an infinite number of equilibrium trajectories, indexed by initial shadow prices, satisfying the soft landing criterion.

3. Initial shadow prices are exogenous. The set of admissible shadow prices is, however, restricted. More specifically, shadow prices must satisfy two conditions: (i) they must be fundamentally warranted, i.e. equal the present value of expected earning differentials over a limited period and (ii) they must be compatible with equilibrium, i.e. induce a trajectory that satisfies the soft landing condition.

[12] Schmitt-Grohé (1997) reviews the empirical evidence on IRS at the level of industries. She finds that the degree of IRS ranges from 1.03 to 1.4 (Schmitt-Grohé, 1997, Table 4; see also the literature cited in Graham and Temple, 2006). Moreover, empirical evidence indicates that $\beta \in [0.16, 0.39]$ (Aschauer, 1989; Finn, 1993).

[13] Figure 9 in the appendix demonstrates that the number of eigenvalues (at the interior steady state) with positive real parts does not change in response to (admissible) variations in α and β.

Both history (initial state variables) and expectations (initial shadow prices) determine to which equilibrium the region ultimately converges. If the region starts inside a specific (K, L)-set, expectations (initial shadow prices) determine whether the region moves towards the superior or inferior steady state, i.e. the model exhibits global indeterminacy. Within this overlap, a term coined by Krugman (1991), knowledge about initial state variables is not sufficient to determine the final outcome. If the economy starts with comparably unfavorable initial conditions (i.e. south-west of the overlap) it converges to the inferior steady state. In contrast, if it starts with comparably favorable initial conditions (i.e. north-east of the overlap) it converges to the superior steady state.

Is a large overlap good or bad? The answer is that it is neither good nor bad. A large overlap may imply that, even under unfavorable initial conditions, the economy is capable, due to strong optimism, of moving towards the superior steady state. In contrast, even under favorable initial conditions there is the risk that, due to a high degree of pessimism, the inferior steady state is ultimately realized. In this sense, the economy becomes more vulnerable against bad moods. Hence, an adequate, although fairly general, proposition states that the relative importance of expectations vis-à-vis history increases with the size of the overlap.

Figure 4 illustrates the basic logic of the model. The parameters ($\bar{K}$,$\bar{L}$) have been chosen such that the interior steady state is centered (in addition, the maximal K and L values are normalized to one). The overlap is represented by the shaded area. Consider a region starting in, say, point D. Provided that agents are optimistic ("good mood"), i.e. $\lambda_L(0), \lambda_K(0) > 0$, both workers and capitalists increasingly engage in the region's domestic market economy. The economy moves towards the upper border $L = \bar{L}$. In contrast, if the agents are pessimistic ("bad mood"), i.e. $\lambda_L(0), \lambda_K(0) < 0$, both labor and capital leaves the region's domestic market sector and the economy is heading towards the lower L-border and eventually approaches the inferior equilibrium point C.[14]

Figure 5 demonstrates the size of the overlap in response to changes in q. Extensive experimentation has shown that the overlap increases with q. This makes good economic sense. A large overlap basically means that a region with, say, a comparably low initial level of capital and labor is capable of moving towards the superior steady state. This requires a sufficiently high degree of optimism, i.e. a sufficiently high value of $\lambda_{i,L}(0)$ and $\lambda_{i,K}(0)$. Recall that shadow prices represent the difference between factor rewards and outside options in present value terms. Since factor rewards increase with q, a higher q-value enables a degree of optimism to be fundamentally warranted such that, despite unfavorable fundamentals, the economy may follow a favorable development path.

[14] Equilibrium trajectories could, of course, also hit the (lower or upper) K-boundary for interior L-values. This pattern is, however, rarely observed for plausible calibrations (i.e. $\bar{K} >> \bar{L}$ and $\gamma_L = 0.5\gamma_K$). A description of the underlying numerical procedure which has been employed to visualize the overlap is available upon request.

