Unemployment dynamics at the zero lower bound nominal interest rate

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Abstract

This paper studies the labor market dynamic when the nominal interest rate hits the zero lower bound (ZLB). We build a DSGE New Keynesian model with frictional unemployment. Our framework takes into account explicitly the non-linearities induced by the ZLB which can not been accurately studied when the model is solved using linear-approximation methods. We show that when the economy enters in a liquidity trap the unemployment rate can increase dramatically which amplify the deflation, consistent with recent observations. We show that search and matching frictions (SaM) have strong effects on the way variables respond to shocks if the economy is at the ZLB. At the ZLB, the government spending multipliers is about two times higher than in normal times. If the labor market is flexible it is about three but still less than unity. The taxes multipliers are found to be very low. In a economy characterized by search and matching frictions, tax cuts on labor income reduce, not increase, output.

Keywords: Zero lower bound, New Keynesian models, search and matching frictions, monetary and fiscal policies, Government spending and tax multipliers.

JEL Classification: E24, E31, E32, E43, E52, E62

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1 Introduction

The dramatic persistence of high unemployment rates that many countries have experienced following the Great Recession has raised an intense debate on what drives the observed weak labor market performance. Indeed, the financial crisis has driven unemployment rates up to 10% in the US and in many European countries. The average unemployment duration has reached historic highs and the job prospects have unconventionally been low. Usually, monetary policy plays an important role for short-run stabilization and economic stimulus but in deep recessions the nominal interest rate can reach the zero lower bound (ZLB for short). Actually, several central banks slashed interest rates close to zero like the FED, the European Central Bank, the Bank of Japan, the Bank of Canada, the Bank of England (see figure 1). This situation, referred as a *liquidity trap*, makes the monetary policy ineffective in the sense that it cannot provide the appropriate stimulus to the economy. When the nominal rate binds to zero, a low or negative inflation rate implies a real rate that is to high compared to the equilibrium value that clears the market. As shown by Hall (2011), the excess supply shows up as diminished output, lower employment, and higher unemployment. But how much of the rise in unemployment is due to the ZLB? How does the labor market behaves when the nominal interest rate hits the ZLB?

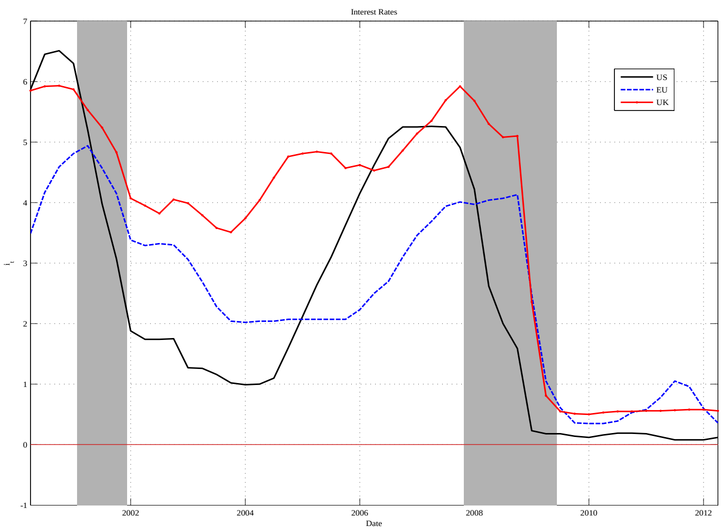

Figure 1: ***Nominal interest rate***. *Interest rate at which depository institutions lend balances to each other overnight*

The Japanese experience with liquidity trap as well as the recent financial crisis have spark off and intense debate on the consequences for the monetary policy of binding interest rate like Krugman (1998), Benhabib and *al.* (2002), Eggerston and Woodford (2003), Werning (2011). They shed light the consequences of policy commitments to achieve central bank's target. The design of the optimal monetary policy under commitment with a zero bound has been investigated by Adam and Billi (2006). They show that the policy should reduce nominal interest rates more aggressively than suggested by a model without lower bound. More recently, the literature has focused on budget and fiscal policies as a way to overcome the limited role of monetary policy. It echoes to the expansionary interventions that many economies have triggered in 2009 to fight the recession (ARRA in the US). It is often argued that tax cuts and increases in government spending have larger positive effect on output when the ZLB on nominal interest rates binds. The underlying mechanism lies in the reaction of nominal interest rate following a fiscal stimulus. In normal times, the nominal interest rate rises against the expansionary policy which reduces its impact. With interest rate unresponsive, there is no force that limit the propagation of government spending shock. Therefore, the multiplier is higher.

In DSGE model, Hall (2009), Eggerston (2010) and Christiano and *al.* (2011), found that the government spending multiplier may exceed unity when the zero interest rate binds. Fernandes-Villaverde and *al.* (2012) also found results in line with previous author with a government purchases multiplier of 0.5 in normal times and 1.5 at the ZLB. On the contrary, Drautzburg and Ulhig (2011) found a much more modest value. The government spending multiplier can even becomes negative if taxes balance the budget. Dong (2012) found that the government spending shock does not matter for determining the bindings of the ZLB. He concludes that fiscal stimulus policies have limitations. Coenen and *al.* (2012) evaluate the size of the different multipliers in seven popular large scale DSGE models. They show that the size of many multipliers is large, particularly for spending and targeted transfers. The empirical literature also delivers a broad range of government and fiscal multipliers. Blanchard and Perotti (2002) investigates the response of output to spending and taxes increases using a structural VAR. They point out that the multipliers are small, often close to one. In addition, they show that while private consumption increases following spending shocks, private investment is crowded out to a considerable extent. This is also the result of Galí, López-Salido, and Vallés (2007). Hall (2009) highlights that the multiplier linking government purchases to GDP may be in the range of 0.7 to 1.0 using vector autoregressions. Based on similar econometric models, Ramey (2011) found that the government spending multipliers range from 0.6 to 1.1 if anticipation effects are taking into account.

However, none of the aforementioned studies incorporate features to explicitly analyze flows in the labor market. They are therefore silent on the consequences of the ZLB on unemployment over the business cycle. They also are mute on the impact of labor market frictions on the size of the fiscal multipliers at the ZLB. In this paper we investigate how the labor market behaves at the ZLB. Our framework takes into account explicitly the non-linearities induced by the ZLB which can not been accurately studied when the model is solved using linear-approximation methods. We show that matching frictions matters for inflation dynamics at the ZLB and have nontrivial consequences for the policy analysis. We show that when the economy enters in a liquidity trap the unemployment rate can increase dramatically which amplify the deflation, consistent with recent observations. In the spirit of Blanchard and Gali (2010) we analyze the role of unemployment rigidities. We show they have strong effects on the way variables respond to shock if the economy is at the ZLB. They influences quantitatively **and** qualitatively the government spending and taxes multipliers. At the ZLB, the government spending multipliers is about two times higher than in normal times. If the labor market is flexible it is about three but still less than unity. The taxes multipliers are found to be very low. In a economy characterized by search and matching frictions, tax cuts on labor income reduce, not increase, output.

The rest of the paper is organized as follows. Section 2 presents the New keynesian DSGE model. The calibration and a quantitative evaluation of the model are presented in section 3. Section 4 is devoted to the understanding of the labor market dynamic at the ZLB and how frictions impact the government purchases and fiscal multipliers. Section 5 concludes.

2 The model

We build a New Keynesian DSGE model with search and matching frictions based on Moyen and Sahuc (2005), Walsh (2005), Krause and Lubik (2007, 2008) and Gertler, Sala and Trigari (2008). The model includes a non-Walrasian labor market with matching frictions and hiring costs in the spirit of Mortensen and Pissarides (1994, 1999). We focus on the flow of workers between employment and unemployment. We allow the model to account for nominal wage rigidities. The time is discrete and our economy is populated by homogeneous workers and firms. Producing firms are large and employ many workers as their only input into the production process. Labor may be adjusted at the extensive margin (employment), individual hours are fixed. Wages are the outcome of a bilateral Nash bargaining process between the large firm and each worker. We assume that retailers set prices and face quadratic adjustment cost as in Rotemberg (2008). Economic fluctuations come from a risk premium shock.

2.1 The labor market

The search process and recruiting activity are costly and time-consuming for both firms and workers. A job may either be filled and productive, or unfilled and unproductive. To fill their vacant jobs, firms publish adverts and screen workers, incurring hiring expenditures. Workers are identical, and they may either be employed or unemployed. The number of matches, m_t, is given by the following Cobb-Douglas matching function:

$$m_t = \chi s_t^{\nu} v_t^{1-\nu} \qquad \text{with } \nu \in [0,1],\ \chi_{>}0 \tag{1}$$

where χ is the matching efficiency shock, $v_t \geq 0$ denotes the mass of vacancies and $s_t \geq 0$ represents the mass of searching workers. Then, ν stands for the elasticity of the matching function with respect to the number of job seekers. The labor force, L, is assumed to be constant over time. Assuming $L = 1$ allows us to treat aggregate labor market variables in number and rate without distinction. The matching function (1) satisfies the usual assumptions: it is increasing, concave and homogenous of degree one. A vacancy is filled with probability $q_t = m_t/v_t$ and the job finding probability $f_t = m_t/s_t$.

2.2 The sequence of events

As Hall (2005) demonstrates, fluctuations in labor market flows are mainly driven by job creation. So, we abstract from job destruction decisions by assuming that in each period a fixed proportion of existing jobs are exogenously destroyed at rate ρ^x. n_t denotes employment in period t. It has two components: new and old workers. New employment relationships are formed through the matching process in period t. The number of job seekers is given by:

$$s_t = 1 - (1 - \rho^x) n_{t-1} \tag{2}$$

This definition has two major consequences. First, it allows workers who lose their job in period t to have a probability of being employed in the same period. Second, it allows us to make a distinction between job seekers and unemployed workers $u_t = (1 - n_t)$. The latter receives unemployment benefits. The employment law of motion is given by:

$$n_t = (1 - \rho^x) n_{t-1} + m_t \tag{3}$$

2.3 The representative household

There is a continuum of identical households of measure one indexed by $i \in [0,1]$. Each household may be viewed as a large family. There is a perfect risk sharing, family members pool their incomes (labor incomes and unemployment benefits) that are equally redistributed. We suppose that, households have preference over different consumption varieties. Good varieties are indexed by $j \in [0,1]$. Each households maximizes the aggregate consumption a Dixit-Stiglitz aggregator of differentiated goods c_{jt}:

$$\max_{c_{jt}} \left[\int_0^1 c_{jt}^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{4}$$

subject to the total consumption expenditure constraint:

$$p_t c_t = \int_0^1 p_{jt} c_{jt}\, dj \tag{5}$$

We obtain the following first-order condition after maximizing equation (4) over the resource constraint (5).

$$\left[\int_0^1 \left(c_{jt}\right)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{1}{\epsilon-1}} \left(c_{jt}\right)^{\frac{1}{\epsilon}} = \vartheta_t p_{jt} \tag{6}$$