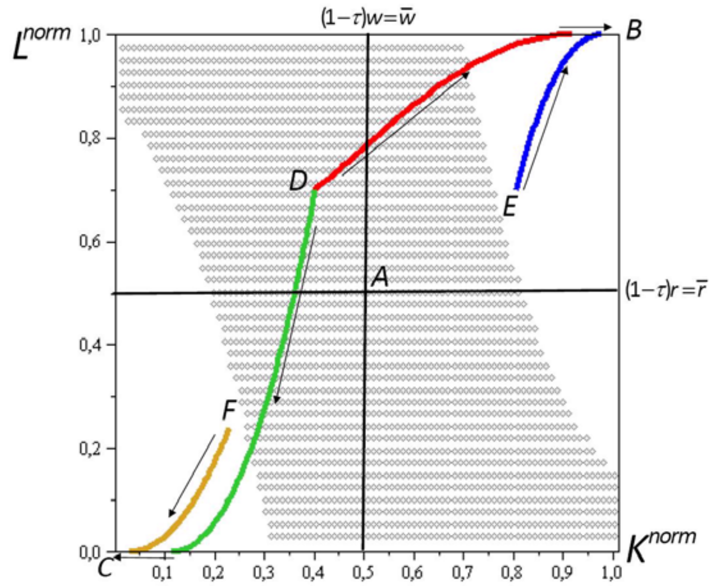

Figure 4: Three steady states and four possible equilibrium trajectories.

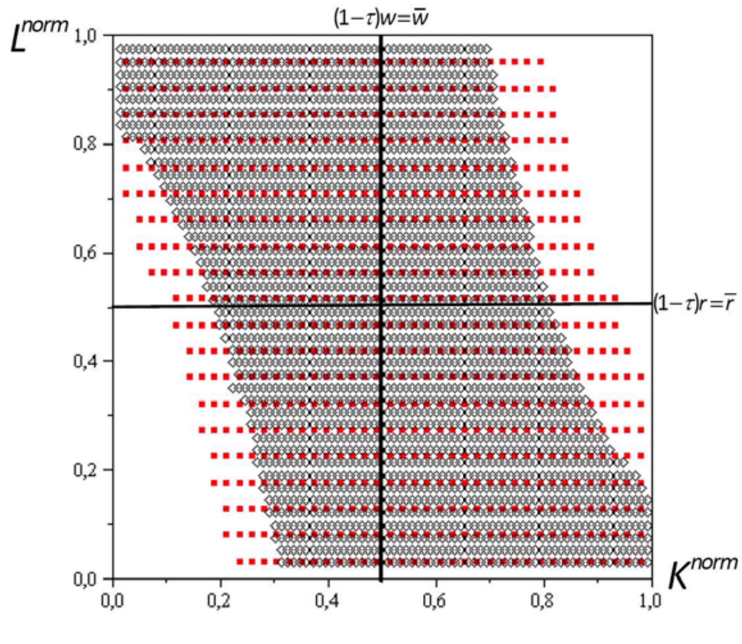

Figure 5: The size of the overlap in response to changes in q.

4 Understanding implications due to numerical exploitation

4.1 Sketch of the underlying procedure

The model is exploited in the following manner. First, we consider a "large number" of regions $i \in \{1, ..., n\}$. Every region starts with a specific combination $\{K_i(0), L_i(0)\}$, which are restricted to fall inside the upper left of the state plane. This assumption implies $(1-\tau)w_i < \bar{w}$ and $(1-\tau)r_i > \bar{r}$ being in line with the empirical observations of (aggregate) labor outflows and (aggregate) capital inflows (see Figure 6 for an illustration). Second, initial shadow prices $\lambda_{i,L}(0)$ are drawn from a normal distribution with $E[\lambda_{i,L}(0)] = -5.9$ and $V[\lambda_{i,L}(0)] = 2$ (these numbers are motivated in the section on calibration). The values of $\lambda_{i,K}(0)$ are then determined by the soft landing criterion (i.e. $\lambda_{i,L}(T) = 0$ or $\lambda_{i,K}(T) = 0$). Once, $K_i(0), L_i(0)$, $\lambda_{i,L}(0)$ and $\lambda_{i,K}(0)$ are specified, one can trace out $K_i(t), L_i(t)$ for all $t \in [0, T]$ and $i \in \{1, ..., n\}$. Third, we calculate the time path of average per capita income $y(t) := \frac{\sum Y_i(t)}{\sum L_i(t)}$. This is done for the baseline set of parameters. Next we change crucial parameters (initial mood $E[\lambda_{i,L}(0)]$ and policy parameter q) to better understand the implications of the model.

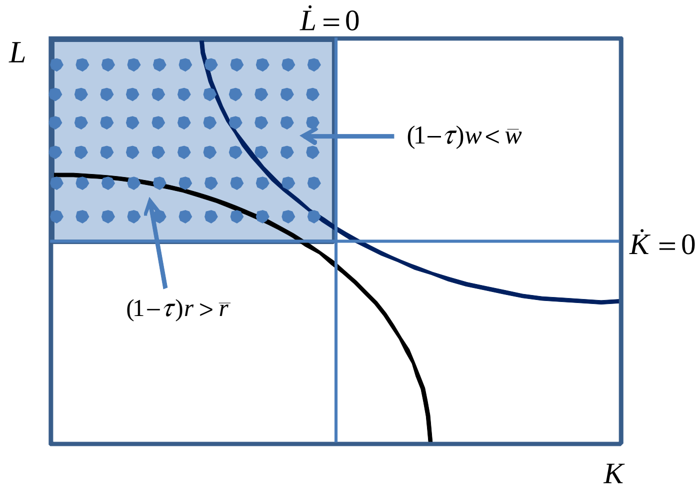

Figure 6: Illustration of the underlying procedure.

4.2 Calibration

Given $\alpha = \beta = 0.5$ we aim at a careful specification of the remaining parameters. This procedure demonstrates that the model under study can be calibrated to a specific real-world economy. The baseline set of parameters is described by the following table.

Table 1: Baseline set of parameters.

Technology and preferences	$\alpha = 0.5;\ \beta = 0.5;\ \gamma_L = 0.007;\ \gamma_K = 2\gamma_L;\ \rho = 0.02$
Policy and outside option	$\tau = 0.36;\ q = 0.5;\ \bar{r} = 0.05;\ \bar{w} = 1$
Initial moods	$E[\lambda_L(0)] = -5.9;\ V[\lambda_L(0)] = 2$

The time preference rate $\rho = 0.02$ is in line with usual calibrations ($0.01 \leq \rho \leq 0.03$). The tax rate $\tau = 0.36$ is the average between the (marginal) capital income tax rate in 2000 (about 0.31) and the (marginal) labor income tax rate for the average earner in 2000 (about 0.4) (OECD, 2007). The policy parameter $q = 0.5$ results from the following observation: The ratio of average public investment across all East German regional authorities (excluding Berlin) from 1992 to 2007 and average tax receipts across all Eastern German regional authorities (excluding Berlin) from 1992 to 2007 amounts to 0.48.[15] The rate of return on capital (net of taxes) $\bar{r}$ is set to 5 percent and the wage rate in the outside option is normalized to one.

The adjustment cost parameters γ_L and γ_K are determined as follows. First notice that we set the size of the factor endowment box such that the interior steady state is centered along both the K-dimension and the L-dimension. Given the parameters we have specified so far this leads to $\{L^* \cong 0.87, K^* \cong 17.36\}$, i.e. $\{\bar{L} = 2L^*, \bar{K} = 2K^*\}$. Moreover, assume that the domestic market wage (net of taxes) converges to the wage rate given by the outside option $\bar{w}$ at a constant rate of convergence 0.065:[16]

$$(1-\tau)w(t) - \bar{w} \simeq [(1-\tau)w(0) - \bar{w}]\, e^{-0.065t}$$

Plugging the RHS into $\lambda_L(0) = \int_0^\infty [(1-\tau)w - \bar{w}]\, e^{-\rho t} dt$ and noting that $(1-\tau)w(1991) - \bar{w} \simeq 0.5$ (Burda, 2006, Table 1) yields

$$\lambda_L(0) = \int_0^\infty [(1-\tau)w(0) - \bar{w}]\, e^{-0.065} e^{-0.02t} dt = \frac{-0.5}{0.085} \cong -5.9$$

Moreover, in 1991 about 2.4 percent of the East German labor force left Eastern Germany.[17] Now we can determine $\dot{L}$ in our model, which is $\hat{L}(91) * \bar{L} = -0.024 * 1.71$ ($\hat{L}$ denotes the growth rate). Taken together this gives the following value for γ_L

$$\gamma_L = \frac{\dot{L}}{\lambda_L} = \frac{-0.024 * 1.74}{-5.9} \cong 0.007$$

[15] Data on public investment are taken from Statistisches Bundesamt (2008a), data on tax receipts are taken from Statistisches Bundesamt (2008b).