Where ϑ_t corresponds to the Lagrangian multiplier. We can rewrite this condition as

$$c_{jt} = \left(\frac{p_{jt}}{p_t}\right)^{-\epsilon} c_t \tag{7}$$

which describes the optimal level of c_{jt} and where c_t is aggregate consumption. The nominal price index is defined by $p_t = \left[\int_0^1 p_{j,t}^{1-\epsilon}\right]$. The second problem that households solve is the maximization of aggregate consumption c_t:

$$\max_{\Omega_t^H} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{{c_t}^{1-\sigma}}{1-\sigma} - \ell \frac{{n_t}^{1+\phi}}{1+\phi}\right] \tag{8}$$

n_t is the level of employment supplied by households. The parameters $\sigma > 0$ and $1+\phi > 0$ denote the coefficient of risk aversion and the Frisch elasticity respectively. The representative household chooses the set of processes $\Omega_t^H = \{c_t, d_t\}_{t=0}^{\infty}$ taking as given the set of processes $\{p_t, w_t, i_t, f_t\}_{t=0}^{\infty}$, initial wealth (d_0) so as to maximize households utility subject to the budget constraint:

$$(1+\tau_t^c)p_t c_t + \frac{d_t}{\eta_t} = d_{t-1} i_{t-1} \quad + \quad w_t(1-\tau_t^s)n_t + (1-n_t)\, b_t + \Pi_t + T_t \tag{9}$$

where τ_t^c is the consumption tax. d_t is the household's holding of one period domestic bonds at date t. The corresponding interest rates is i_t. w_t is the nominal wage level. Π_t represent profits from holding shares in domestic goods firms. We distinguish between employers payroll tax and employees labor tax. τ_t^s denotes the wage tax paid by the labor force and τ_t^w denotes the payroll tax. T_t is a lump-sum tax. Assuming that ρ^b denotes the replacement rate to the unemployment insurance, unemployment benefits b_t are indexed on nominal wage w_t by the following relation:

$$b_t \;=\; \rho^b w_t \tag{10}$$

η_t denote the first-order autoregressive shock in the risk premium. As shown in Smets and Wouters (2007), it reflects inefficiencies in the financial sector. They are some premium on the deposit rate versus the risk free rate set by the central bank. In other words, there is a risk premium that households require to hold the one period bond. The AR(1) process characterizing this shock is

$$\log \eta_t = \rho^\eta \log \eta_{t-1} + \sigma_t^\eta \tag{11}$$

and the law of motion of employment:

$$n_t \;=\; (1-\rho^x)n_{t-1} + f_t s_t \tag{12}$$

The optimality conditions of the household's problem are:

$$\varphi_t \;=\; \lambda_t \left(w_t^R (1-\tau_t^s) - b_t^R \right) - \ell n_t^\phi + \beta E_t (1-\rho^x)(1-f_{t+1})\varphi_{t+1} \tag{13}$$

$$\lambda_t \;=\; \frac{c_t^{-\sigma}}{(1+\tau_t^c)} \tag{14}$$

$$\lambda_t \;=\; \beta \eta_t (1+i_t) E_t \lambda_{t+1} \frac{p_t}{p_{t+1}} \tag{15}$$

Equation (13) is the expected value of employment minus the expected value of unemployment. λ_t is the Lagrange multiplier on the budget constraint and equation (15) defines the standard Euler equation. $w_t^R = w_t / p_t$ and $b_t^R = b_t / p_t$ denotes the real wage rate and the real unemployment benefits level.

2.4 Firms

There is a continuum producers in a monopolistically competitive market indexed by j. They use labor as their only input and sell output to the representative household. They set the price under quadratic price adjustment cost (Rotemberg-style). The production function of a firm j using a fraction n_{jt} of total employment, such that $\int_0^1 n_{jt} dj = n_t$, is given by:

$$y_{jt} = n_{jt}^\alpha \tag{16}$$

α is the employment share of production in the consumption good. The optimization problem of the firm j is to choose a set of processes $\Omega_{jt}^F = \{v_{jt}, p_{jt}\}_{t=0}^\infty$

taking as given the set of processes $\{p_t, w_{jt}, q_t\}_{t=0}^{\infty}$. Each j producer maximizes the following intertemporal function:

$$\max_{\Omega_t^P} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \Pi_{jt} \tag{17}$$
$$\text{where } \Pi_{jt} = \left[\frac{p_{jt}}{p_t} y_{jt} - \frac{w_{jt}}{p_t} n_{jt}(1+\tau_t^w) - \kappa v_{jt} + y_t \Gamma^{\pi}(p_{jt}) \right]$$

subject to the production function (16) and the following evolution of employment,

$$n_{jt} = (1-\rho^x) n_{jt-1} + q_t v_{jt} \tag{18}$$

$\frac{p_{jt}}{p_t}$ is the relative price which coincides with the marginal cost of the producing firm. As in Rotemberg, adjusting prices incurs a cost :

$$\Gamma^{\pi}(p_{jt}) = \frac{\psi_p}{2} \left(\frac{p_{jt}}{\pi p_{jt-1}} - 1 \right)^2 \tag{19}$$

This cost is assumed to be proportional to the output level y_t. Inflation is defined as the gross inflation rate $\pi_t = p_t / p_{t-1}$. ψ_p is the price adjustment cost parameter and π is the steady state inflation. Hiring is costly and incurs a cost per vacancy posted κ (with $\int_0^1 v_{jt} dj = v_t$). It is paid by the firm as long as the job remains unfilled. Since all firms chose the same price and the same number of vacancies in equilibrium we can drop the index j by symmetry. The optimality conditions of the above problem are:

$$q_t \mu_t = \kappa \tag{20}$$
$$\mu_t = mc_t \alpha \frac{y_t}{n_t} - w_t^R (1+\tau_t^w) + \beta(1-\rho^x) E_t \frac{\lambda_{t+1}}{\lambda_t} \mu_{t+1} \tag{21}$$
$$0 = (1-\epsilon) + \epsilon mc_t - \psi_p \frac{\pi_t}{\pi} \left(\frac{\pi_t}{\pi} - 1 \right) + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \psi_p \frac{\pi_{t+1}}{\pi} \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \frac{y_{t+1}}{y_t} \tag{22}$$

where μ_t is the Lagrangian multiplier associated with the employment that gives the expected marginal value of a job for the firm[2]. mc_t is the Lagrange multiplier associated to the individual consumption demand. Combining the two first-order conditions (20) and (21) gives the job creation condition:

$$\frac{\kappa}{q_t} = mc_t \alpha \frac{y_t}{n_t} - w_t^R (1+\tau_t^w) + \beta(1-\rho^x) E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\kappa}{q_{t+1}} \tag{23}$$

[2]It is obtained by deriving the program (17) with respect to n_t.

This condition shows that the expected gain from hiring a new worker is equal to the average cost of search (which is the marginal cost of a vacancy times the average duration of a vacancy $1/q_t$).

2.4.1 Wage setting

We now turn to the wages setting mechanism. At equilibrium, filled jobs generate a return (the firm marginal value of the job μ_t plus the worker marginal value of the job φ_t) greater than the values of a vacant job and of an unemployed worker. The net gain issued from a filled job is the total surplus of the match:

$$S_t = \frac{\varphi_t}{\lambda_t} + \mu_t \tag{24}$$

Nominal wages are determined through an individual Nash bargaining process between each worker and his employer who share the total surplus of the match. The outcome of the bargaining process is given by the solution of the following maximization problem:

$$\max_{w_t} \left(\frac{\varphi_t}{\lambda_t}\right)^{1-\xi} \mu_t^{\xi} \tag{25}$$

where $\xi \in [0, 1]$ and $1 - \xi$ denote the firms and workers bargaining power respectively. We assume that bargaining power is not constant over time and follows a stochastic process. The optimality conditions of the above problems are given by:

$$\xi \varphi_t \frac{\partial \mu_t}{\partial w_t} = -(1-\xi)\mu_t \frac{\partial \varphi_t}{\partial w_t} \tag{26}$$

which gives

$$\begin{aligned} w_t^R &= \frac{1-\xi}{1+\tau_t^w} mc_t \alpha \frac{y_t}{n_t} + \frac{\xi}{1-\tau_t^s}\left(b_t^R + \frac{\ell n_t^{\phi}}{\lambda_t}\right) \\ &+ (1-\xi)\beta(1-\rho^x)E_t \frac{\lambda_{t+1}}{\lambda_t}\frac{\kappa}{q_{t+1}}\left(\frac{1}{1+\tau_t^w} - \frac{(1-f_{t+1})(1-\tau_{t+1}^s)}{(1-\tau_t^s)(1+\tau_{t+1}^w)}\right) \end{aligned} \tag{27}$$

The real wage is a weighted sum of the worker's outside option and its contribution to the product. The former is represented by the second term to the right hand side of (27). The labor tax τ_t^s increases the wage because employees can use it in the bargaining to make employers bear the burden of the tax. Similarly, employers surplus is reduced by the payroll tax τ_t^w. Then, they use it as a threat to lower wages. This effect is represented by the worker's contribution to the product (the first term on the r.h.s). Finally, any variations of the labor taxes are anticipated and taking into account (last term of the r.h.s of (27)).

2.5 The monetary and fiscal authorities

We assume that the central bank adjusts the nominal interest rate in response to deviations of inflation and output from their steady-state values. The monetary authorities chooses the short-run interest rate i_t according to a Taylor-type rule. Nominal interest rate cannot fall below zero:

$$i_t = \begin{cases} i_{t-1}^{\rho_i} \left[\frac{\pi}{\beta} \left(\frac{\pi_t}{\pi} \right)^{\rho_\pi} \left(\frac{y_t}{\gamma y} \right)^{\rho_y} \right]^{1-\rho_i} & \text{if} \quad i_t > 0 \\ 0 & \text{otherwise} \end{cases} \tag{28}$$

The fiscal authority finance government expenditures g_t and unemployment benefits b_t through taxes assessed on consumption and labor (workers' side and employers' side). Formally the fiscal budget rule satisfies:

$$d_t + \tau_t^c c_t p_t + n_t w_t (\tau_t^w + \tau_t^s) = i_{t-1} d_{t-1} + b_t (1 - n_t) + g_t + T_t \tag{29}$$

2.6 Market clearing

The aggregation of individual profits Π_t is given by:

$$\Pi_t = p_t y_t - n_t w_t (1 + \tau_t^w) - p_t y_t \Gamma_t^\pi \tag{30}$$

Equations (29) together with the budget constraint (9) and the profit (30) give the aggregate resource constraint :

$$y_t = c_t + g_t + \kappa v_t \tag{31}$$

Definition 1 (The competitive equilibrium) *For a given tax processes* $\{\tau_t^c, \tau_t^w, \tau_t^s, g_t, T_t\}$ *and the exogenous stochastic process* η_t*, the competitive equilibrium is a sequence of prices and quantities* n_t, π_t, π_t^w, v_t, w_t^R, μ_t, φ_t, mc_t, i_t, c_t, λ_t, b_t^R *satisfying equations* (3), (10), (13), (14), (15), (20), (21), (22), (27), (28) *and* (31) *and using the definition of* f_t, q_t, θ_t, s_t *and* y_t.