[16] This procedure basically follows Burda (2006).

[17] See Burda (2006, p. 372) and the labor force data on www.statistik-bw.de/Arbeitskreis_VGR/tbls/tab16.asp.

Capital mobility costs are set at 50 percent of labor mobility costs, i.e. $\gamma_K = 2\gamma_L$. Finally, we set $V[\lambda_L(0)]$ equal to 2; recall that $E[\lambda_K(0)]$ and $V[\lambda_K(0)]$ are endogenous.

4.3 Results

Figure 7 shows the time path of average per capita income $y(t) := \frac{\sum Y_i(t)}{\sum L_i(t)}$ under $E[\lambda_{i,L}(0)] = -5.9$ (baseline scenario) and $E[\lambda_{i,L}(0)] = -5.3$ (scenario "less pessimism") for all $i \in \{1, ..., n\}$. Average per capita income at $t = 0$ has been normalized to one. The time paths clearly show that better moods ultimately lead to a higher level of per capita income. The economic intuition for this observation is straightforward. Given the regional economic fundamentals $\{K_i(0), L_i(0)\}$ and public policy (τ and q), comparably favorable moods induce more workers and capital owners to engage in the region's domestic market sector such that more regions follow a favorable economic development heading towards the superior steady state.[18]

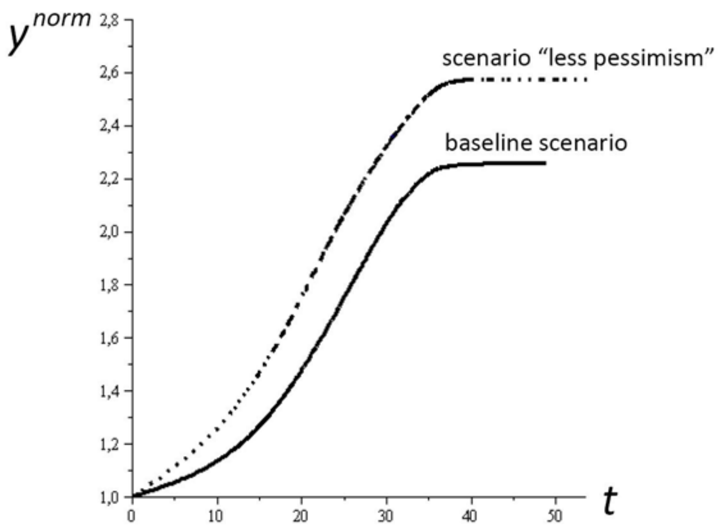

Figure 7: Time path of average per capita income under alternative moods.

Figure 8 shows the time path of average per capita income $y(t)$ for two alternative values of q. Assuming $q = 0.5$ leads to a higher initial level of per capita income compared to $q = 0.45$. This observation is in line with basic economic reasoning. A higher q-value, everything else the same, implies higher factor rewards (recall $r = 0.5q\tau L$ and $w = 0.5q\tau K$) and hence a higher level

[18] Regional per capita income may temporarily increase in regions which move towards the inferior steady state. This requires that labor leaves the region more rapidly than capital. Conversely, regional per capita income may decrease along the transition in regions moving towards the superior steady state.

of income. However, the lower q-value ($q = 0.45$, i.e. less productive government expenditures) turns out to lead to a higher average level of per capita income in the long run. This obviously counterintuitive and surprising result can be explained as follows. As shown above, the size of the overlap (i.e. the (K,L)-set where expectations are crucial) increases with q. Now, given the assumed degree of pessimism, as reflected by $E[\lambda_{i,L}(0)] = -5.9$ for all $i \in \{1, ..., n\}$, a larger overlap induces more regions to follow a comparably unfavorable development path heading towards the inferior steady state. To sum up, this observation demonstrates the important point that a strong macroeconomic supply side policy may turn out unfavorable provided that expectations of agents are comparably pessimistic.