3 Model solution and calibration

Quarterly frequencies are assumed in our calibration. For the sake of our purpose the benchmark calibration is the most commonly one used in recent papers on ZLB in the US: Christiano and *al.* (2010), Fernandez-Villaverdé and *al.* (2012), Dong (2012), Rendhal (2012), Aruoba and Schorfheide (2012). We follow Blanchard and Gali (2010) and Abbriti and Weber (2012) for parameters in the labor market. They assume a calibration that typically mimic a flexible labor market and investigate an alternative calibration to capture the more sclerotic European labor market. The model is solved using a version of the Parameterized expectation algorithm (PEA) with regime switching[3].

Preferences, production and shocks: We set the discount factor to 0.994 and the Frisch elasticity is set to 1. The risk aversion coefficient σ is set to 1 and following the standard approach, the elasticity of substitution between goods is $\varepsilon = 6$, which gives a gross markup of about 1.2. The elasticity of output with respect to employment α is equal to 1. The aggregate risk premium shock follows a first-order autoregressive process where the autocorrelation coefficient ρ_η is equal to 0.80. $\varepsilon_t^\eta \sim iid\mathcal{N}(0, \sigma_\eta^2)$. As in previous studies the standard deviation is 0.0025. As in Fernández-Villaverde and *al.* (2012), the demand shock has a half-life of roughly 3 quarters and an unconditional standard deviation of 0.42 percent.

Variables	Symbol	Value	Source
Discount factor	β	0.994	Standard
Risk aversion coefficient	σ	1	Standard
Elasticity of subst. between goods	ϵ	6	Standard
Frisch elasticity	ϕ	1	Standard
Annual steady state inflation	π	0.02	Target
Production function elasticity	α	1	Standard
Autocorrelation coefficient	ρ_η	0.8	CER
Std. dev. of aggregate shock	σ_η	0.0025	CER
Vacancy posting costs	κ	0.05	Blanchard Gali
Replacement rate	ρ^R	0.40	DOLETA
Matching elasticity	ν	0.5	PP
Matching efficiency	χ	0.43	recalculated
Worker bargaining power	ξ	0.5	Hosios
Price adjustment	ϕ_p	220	Aruoba and Schorfheide
Response to inflation	ρ_π	1.5	Aruoba and Schorfheide
Response to output	ρ_y	0.8	Aruoba and Schorfheide
Interest rate smoothing	ρ_i	0	CER
Value added tax	τ^c	0.05	OCDE
Payroll tax	τ^w	0.14	OCDE
Social tax	τ^s	0.14	OCDE
Government spending	g	0.2y	OCDE

Table 1: **PARAMETERS**

Labor market: stocks and flows Consistent with the US labor market, we impose the equilibrium unemployment rate u to be equal to 5.5%. The labor

[3]see appendix A for details.

market is assumed to be quite flexible: the labor turnover is high. We set the probability of being unemployed $\rho^x = 10\%$, which involves a job finding rate of 0.63. As in Blanchard and Gali (2010) we investigate an alternative calibration to capture the more sclerotic European labor market. Thus we assume a low job finding rate of 0.26 (which is roughly consistent with a monthly rate of 0.1) and a steady state unemployment rate $u = 0.1$, values in line with evidence for the European Union over the past two decades. The implied separation rate is $\rho^x = 0.04$. The steady state number of matches must be equal to the number of separations: $m = \rho^x n$ with $n = 1 - u$. We also deduce the number of job seekers from the definition $s = 1 - (1 - \rho^x)n$. χ is calculated in such a way that $m = \chi s^{\psi} v^{1-\psi}$. Following the standard approach we impose the Hosios condition to be satisfied $\xi = 1 - \nu = 0.5$. Following the previous authors, the costs of posting vacancies is about 1% of GDP. The remaining parameters ℓ are set to balance the steady state wage equation.

Variables	Symbol	Labor market Flexible	Labor market Rigid
Unemployment rate	u	0.055	0.1
Job finding rate	f	0.63	0.26
Job filling rate	q	0.71	0.71
Separation rate	ρ^x	0.1	0.04

Table 2: **STEADY STATE: FLEXIBLE VS RIGID LABOR MARKET.** Blanchard and Gali calibration

Monetary and fiscal policy Steady state inflation is assumed to be equal to 2%. For a discount factor of 0.994, the Euler equation involves a steady state nominal interest rate equal to 4.5% annual and a real interest rate of about 2.5% annual. Price adjust infrequently. We pick the value of Aruoba and Schorfheide (2012) estimated on a DSGE using non-linear methods. Then, $\psi_p = 220$ and the reaction coefficient to inflation is $\rho_\pi = 1.5$ and the reaction to output deviation from long run trend is $\rho_y = 0.8$. To make the analysis tractable enough and save on the dimensionality of the problem, Fernandez-Villaverdé and *al.* (2012) and Christiano et *al.* (2011) set the persistence of the nominal interest rate equal to zero. We follow them and assumed $\rho^i = 0$. We will come back later on this assumption.

Finally, for the fiscal shocks we assume that g_t, τ_t^c, τ_t^w and τ_t^s follow a first auto-regressive process with autocorrelation coefficient 0.8 and deviation 0.0025. The steady state government spending g is equal to 0.2 of output. Following Drautzburg and Uhlig (2011), Correia and *al.* (2013) and Mendoza and *al.* methodology, the consumption tax is on average 0.05 in the US. They also found that the labor tax rate is about 0.28. However, the labor tax includes employers' and employees' social security contributions as well as taxes on payroll and workforce. They are calculated to cover all of a tax-payer's income, regardless of its origin. There is no distinction between what an employer pay and what an employee pay. The employers' and employees social security contribution rate are both on average (OECD taxing wage report) 7.65%. We also assume that tax on workforce are shared similarly between workers and firms. Then, $\tau^w = 0.14$ and $\tau^s = 0.14$. We impose b to be consistent with the average net replacement rate of 40% ($b/w = \rho^R = 0.4$) according to DOLETA[4] over the period 1988-2011. The calibration is summarized in Table 1 and Table 2.

4 Quantitative evaluation of the model

4.1 How does the labor market behaves at the ZLB?

To understand how the ZLB influences the way variables respond to shock, we decompose our analysis in two steps. First, we compute the policy rules of the benchmark model and compare them against the ones obtained with the standard linear-approximation method. This exercise will allows us to determine the inaccuracy and the misleading properties of linear-approximation methods. Second, we compute the response of variables following a demand shock sufficiently large to drive the economy to the ZLB. We perform a counterfactual analysis in which we compare the case where the nominal interest rate i_t is binding against the case where it might fall under zero.

The results of the first experiment are shown in Figure 2. The policy rules derived from the PEA method exhibit kinked curves when the ZLB is reached. Indeed, in the liquidity trap the slope of inflation is a little bit more pronounced which makes easier the economy to enter in a deflationary spiral. Similarly, vacancies fall more with respect to the demand shock than in normal times. It results in a fall of employment that can be dramatic for high levels of risk premia. The intuition is that deflationary pressure makes the real interest rate too high compare to the value that clears the market in normal time. It entails a strong decline in output and the vacancy to unemployment ratio in a liquidity trap which lower real wages. While the linear method displays policy rules that are fairly similar to the ones from the PEA method outside the ZLB, it clearly fails to reproduce the kink. Our results are in line with those from Auruoba and Schorfheide (2012), Braün and *al.* (2012), Fernández-Villaverde and *al.* (2012) and Dong (2012) but we document that when employment and the vacancy posting process are taken into account the misleading properties of linear-approximation methods are even bigger.

To get a precise measure of unemployment and deflation effects induced by the ZLB we perform a counterfactual analysis. We compare the path of

[4]Department Of Labor, Employment and Training Administration

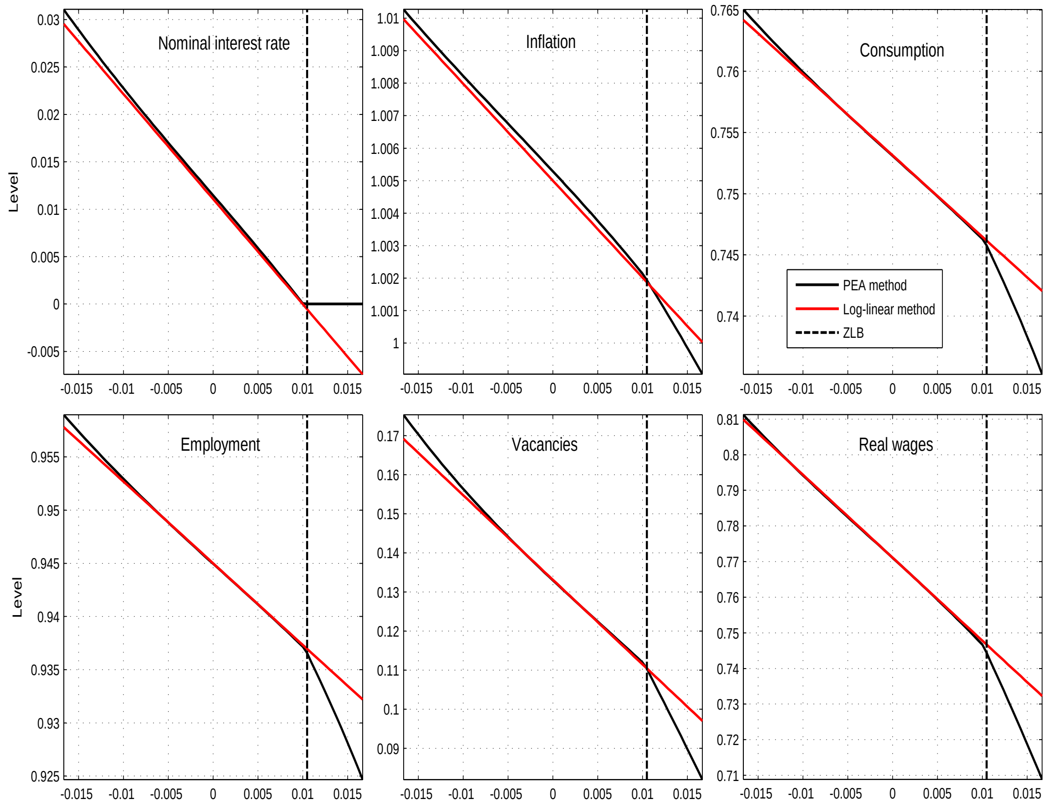

Figure 2: ***Policy rules - PEA versus linear approximation*** *(x axis: size of the risk premium shock)*

macroeconomic variables against an unconstrained economy (no ZLB), or equivalently if the nominal interest rate i_t may fall under 0. In other words, our question is: what would have been the path of aggregate variables in the absence of the ZLB? To answer this question we first compute the impulse response functions following a shock sufficiently large to make the nominal interest rate hits the ZLB during 3 quarters. We plot the response of the economy when i_t is binding by the ZLB and when i_t is allowed to fall bellow zero. Secondly, we simulate the model for a shock series over several periods. We calculate the difference between the two during the periods where the economy is in a liquidity trap. The differences are shown in Figure 3.