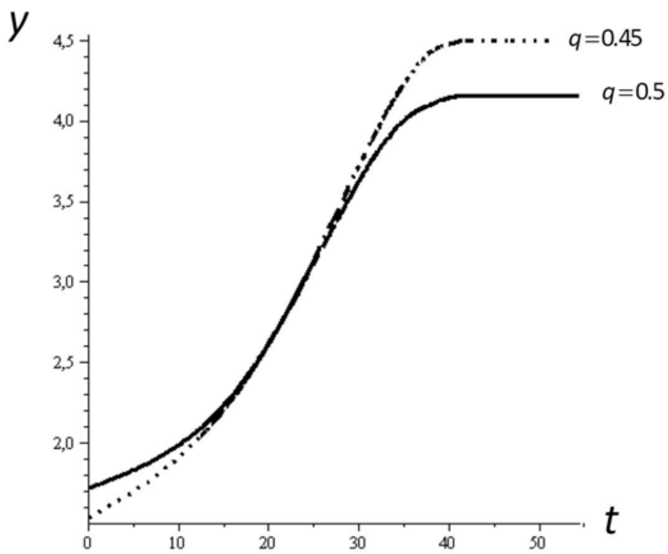

Figure 8: Time path of per capita income in response to changes in q.

5 Summary and conclusion

Motivated by the historical example of Eastern Germany we have set up a dynamic macroeconomic model of a small open economy where both capital and labor are mobile and there are increasing returns to scale at the aggregate level. The model features multiple equilibria and (local and global) indeterminacy. Expectations matter for the resulting equilibrium dynamics implying that "good or bad moods" are crucial for economic development.

This simple model is in line with a number of stylized facts: (i) regional divergence of per capita income; (ii) aggregate inflow of private capital; (iii) aggregate labor outflows. This implies that the model can also account for an asymmetric pattern of aggregate factor movements, as observed by

Burda (2007, p. 5): "This pattern of adjustment with factor accumulation in opposite directions is difficult to account for using a simple neoclassical growth framework". Moreover, the model can account for regional heterogeneity in labor movements, i.e. some regions attract labor while others experience labor outflows, as documented by regularity #3 above.

The paper's main result concerns the observation that the long term success of public policy measures may be conditional on certain side conditions. Specifically, a strong macroeconomic supply side policy (modeled as a high level of productive government expenditures) may be beneficial if optimism prevails. However, the same policy turns out detrimental for long term economic development if expectations of agents are predominantly pessimistic. This surprising and counterintuitive result can be explained within the context of models with multiple equilibria where expectations play a decisive role for economic development. The reason is that public policy affects the relative importance of expectations vis-à-vis history in the process of equilibrium selection. Hence, the consequences of any policy measure depend crucially on the degree of optimism or pessimism.

This aspect is also instructive when it comes to better understanding the pattern of East-West convergence. Uhlig (2006, p. 383) recently noted that "Fiscal transfers into East Germany have been massive, for a total transfer of nearly one trillion EUR from West to East Germany from 1991 to 2003, averaging close to 37 percent of East German GDP, throughout. These transfers may have improved the lives of East Germans, but they do not seem to have accelerated convergence."[19] The trivial explanation for the observation of "limited convergence" would refer to the consumptive nature of a substantial fraction of these fiscal transfers. A more interesting and instructive explanation would be based on the mechanism described above. Specifically, a high level of productive government expenditures may increase the standard of living in the short run but may be detrimental for long term economic development (hence convergence) provided that moods are predominantly pessimistic.

6 Appendix

6.1 Notes on stability ($\alpha = \beta = 0.5$)

We assume that $\alpha = \beta = 0.5$. The Jacobian matrix of system (13) to (14) then reads as follows

[19] Real GDP per capita in Eastern relative to Western Germany stood at 30 percent in 1991, reached about 60 percent in 1996 and approached 70 percent in 2007 (Arbeitskreis "Volkswirtschaftliche Gesamtrechnungen der Laender", 2008(a)).

$$J = \begin{pmatrix} 0 & 0 & \gamma_K & 0 \\ 0 & 0 & 0 & \gamma_L \\ 0 & 0.5q(1-\tau)\tau & \rho & 0 \\ 0.5q(1-\tau)\tau & 0 & 0 & \rho \end{pmatrix}.$$

It can be readily shown that the eigenvalues of the Jacobian matrix are given by

$$r_{1,2} = \frac{1}{2}\left[\rho \pm \sqrt{\rho^2 - 2\sqrt{q^2\gamma_L\gamma_K(\tau-1)^2\tau^2}}\right],$$

$$r_{3,4} = \frac{1}{2}\left[\rho \pm \sqrt{\rho^2 + 2\sqrt{q^2\gamma_L\gamma_K(\tau-1)^2\tau^2}}\right].$$

Several remarks are at order:

1. Eigenvalues $r_{3,4}$ are always real-valued. Moreover, $r_3 > 0$ and $r_4 < 0$. This requires $\rho < \sqrt{\rho^2 + 2\sqrt{q^2\gamma_L\gamma_K(\tau-1)^2\tau^2}}$, which boils down to $q^2\gamma_L\gamma_K(\tau-1)^2\tau^2 > 0$ being always true.