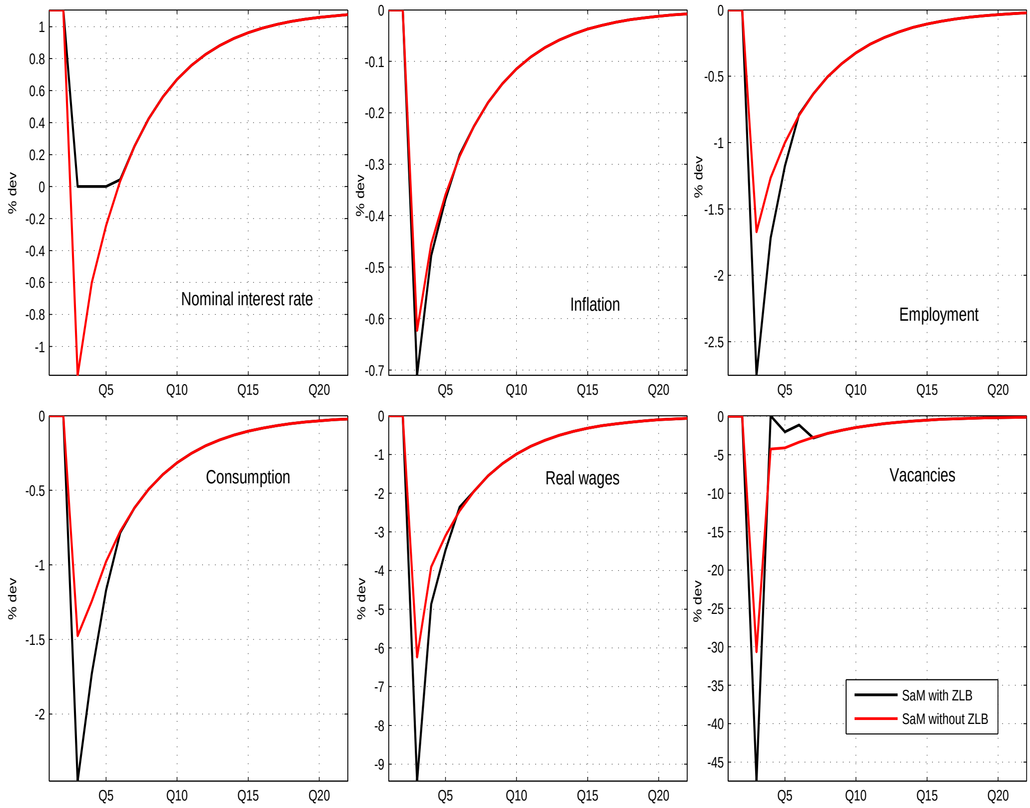

Figure 3: ***Impulse response function - counterfactual analysis.***

The impulse response functions clearly highlight the impact of the ZLB. It amplifies the propagation of the aggregate shock. The nominal interest rate is binding by the ZLB up to three quarters in our benchmark model while it declines up to minus its steady state value in the counterfactual scenario (200%). Consequently, inflation falls by more than without ZLB. The drop of consumption is about 60% higher. The demand shock also amplifies the decreases in vacancies which in turn cause a larger cut in employment. The worker's contribution to the firm's surplus composed by its productivity and the saving of vacancy costs falls. It allows employers to bargain lower real wages. This effect is dampened if the nominal interest rate falls below zero.

We now simulate the model over 10^5 periods (see Figure 4). The unemployment is on average 8% lower when we relax the ZLB restriction. In extreme situation, the ZLB may cause unemployment and vacancies to be 30% higher and 45% lower respectively. Similarly, inflation is about 0.5% lower if we remove the ZLB constraint but might falls up to 0.2%.

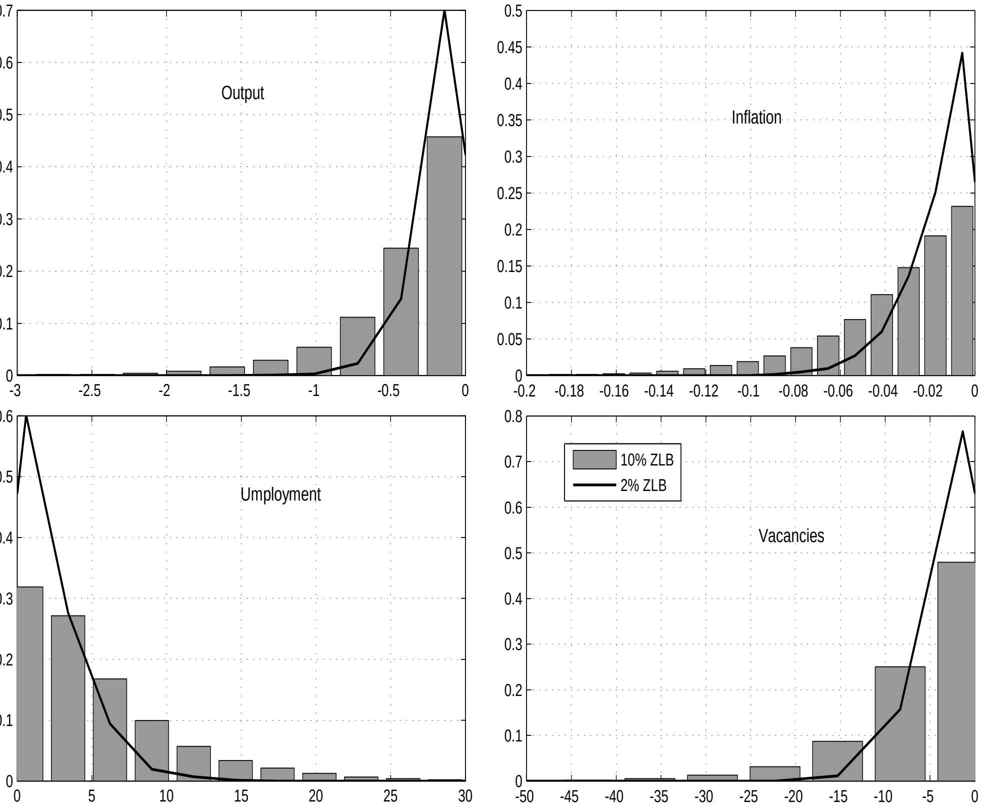

Figure 4: ***Deviations induced by the ZLB*** *Deviations induced by the ZLB in % from the case without bound on nominal interest rate), by average duration at the ZLB.*

4.2 Accounting for labor market frictions and unemployment rigidities

Some questions naturally arise: how do matching frictions affect the economic dynamic at the ZLB? Do frictions in the labor market help to explain some employment pattern? In this section we wonder how the presence of matching frictions affects the results. We then compare the benchmark model against the standard New Keynesian model[5] that we label the Walrasian labor market. We also compare the matching model with a flexible labor market to a rigid labor market as assumed in our calibration strategy (see Section 3). This exercise will provide us a better intuition the role of unemployment rigidities in line with Blanchard and Gali (2010) experiments.

[5]Without the search and matching building bloc. It is detailed in appendix B.

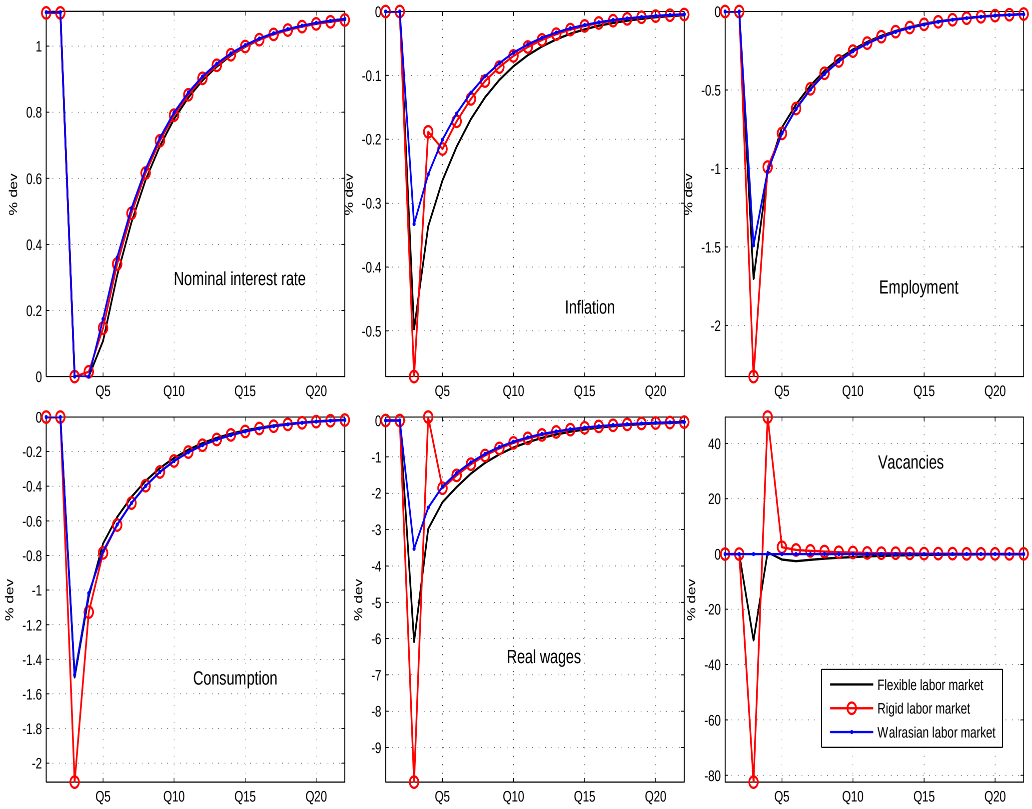

Figure 5: ***Impulse response function - comparing labor market types***

The risk premium shock sends the benchmark economy (flexible labor market) to the ZLB up to two quarters. The NK model experiences the same duration but the rigid labor market only spent 1 quarters to the ZLB. Recall that the size of the shock is the same for the three models. Outside the ZLB the path of the economy is roughly similar except for vacancies which are not defined in the basic NK model. However, employment and inflation in the two search and matching economies react more aggressively. In addition, unemployment rigidities cause a larger drop of vacancies which entail a stronger fall of employment and inflation at the ZLB compared to the the flexible (SaM) labor market. This effect is stronger when unemployment rigidity are high. As mentioned Krause and Lubik (2008) the new Keynesian Phillips curve explains inflation as being mainly driven by current and expected future marginal costs. They show that the contribution of labor market frictions in explaining inflation dynamics is small. Notwithstanding they abstract from the ZLB story and use a linear-approximation method to derive their results. According to Equations (23) and (42) the real marginal cost in the standard NK model and in the SaM model are respectively[6]:

[6]Since $y_t = n_t$ and $\alpha = 1$, we remove the term $\alpha y_t / n_t$ which is equal to 1 by definition.

$$mc_t^{NK} = w_t^R(1+\tau_t^w) \quad (32)$$

$$mc_t^{SaM} = \underbrace{w_t^R(1+\tau_t^w)}_{\substack{\text{Net}\\ \text{real wages}}} + \underbrace{\frac{\kappa}{q_t}}_{\substack{\text{Current}\\ \text{hiring costs}}} \underbrace{-\beta(1-\rho^x)E_t\frac{\lambda_{t+1}}{\lambda_t}\frac{\kappa}{q_{t+1}}}_{\substack{\text{Expected}\\ \text{hiring costs}}} \quad (33)$$

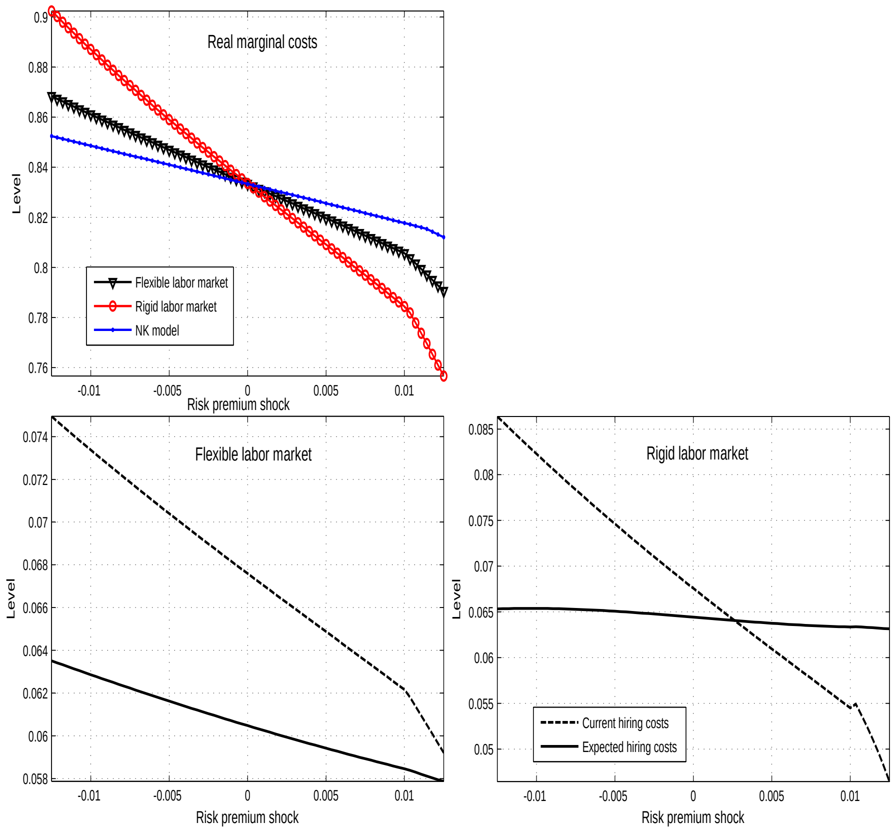

Figure 6: ***Policy rule - hiring and marginal costs*** *by labor market types.*

The general principle is as follow. Both depend on the real wage but only the latter is affected by current and expected hiring costs. In the SaM model high levels of the current labor market tightness reduce the job filling rate q_t which, in turn, increases the average duration of a vacant job. It results in a rise in current hiring costs. On the other side, as mentioned Blanchard and Gali (2010) if the firm increases its current hiring and needs ever less future hiring, a rise in expected labor market tightness implies larger savings in next period's hiring costs. Consequently, real marginal costs decrease. These two effects play in opposite direction. The overall impact depends on whether the former is stronger than the latter or not. How do they vary with respect to the demand shock?

The top graph of Figure 6 illustrates that marginal costs are highly affected by matching frictions, especially when the economy enters in a liquidity trap. It falls more sharply when the cost of generating and maintaining employment relationships are taking into account. The bottom graphs of Figure 6 show that current and expected hiring costs both decline with η_t in the flexible labor market. The former being stronger than the latter it puts downward pressure on real marginal costs. When the labor market is rigid, current hiring costs falls while expected hiring costs remain virtually unchanged. Any fall in the current job posting will not make firms support higher costs in the future. The reason is that the job filling rate increases sufficiently to make firms expect that future costs of attracting an additional workers will remain low. Consequently, expected hiring costs do not temper the decline in current hiring costs. The pressure on real marginal costs are then stronger which explain the large drop of inflation in the rigid labor market case. This effect is absent in the standard frictionless NK model.

To summarize, the presence of search frictions impacts the volatility of aggregate variables but do not really matter for their persistence, whatever the degree of unemployment rigidities. The adjustment of the Walrasian and non-Walrasian labor market ovelap. The model clearly needs to embed additional mechanisms to explain the persistence of unemployment in a liquidity trap. Real wage rigidities as in Shimer (2012) or nominal wage rigidities as in Schmitt-Grohé and Uribe (2012) seem to be the most promising avenues to reproduce the unconventional persistence of the unemployment on following the financial crisis. The authors highlight that such rigidities are needed to explain the jobless recovery.

4.3 The size of the government spending and fiscal multiplier

The size of the government spending multiplier is certainly the most controversial statistic in the recent literature on fiscal policies. Unlike Eggerston and Woodford (2003), Eggerston (2010), Christiano and *al.* (2010), Fernández-Villaverde and *al.* (2012), Drautzburg and Uhlig (2011), our solution method do not suffer from inaccuracy. We can provide a precise value of the fiscal multipliers for the different models (Standard NK, flexible labor market, rigid labor market) and a broad range of parameters value. We compute the fiscal multiplier as in Fernández-Villaverde and *al.*. It measures how many dollars of extra output is gained from one dollar additional spending coming from government purchases or tax cuts. We compute it in normal times (outside the ZLB) and at the ZLB. For the latter, we force the economy to enter in a liquidity trap up to 4 quarters (as in Fernández-Villaverde and *al.* (2012)) using the risk premium shock. We come back letter on this assumption. We also examine the cumulative fiscal multiplier. The description and computational details are provide in appendix D.

The results are shown in Figure 7 to 10. First of all, in these simulations none of the multipliers are found to be higher than one. Outside the ZLB the government spending multiplier is found to be about 0.3 on impact in the three models, which is a very low value as compared to previous studies. At the ZLB it is almost twice as much in the standard NK model and in the SaM model with a rigid labor market. It is however three times higher when the labor market is flexible and peaks at 0.92. Where do that comes from? Again, the behavior of the marginal cost provides some intuitions (see Figures 11 to 18).

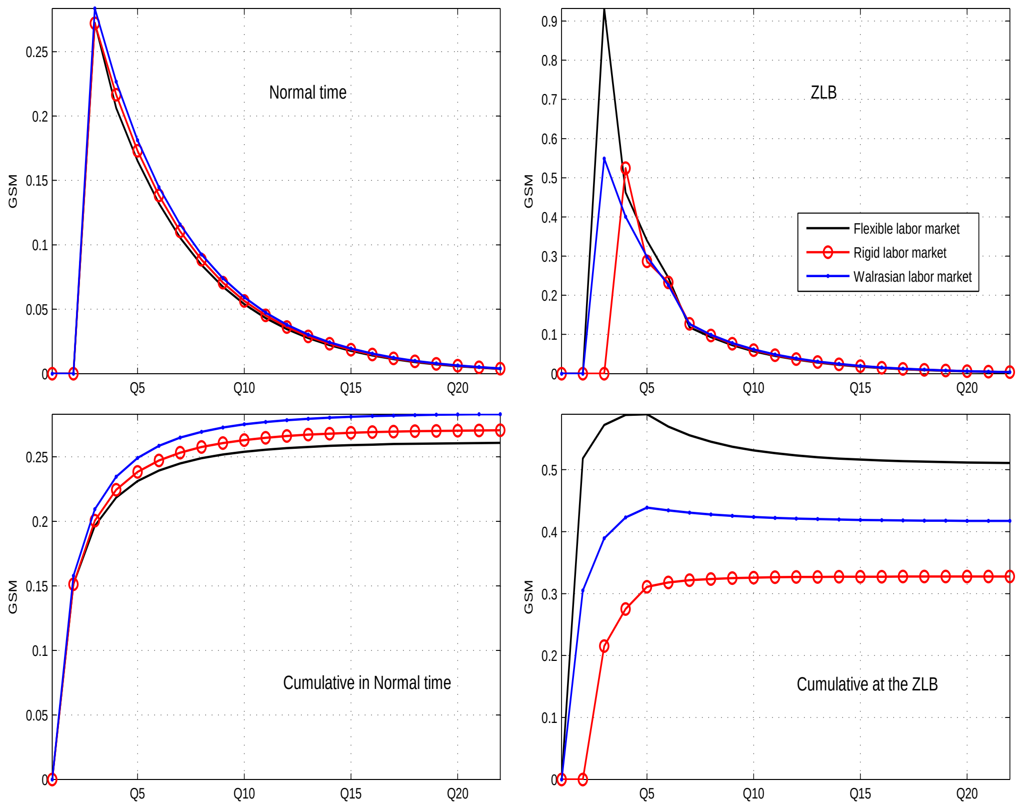

Figure 7: ***Impulse response function - Government spending multiplier.*** *Government spending increases by 1%.*