2. As regards $r_{1,2}$, we need a case distinction: Provided that $\rho^2 < 2\sqrt{q^2\gamma_L\gamma_K(\tau-1)^2\tau^2}$ eigenvalues $r_{1,2}$ are conjugate complex with positive real parts $\frac{1}{2}\rho > 0$.

3. If, on the other hand, $\rho^2 > 2\sqrt{q^2\gamma_L\gamma_K(\tau-1)^2\tau^2}$ eigenvalues $r_{1,2}$ are real valued. Eigenvalue $r_1 > 0$. Eigenvalue $r_2 > 0$ iff $\rho > \sqrt{\rho^2 - 2\sqrt{q^2\gamma_L\gamma_K(\tau-1)^2\tau^2}}$. This condition boils down to $q^2\gamma_L\gamma_K(\tau-1)^2\tau^2 > 0$, which is always true.

The set of eigenvalues of the Jacobian matrix associated with the dynamic system (12), (13), (14), and (15) with w and r given by (4) and (5) exhibits the following pattern: there are three eigenvalues with positive real parts and one eigenvalue with negative real part.[20] Since there are two jump variables and three unstable roots, the interior steady state is unstable.

Local indeterminacy. There is a three-dimensional, unstable manifold which leads the economy away from the interior steady state. Since the state space has dimension two, there is (local) indeterminacy. Given an initial condition $K(0) = K_0$ and $L(0) = L_0$ there are different combinations $\{\lambda_K(0), \lambda_L(0)\}$ which lead the economy to the inferior steady state (superior steady state), i.e. there is a multiplicity of paths leading to the inferior steady state (superior steady state).

Global indeterminacy. Given an initial condition $K(0) = K_0$ and $L(0) = L_0$ inside the overlap there are different possible combinations $\{\lambda_K(0), \lambda_L(0)\}$ such that the economy evolves either to the inferior steady state $\{K = 0, L = 0\}$ or to the superior steady state $\{K = K^{**}, L = \bar{L}\}$.

[20] This can be shown analytically for $\alpha = \beta = 0.5$. In addition, numerical evaluations indicate that this pattern is stable also in the non-linear case.

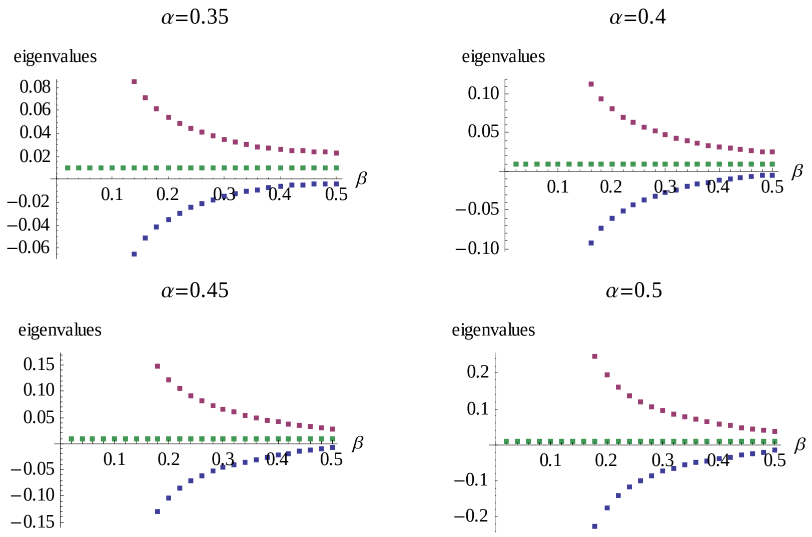

Figure 9: Eigenvalues (real parts) in response to changes in α and β. Notice that the horizontal line represents the two (identical) real parts of a pair of conjugate complex eigenvalues.

6.2 Notes on equilibrium dynamics

Reasoning of Fukao and Benabou (1993). Fukao and Benabou (1993, Proposition 2) have shown that, within the one-factor Krugman (1991) model, equilibrium trajectories must satisfy two conditions: (i) the shadow price of the factor reaching the boundary must approach zero and (ii) once the boundary has been touched, equilibrium implies that the economy remains at the boundary forever. The reasoning relies on an arbitrage condition, which must hold in equilibrium, and applies also to the model under study: assume that the economy hits, say, the lower L-boundary at $t = T$ (i.e. $L(T) = 0$ with $K > 0$) with $\lambda_L(T) < 0$. In this case, each individual worker has an incentive to deviate from the trajectory under consideration since he can realize the gain, reflected by $\lambda_L(T) < 0$, an instant in time later and thereby avoid all reallocation costs (the individual is of measure zero) by moving one period later. Hence, any equilibrium trajectory must hit the L-boundary such that $\lambda_L(T) = 0$.

A similar reasoning applies to the case when the economy is located at the boundary and remains there forever. Assume the economy is located at the lower L-boundary (i.e. $L = 0$ and $K > 0$). In this case $w > \bar{w}$ applies. It would indeed be optimal for workers to return into the domestic market sector. This will, however, never happen. Each individual worker has an incentive to realize the gain, reflected by $w > \bar{w}$, an instant in time later by moving alone and thereby avoiding reallocation costs. Hence, the fact that the economy does not return into the interior of the state region is essentially due to a coordination failure in market equilibrium.

The arbitrage argument used here relies on one crucial assumption, namely that the individual agent is of measure zero. This guarantees that the deviation of any individual from a given trajectory does not change competitive factor rewards and hence leaves λ_K and λ_L unchanged. Moreover, this assumption guarantees that reallocation costs are zero if one agent moves in isolation.[21] Therefore, this reasoning extends to the two-factor case under consideration with atomistic agents implying that equilibrium trajectories must approach the border of the state region tangential, i.e. satisfying either $\dot{L}(T) = \lambda_L(T) = 0$ or $\dot{K}(T) = \lambda_K(T) = 0$ and, in addition, remains at the border of the state space once the economy hits the boundary.

Boundary dynamics. Assume that the economy hits, say, the L-border at $t = T$, i.e. $L(T) = 0$ or $L(T) = \bar{L}$, with $0 < K(T) < \bar{K}$. The dynamics of the economy are then governed by (13) and (15) (noting that $L(T) = 0$ or $L(T) = \bar{L}$). The shadow price λ_K at $t = T$ jumps in order to satisfy the transversality condition. . Next assume that the economy hits the K-border at $t = T$, i.e. $K(T) = 0$ or $K(T) = \bar{K}$, with $0 < L(T) < \bar{L}$. The dynamics of the economy are then governed by (12) and (14) (noting that $K(T) = 0$ or $K(T) = \bar{K}$). The shadow price λ_L at $t = T$ jumps in order to satisfy the transversality condition. A non-formal sketch of equilibrium dynamics at the border of the state space is as follows. Assume that the economy touches the K-axis at $t = T$, i.e. $L(T) = 0$. The rate of return then is $r(T) = 0$ and, hence, capitalists leave the domestic market sector (in finite time). This movement is sluggish because of convex adjustment costs. An equivalent reasoning applies for $K(T) = 0$ and $w(T) = 0$. Now assume that the economy touches the upper border of the state region, i.e. $L(T) = \bar{L}$ for some $t = T$. Assume further that $(1-\tau)r(\bar{L}, \bar{K}) > \bar{r}$. Capitalists then increase the amount of capital allocated to the domestic market sector until $(1-\tau)r = \bar{r}$. An equivalent reasoning holds true if the economy hits the right border of the state region $K = \bar{K}$.

6.3 An alternative interpretation of the "typical capital owner"

The problem of the typical East German capital owner may be expressed as follows

$$\max_{\{v_K^E\}} \int_0^\infty \left[(1-\tau) r K_E + \bar{r} \underbrace{(\bar{K}_E - K_E)}_{=0 \text{ at } t=0} - \frac{1}{2\gamma_K} \left(v_K^E\right)^2 \right] e^{-\rho t} dt$$
$$s.t.\ \dot{K}_E = v_K^E$$
$$K_E(0) = \bar{K}_E;\ 0 \leq K_E \leq \bar{K}_E,$$

This modeling assumes that the East German capital owner has all his capital $\bar{K}_E$ allocated

[21] Notice that "reallocation costs" are essentially congestion costs, i.e. marginal moving costs are zero at the origin.

to the East German region initially ($K_E(0) = \bar{K}_E$). Thus he can either keep all his capital inside the region's domestic market sector or leave the region's market sector. Notice that this modeling implies that an increase in the region's stock of capital requires foreign capital inflows.