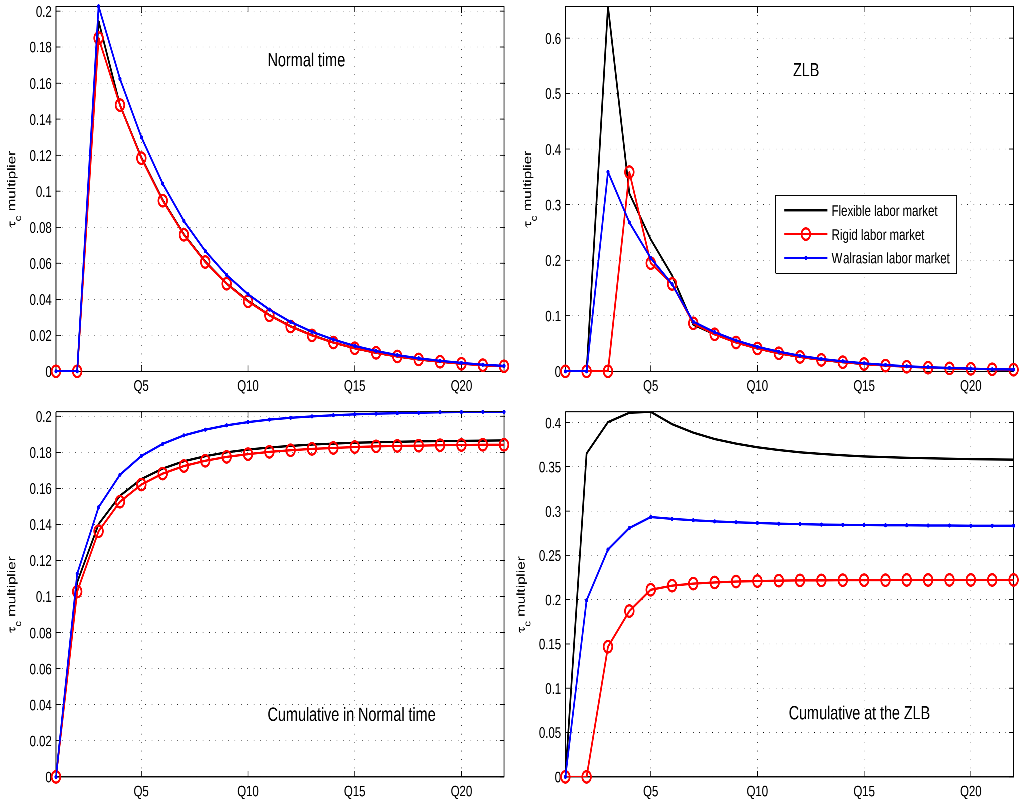

Figure 8: ***Impulse response function - Consumption tax multiplier.*** *The tax decreases by 1%.*

An increase in the government spending raises the marginal cost of firms. In the standard NK model, it is only governed by the real wage rate. In the SaM model it is affected by current and expected hiring costs. The former increases in the two cases since it increases firms' vacancy posting, which tightens the labor market. The latter falls when the labor market is rigid but increases if the labor market is flexible. In the flexible labor market model the jump of expected hiring costs partly offset the rise in current hiring costs, which mitigates the effect on the marginal cost and temper the deflationary pressure cause by the risk premium shock. In the rigid labor market, the fall in expected hiring costs amplify the initial drop of current hiring costs. It results in a strong positive effect on the marginal cost which, in turn, limit the propagation of the government spending shock. Note that the multiplier in a liquidity trap starts to move with a one-lag period. The reason is that vacancies fall almost to zero during one quarter due to the risk premium shock that sends the economy to the ZLB. Any (small) increase in government purchases is not able to offset the strong decline of vacancies. Then, the path of output[7] is not different from its

[7]Which is directly connected to the one of vacancies since $y_t = n_t$.

unconditional mean. This also explain why the government spending is lower under the rigid labor market model. Indeed, the fall in vacancies near zero involves very low current hiring costs. Since the job filling rate is huge, the future costs of hiring a new workers are expected to remain high and so on.

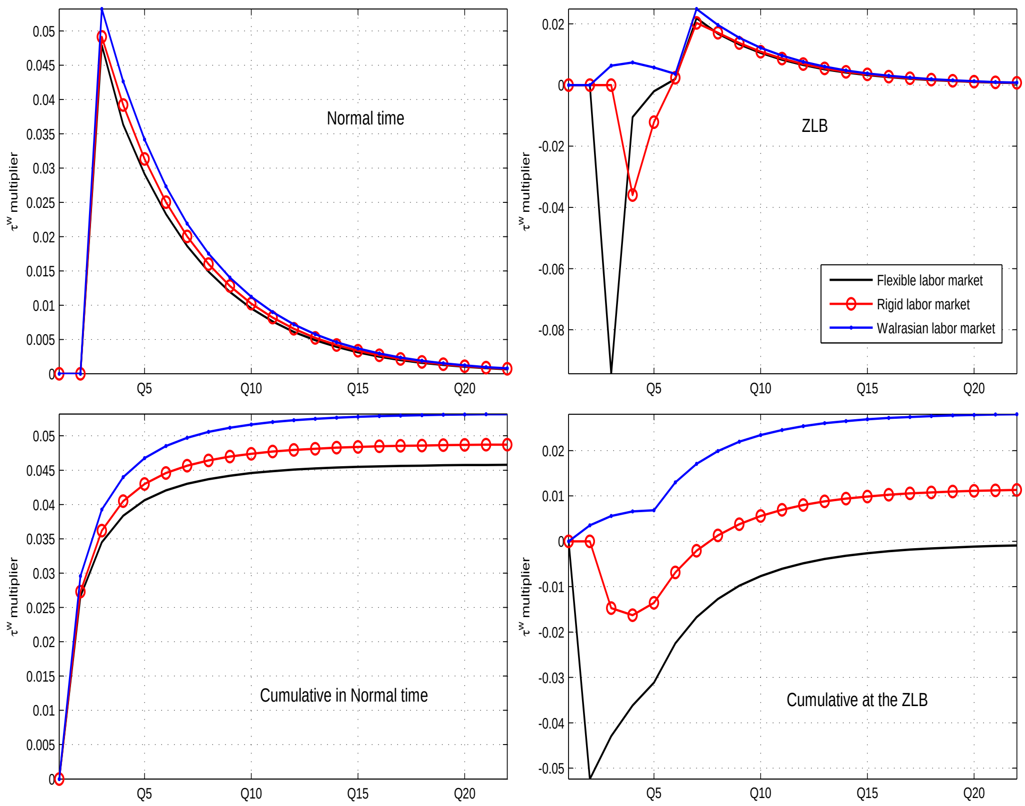

Figure 9: ***Impulse response function - Employers labor tax multiplier.*** *The tax decreases by 1%.*

The multipliers associated to the tax cut on consumption display very similar patterns except that it gives rise to lower quantitative effects. More interestingly tax cuts on labor, either from the employers' side or employees' side, involve negative multipliers in a liquidity trap. This effect is not present in the standard NK model where it remains almost unchanged during the first four periods following the shock and then increases as the economy escapes from the ZLB. The behavior of the marginal cost still explains the story. Contrary to the government spending shock, the tax cuts (τ_t^s or τ_t^w) cause deflationary pressures by reducing marginal costs of firms. Consequently, the real interest rate increases too much to make production attractive. The tax cuts result therefore in an output contraction. We conclude that no labor tax cut should be implemented in a liquidity trap, especially from the workers' side whose negative effects on output are more than two times stronger than if they come from the employers' side. This last result is in line with the one obtained by Eggerston (2010). He found a negative multiplier for a cut on labor tax. However, in our simulations it only happened if frictions in the labor market are taken into account. They strongly modifies the composition of the marginal cost. As for the Walrasian labor market, the SaM model increases tax multiplier outside the ZLB. Finally, the cumulative multiplier are shown to be low.

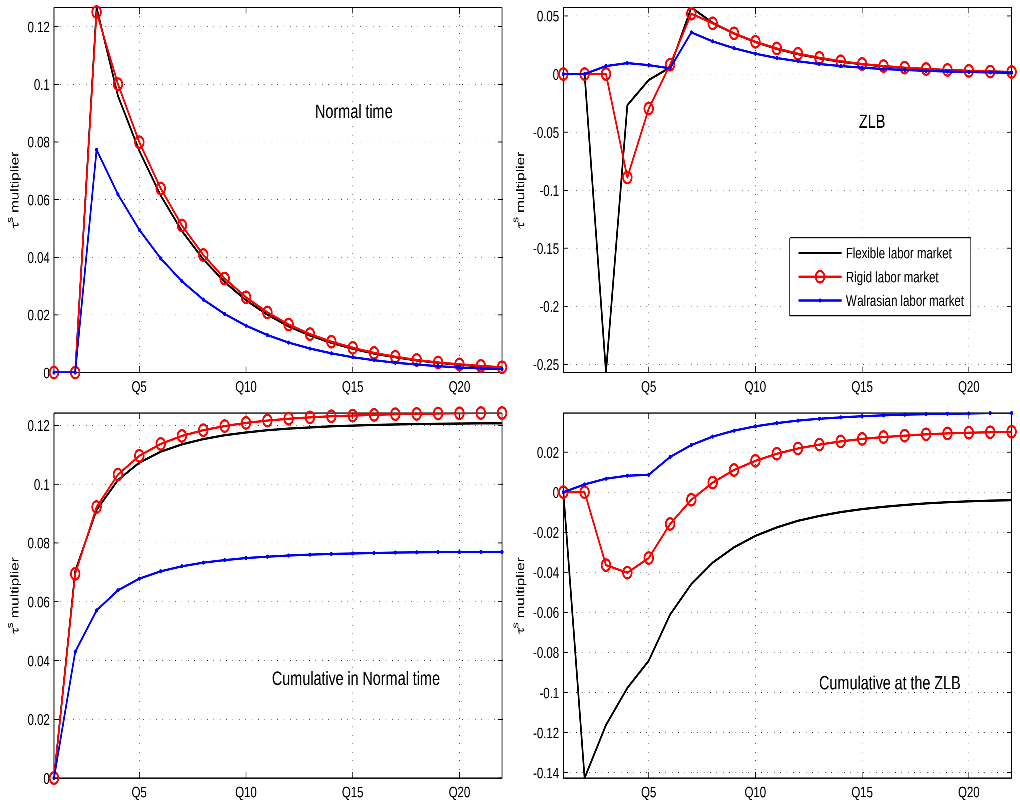

Figure 10: ***Impulse response function - Employees labor tax multiplier.*** *The tax decreases by 1%.*

5 Conclusion and discussion

This paper studies the labor market dynamic when the nominal interest rate hits the zero lower bound (ZLB). We wonder how the labor market behaves during these unconventional recessionary periods. We build a DSGE New Keynesian model with frictional unemployment. Our framework takes into account explicitly the non-linearities induced by the ZLB which can not been accurately studied when the model is solved using linear-approximation methods. We show that when the economy enters in a liquidity trap the unemployment rate can increase dramatically which amplify the deflation, consistent with recent observations. We show that search and matching frictions (SaM) have strong effects on the way variables respond to shocks if the economy is at the ZLB. They influences quantitatively **and** qualitatively the government spending and taxes multipliers through the marginal cost. At the ZLB, the government spending multipliers is about two times higher than in normal times. If the labor market is flexible the government spending multiplier the scaling factor is about three but its level is far less than what many other studies have found. The tax multipliers are found to be very low. In a economy characterized by search and matching frictions, tax cuts on labor income reduce, not increase, output.

During the paper we have extensively assumed that wage are (almost) flexible. Wage rigidities, either nominal or real, are an important source of propagation. As mentioned Shimer (2012) and Schmitt-Grohé and Uribe (2012) wage rigidities seem to be the most promising avenues to reproduce the stylized facts on the labor market following the financial crisis. The authors highlight that such rigidities are able to explain the jobless recovery. The introduction of real and nominal wage rigidities is in our research agenda.