The problem of the typical West German capital owner may be expressed as follows

$$\begin{aligned}
&\max_{\{v_K^W\}} \int_0^\infty \left[\bar{r} K_W + (1-\tau) r \underbrace{(\bar{K}_W - K_W)}_{=0 \text{ at } t=0} - \frac{1}{2\gamma_K} \left(v_K^W\right)^2 \right] e^{-\rho t} dt \\
&s.t. \ \dot{K}_W = v_K^W \\
&K_W(0) = \bar{K}_W; \ 0 \leq K_W \leq \bar{K}_W,
\end{aligned} \tag{18}$$

The typical West German capital owner has a total wealth $\bar{K}_W$, which is completely invested in West Germany (or the rest of the world) initially. This implies that initial investments in East Germany are zero.

Capital in the East German Region (K_{EE}) is given by

$$\begin{aligned}
K_{EE} &= K_E + (\bar{K}_W - K_W) \\
\dot{K}_{EE} &= \dot{K}_E - \dot{K}_W
\end{aligned} \tag{19}$$

It can be readily shown that the following relations must hold in equilibrium (from $\frac{\partial H_K}{\partial v_K^E} = 0$, $K_E \leq \bar{K}_E$ and $\frac{\partial H_K}{\partial v_K^W} = 0$, $K_W \leq \bar{K}_W$)

$$\dot{K}_E = \begin{cases} \gamma_K \lambda_K^E & \text{for} \quad \lambda_K^E < 0 \\ 0 & \text{for} \quad \lambda_K^E = 0 \\ 0 & \text{for} \quad \lambda_K^E > 0 \end{cases} \tag{20}$$

$$\dot{K}_W = \begin{cases} \gamma_K \lambda_K^W & \text{for} \quad \lambda_K^W < 0 \\ 0 & \text{for} \quad \lambda_K^W = 0 \\ 0 & \text{for} \quad \lambda_K^W > 0 \end{cases} \tag{21}$$

Notice that λ_K^E is the shadow value placed on capital in East Germany and λ_K^W is the shadow value placed on capital invested in the rest of the world. Since there are only these two investment possibilities (investments in East Germany and investments outside East Germany) the following relation must hold $\lambda_K^E = -\lambda_K^W$.

Case distinction

1. Assume $\lambda_K^E < 0$ (implying $\lambda_K^W > 0$). Noting (19), (20), (21), and $\lambda_K^E = -\lambda_K^W$ this gives $\dot{K}_{EE} = \dot{K}_E - \underbrace{\dot{K}_W}_{=0} = \gamma_K \lambda_K^E$.

2. Assume $\lambda_K^E = 0$ (implying $\lambda_K^W = 0$). Noting (19), (20), (21), and $\lambda_K^E = -\lambda_K^W$ this gives $\dot{K}_{EE} = \underbrace{\dot{K}_E}_{=0} - \underbrace{\dot{K}_W}_{=0} = 0$.

3. Assume $\lambda_K^E > 0$ (implying $\lambda_K^W < 0$). Noting (19), (20), (21), and $\lambda_K^E = -\lambda_K^W$ this gives $\dot{K}_{EE} = \underbrace{\dot{K}_E}_{=0} - \dot{K}_W = -\gamma_K \lambda_K^W = \gamma_K \lambda_K^E$.

In summary, the equation of motion for K_{EE} reads

$$\dot{K}_{EE} = \gamma_K \lambda_K^E \text{ for } \lambda_K^E \lesseqgtr 0$$

This equation of motion is the same as the equation of motion for K in the main part of the paper (equ. (13)).[22]

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[22] A final clarification: Do foreign investors invest at first in the region where the "rate of return" is highest? Does this mean that this region develops first and then other regions develop at next? No, there is enough capital outside the economy for a simultaneous development of all regions heading towards superior equilibrium.

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