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A PEA: computation algorithm

The parameterized expectation algorithm consists of approximating the conditional expectations of the system described previously. We approximate the expectation functions of the model using Chebyshev polynomials of the state variables. This parametric function displays suitable orthogonality and convergence properties to minimize the error distance approximation. We consider a third-order Chebyshev polynomial over a fixed grid obtain by collocation methods. Our strategies is more accurate than FVGR since we approximate two policy functions: the ZLB and outside the ZLB. Then, one can approximate kink in all decision rules accurately. The basic mechanism is as follow:

When the expectation functions are evaluated at each point of the grid, we check if the interest rate i_t hits the ZLB. If it is the case $i_t = 0$. Otherwise the next period nominal interest rate is computed according to the Taylor rule. Once the interest rate regime is determined, we are able to evaluate the expectation functions on the state-space representation of the model using Gauss-hermite quadratures with 30 nodes. The competitive equilibrium can be summarized by the following reduced form bloc:

Backward looking dynamics

$$\begin{aligned}
n_t &= n_{t-1}(1-\rho^x) + v_t q_t \\
i_t &= i_{t-1}^{\rho_i}\left[\frac{\pi}{\beta}\left(\frac{\pi_t}{\pi}\right)^{\rho_\pi}\left(\frac{y_t}{y}\right)^{\rho_y}\right]^{1-\rho_i} \\
\log \eta_t &= \rho_\eta \log \eta_{t-1} + \varepsilon_t^\eta \\
\log g_t &= \rho_g \log g_{t-1} + (1-\rho_g)\log g + \varepsilon_t^g \\
\log \tau_t^c &= \rho_c \log \tau_{t-1}^c + (1-\rho_c)\log \tau^c + \varepsilon_t^c \\
\log \tau_t^w &= \rho_w \log \tau_{t-1}^w + (1-\rho_w)\log \tau^w + \varepsilon_t^w \\
\log \tau_t^s &= \rho_s \log \tau_{t-1}^s + (1-\rho_s)\log \tau^s + \varepsilon_t^s
\end{aligned}$$

Forward looking dynamics

The two regimes deviate from one another according through the Euler equation

$$\begin{aligned}
\lambda_t = (1+i_t)\eta_t \Psi_t^1 \quad &\text{if} \quad i_t > 0 \\
\lambda_t = \eta_t \Psi_t^1 \quad &\text{if} \quad i_t = 0
\end{aligned}$$

The rest of the forward looking equation are similar in the two regimes

$$\begin{aligned}
&\frac{\kappa}{q_t} = mc_t\alpha\frac{y_t}{n_t} - w_t^R(1+\tau_t^w) + \frac{\kappa}{\lambda_t}\Psi_t^3 \\
&w_t^R = \frac{1-\xi}{1+\tau_t^w}mc_t\alpha\frac{y_t}{n_t} + \frac{\xi}{1-\tau_t^s}\left(b_t + \frac{\ell n_t^\phi}{\lambda_t}\right) \\
&+(1-\xi)\frac{\kappa}{\lambda_t}\left(\frac{\Psi_t^3}{1+\tau_t^w} - \frac{\Psi_t^4}{1-\tau_t^s}\right) \\
&0 = (1-\epsilon) + \epsilon mc_t - \psi_p\frac{\pi_t}{\pi}\left(\frac{\pi_t}{\pi} - 1\right) + \frac{\Psi_t^2}{\lambda_t y_t}
\end{aligned}$$

where:

$$
\begin{aligned}
q_t &= \chi \left(\frac{v_t}{1-(1-\rho^x)n_{t-1}} \right)^{-\nu} \\
y_t &= n_t^{\alpha} \\
c_t &= y_t(1-\Gamma_t^{\pi}) - \kappa v_t - g_t \\
\lambda_t &= \frac{c_t^{-\sigma}}{1+\tau_t^c} \\
\Gamma_t^{\pi} &= \frac{\psi_p}{2}\left(\frac{\pi_t}{\pi}-1\right)^2
\end{aligned}
$$

For a given process $\tau_t^c, \tau_t^w, \tau_t^s, g_t$ and η_t the state variables are: $\{n_{t-1}, i_{t-1}\}$ and the control variables are: $\{\pi_t, v_t, w_t^R, mc_t\}$.

The expectation functions are the following ones:

$$
\begin{aligned}
\Psi_t^1 &= \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \\
\Psi_t^2 &= \beta E_t \lambda_{t+1} \psi_p \frac{\pi_{t+1}}{\pi} y_{t+1} \left(\frac{\pi_{t+1}}{\pi}-1\right) \\
\Psi_t^3 &= \beta(1-\rho^x) E_t \frac{\lambda_{t+1}}{q_{t+1}} \\
\Psi_t^4 &= \beta(1-\rho^x) E_t \frac{\lambda_{t+1}}{q_{t+1}} \frac{(1-f_{t+1})(1-\tau_{t+1}^s)}{1+\tau_{t+1}^w}
\end{aligned}
$$

The PEA procedure consists of approximating the unknown functions Ψ_i and the decision rules by Chebyshev polynomials. The algorithm is as follows:

Step 1 *Set the parameters values and the deterministic steady state of all endogenous variables in the unconstrained model.*

Step 2 *Choose the order of the Chebyshev polynomial and the number of nodes (which are at least equal to the order of the Chebyshev polynomial plus one). Build the Chebyshev polynomials using the following recursion:*

$$
T_n(x) = \cos(n \arccos(x))
$$

Step 3 *Compute the grid of the six state variables: (n, i and the four stochastic processes), imposing the steady states to be equidistant from the upper bound and the lower bound of the grid. Use the Kronecker product to get the tensor product base.*

Step 4 *Initialize the policy rules of the forward-looking variables using their steady state level on the first row.*

Step 5 *Initialize the expectation functions* Ψ_i*. As a first guess, we evaluate them at the deterministic steady state.*

Step 6 *Given the value of expectation functions, determine the policy rules of the two models using a Newton algorithm.*

Step 7 *Given the policy rules, compute the next period states variables at each node of the grid and the next period forward-looking variables.*

Step 8 *Check if nominal interest rate* i_t *hits the ZLB to define a unique policy rule for each variable that takes into account the regime-switching.*

Step 9 *Given the new policy rules, compute the new expectation functions using ordinary least square.*

Step 10 *Check if the expectation functions are the same as in step 5 using an Euclidian norm. Otherwise, define the new expectation functions as the initial value and return to step 6. Repeat this procedure until convergence.*

B The basic New-Keynesian model

We build an New-Keynesian model with now walrasian labor market. We assume, as before, that retailers are facing quadratic adjustment cost to reset their prices each periods.

B.1 The representative households

From the maximization program of households over good varieties we obtain the following condition:

$$c_{jt} = \left(\frac{p_{jt}}{p_t}\right)^{-\epsilon} c_t \tag{34}$$

which describe the optimal level of c_{jt}. Households maximizes aggregate consumption:

$$\max_{\Omega_t^{F\prime}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{{c_t}^{1-\sigma}}{1-\sigma} - \ell \frac{{n_t}^{1+\phi}}{1+\phi}\right] \tag{35}$$

subject to the budget constraint and taking as given the set of process $\{p_t, w_t, i_t\}$ and initial wealth and debt.

$$(1+\tau_t^c)p_t c_t + \frac{d_t}{\eta_t} = d_{t-1}i_{t-1} \quad + \quad w_t(1-\tau_t^s)n_t + \Pi_t + T_t \tag{36}$$

the optimality conditions are given by the following equations :

$$\lambda_t = \frac{c_t^{-\sigma}}{(1+\tau_t^c)} \tag{37}$$

$$\lambda_t = \beta\eta_t(1+i_t)E_t\lambda_{t+1}\frac{p_t}{p_{t+1}} \tag{38}$$

and (with $w_t/p_t = w_t^R$)

$$\ell n_t^\phi c_t^{-\sigma} = w_t^R \frac{1-\tau_t^s}{1+\tau_t^c} \tag{39}$$

B.2 Firms

The optimization problem of the firm j is to choose a set of processes $\Omega_{jt}^{F\prime} = \{p_{jt}\}_{t=0}^{\infty}$ taking as given the set of processes $\{p_t\}_{t=0}^{\infty}$. Each j producer maximizes the following intertemporal function:

$$\max_{\Omega_t^{F\prime}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0}\Pi_{jt} \tag{40}$$

$$\text{where } \Pi_{jt} = \left[\frac{p_{jt}}{p_t}y_{jt} - \frac{w_{jt}}{p_t}n_{jt}(1+\tau_t^w) + y_t\Gamma^\pi(p_{jt})\right]$$

subject to the production function $y_{jt} = n_{jt}^\alpha$. The optimality conditions of the above problem are:

$$\begin{aligned} 0 &= (1-\epsilon) + \epsilon mc_t - \psi_p\frac{\pi_t}{\pi}\left(\frac{\pi_t}{\pi}-1\right) \\ &+ \beta E_t\frac{\lambda_{t+1}}{\lambda_t}\psi_p\frac{\pi_{t+1}}{\pi}\left(\frac{\pi_{t+1}}{\pi}-1\right)\frac{y_{t+1}}{y_t} \end{aligned} \tag{41}$$

mc_t is the Lagrange multiplier associated to the individual consumption demand. We can rewrite the wage as follow :

$$w_t^R = \gamma mc_t(1+\tau_t^w) \tag{42}$$

B.3 The monetary and fiscal authorities

The central bank follows the same rule as described previously The fiscal authority finance the government expenditure and the fiscal rule satisfy :

$$d_t + \tau_t^c c_t p_t + n_t w_t(\tau_t^w + \tau_t^s) = i_{t-1} d_{t-1} + g_t + T_t \tag{43}$$

B.4 The market clearing

The aggregation of individual profits Π_t is given by:

$$\Pi_t = p_t y_t - n_t w_t(1 + \tau_t^w) - p_t y_t \Gamma_t^\pi \tag{44}$$

Equations (43) together with the budget constraint (36) and the profit (44) give the aggregate resource constraint :

$$y_t = c_t + g_t \tag{45}$$

B.5 PEA: computation algorithm for basic New-Keynesian model

Backward looking dynamics

$$\begin{aligned}
i_t &= i_{t-1}^{\rho_i} \left[\frac{\pi}{\beta} \left(\frac{\pi_t}{\pi} \right)^{\rho_\pi} \left(\frac{y_t}{y} \right)^{\rho_y} \right]^{1-\rho_i} \\
\log \eta_t &= \rho_\eta \log \eta_{t-1} + \varepsilon_t^\eta \\
\log g_t &= \rho_g \log g_{t-1} + (1 - \rho_g) \log g + \varepsilon_t^g \\
\log \tau_t^c &= \rho_c \log \tau_{t-1}^c + (1 - \rho_c) \log \tau^c + \varepsilon_t^c \\
\log \tau_t^w &= \rho_w \log \tau_{t-1}^w + (1 - \rho_w) \log \tau^w + \varepsilon_t^w \\
\log \tau_t^s &= \rho_s \log \tau_{t-1}^s + (1 - \rho_s) \log \tau^s + \varepsilon_t^s
\end{aligned}$$

Forward looking dynamics

The two regimes deviate from one another according through the Euler equation

$$\begin{aligned}
\lambda_t = (1 + i_t)\eta_t \Psi_t^{1\prime} \quad &\text{if} \quad i_t > 0 \\
\lambda_t = \eta_t \Psi_t^{1\prime} \quad &\text{if} \quad i_t = 0
\end{aligned}$$

The rest of the forward looking equation are similar in the two regimes

$$\begin{aligned}
&w_t^R = \gamma m c_t (1 - \tau_t^w) \\
&0 = (1 - \epsilon) + \epsilon m c_t - \psi_p \frac{\pi_t}{\pi} \left(\frac{\pi_t}{\pi} - 1 \right) + \frac{\Psi_t^{2\prime}}{\lambda_t y_t}
\end{aligned}$$

where:

$$
\begin{aligned}
y_t &= \gamma n_t^{\alpha} \\
c_t &= y_t(1-\Gamma_t^{\pi}) \\
\lambda_t &= \frac{c_t^{-\sigma}}{1+\tau_t^c} \\
\Gamma_t^{\pi} &= \frac{\psi_p}{2}\left(\frac{\pi_t}{\pi}-1\right)^2
\end{aligned}
$$

For a given process $\tau_t^c, \tau_t^w, \tau_t^s, g_t$ and η_t the state variables are: $\{i_{t-1}\}$ and the control variables are: $\{\pi_t, w_t^R, mc_t\}$.

The expectation functions are the following ones:

$$
\begin{aligned}
\Psi_t^{1\prime} &= \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} \\
\Psi_t^{2\prime} &= \beta E_t \lambda_{t+1} \psi_p \frac{\pi_{t+1}}{\pi} y_{t+1} \left(\frac{\pi_{t+1}}{\pi}-1\right)
\end{aligned}
$$

C Computing and estimating the Markov chains of transition probabilities

To estimate the probability of moving from one normal time to the ZLB we use the Maximum likelihood function. Assuming i_t corresponds to the case where $\tilde{i}_t > 0$, the states of the Markovian matrix are : $\{\tilde{i}_t, 0\}$. We denote by $\mathcal{P}$ the transition matrix with no restriction and $\mathcal{P}_{ij}$ the probability of moving from state i to state j. It is defined by

$$\mathcal{P}_{ij} = Pr(i_{t+1} = j | i_t = i)$$

Defining the transition counts $\mathcal{K}_{ij}$ as the number of times the state i is followed by j, the log-likelihood function can be written in the following manner:

$$\mathcal{L}(\pi) = \sum_{i,j} k_{ij} \log \mathcal{P}_{ij} \qquad \text{with} \quad k_{ij} = \log \mathcal{K}_{ij}$$

The estimation procedure consists of choosing the value of $\mathcal{P}_{ij}$ that maximizes the log-likelihood function subject to:

$$\sum_j \mathcal{P}_{ij} = 1$$

With $m = 3$ states, the above optimization problem is characterized by 3 Lagrange multipliers (λ_i) and takes the following form:

$$\hat{\mathcal{P}}_{ij} = \arg\max_{\mathcal{P}_{ij}} \quad \mathcal{L}(\mathcal{P}) - \sum_{i=1}^{j} \lambda_i \left(\sum_j \mathcal{P}_{ij} - 1 \right)$$

The first-order conditions with respect to $\mathcal{P}_{ij}$ are:

$$0 = \frac{k_{ij}}{\hat{\mathcal{P}}_{ij}} - \lambda_i \iff \hat{\mathcal{P}}_{ij} = \frac{k_{ij}}{\lambda_i}$$

Using the constraint we have $\lambda_i = \sum_{j=1}^{m} k_{ij}$. By replacing it in the first-order conditions we obtain the maximum likelihood estimator of the transition probability $\hat{\mathcal{P}}_{ij}$ from state i to state j:

$$\hat{\mathcal{P}}_{ij} = \frac{k_{ij}}{\sum_{j=1}^{m} k_{ij}}$$

Using Chapman-Kolmogorov equation one can easily compute annual transition probabilities of the transition matrix $\mathcal{P}$ as $\mathcal{P}^n(i,j)$ with n being equal to 4.

D Government spending and fiscal multiplier

We follow Fernández-Villaverde and al. (2012), Mertens and Ravn (2012) to compute the government spending and tax multipliers and Ulhig (2012) to compute the cumulative government spending and tax multipliers. We make a distinction between the multiplier in normal time and at the ZLB.

D.1 Multiplier in normal time

• The government spending and tax multipliers write as follow. Let X_t be a stochastic process (government spending or tax) with unconditional mean X and y_t the equilibrium path for output where fiscal instrument are constant. Note that y_t can be viewed as the unconditional mean of output in the absence of additional shock. The multiplier is the increase of output generated by an increase in X. We denote by $y_{x,t}$ and $X_{x,t}$ the simulated path of output and the fiscal instrument resulting from an increase in X of ε^x. Subscript x denotes the type of fiscal instrument $\{g, \tau^w, \tau^c, \tau^s\}$ that buffeted the economy. The multiplier is:

$$m_t^x = \frac{y_{x,t} - y_t}{\varepsilon^x}$$

Since the fiscal shock is temporary, $y_{x,t}$ deviates from its unconditional mean on impact and return to its mean as time goes by. This calculation involves that the multiplier is computed on a constant basis (the size of the increase of X) and ensures that the government or tax multiplier return to zero in the long-run.

• The cumulative government spending and tax multipliers correspond to the present value of xm_t. Formally it writes,

$$cm_t^x = \frac{\sum_{s=1}^t R^{-j}(y_{x,s} - y_s)}{\sum_{s=1}^t R^{-j}(X_{x,s} - X)}$$

The main difference here is that both term are discounted using the steady state gross nominal interest rate $R_t = 1 + i_t$. The basis on which the multiplier is calculated (the denominator) is not constant as previously. Since both terms tend to zero in the long-run, cm_t^x tend to a positive value.

D.2 Multiplier at the ZLB

• The multiplier at the ZLB is computed in the same way except that we allow for an additional shock to force the economy to enters in a liquidity trap. We use the risk premium shock to proxy the recessionary shock. Let $y_{x,\eta,t}$ be the path of output following a fiscal shock x and a positive risk premium shock η and $y_{\eta,t}$ the path of output following a positive risk premium shock η (no fiscal shock). The multiplier is equal to the difference between both:

$$xm_t^{ZLB} = \frac{y_{x,\eta,t} - y_{\eta,t}}{\varepsilon^x}$$

• Similarly, we compute the cumulative one by discounting over t period:

$$xm_t^{ZLB} = \frac{\sum_{s=1}^t \beta^{-j}(y_{x,\eta,s} - y_{\eta,s})}{\sum_{s=1}^t \beta^{-j}(X_{x,s} - X)}$$

E Additional figures

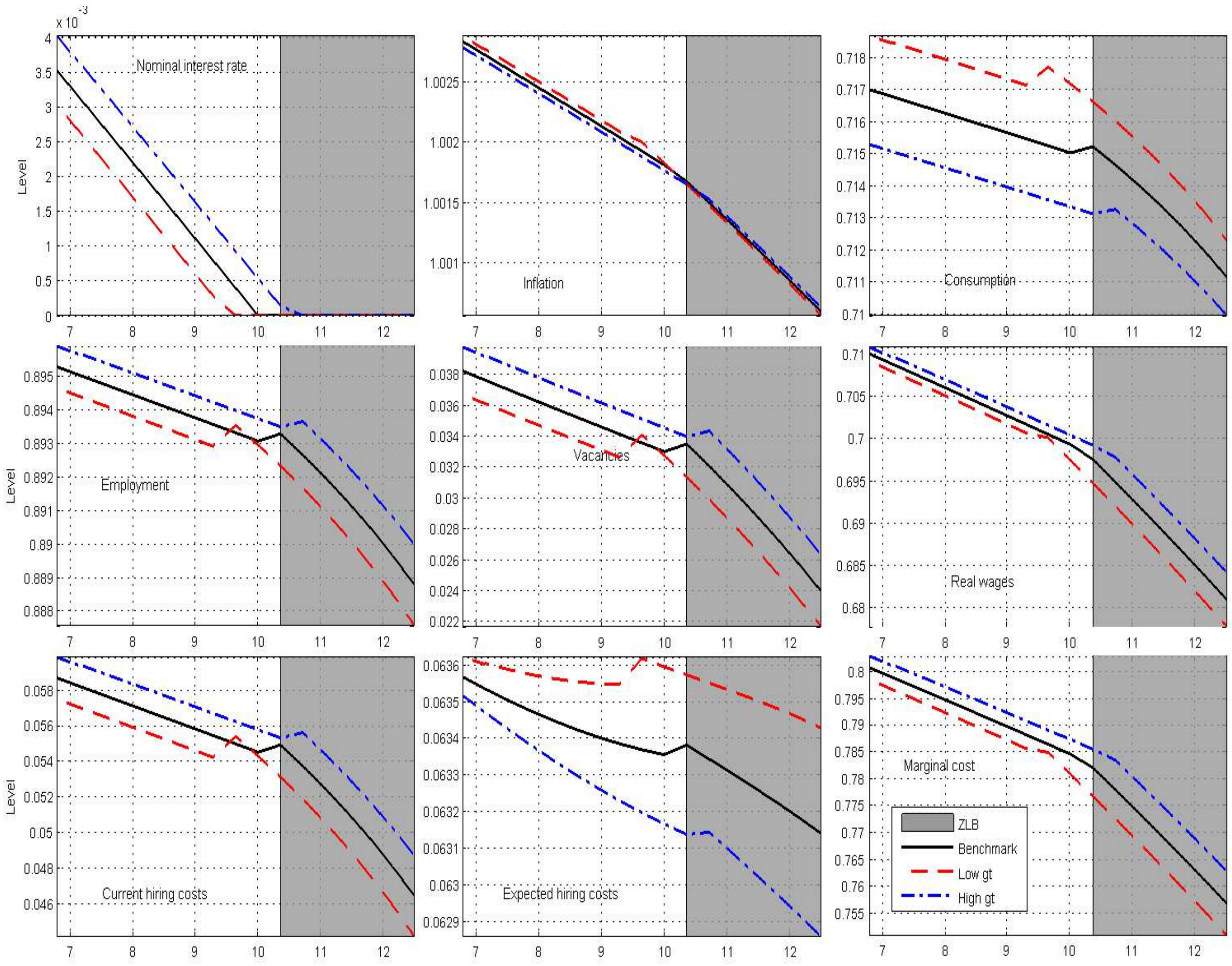

Figure 11: ***Policy rules - rigid labor market*** *The policy shock is equal to minus and plus five times its standard deviation.*

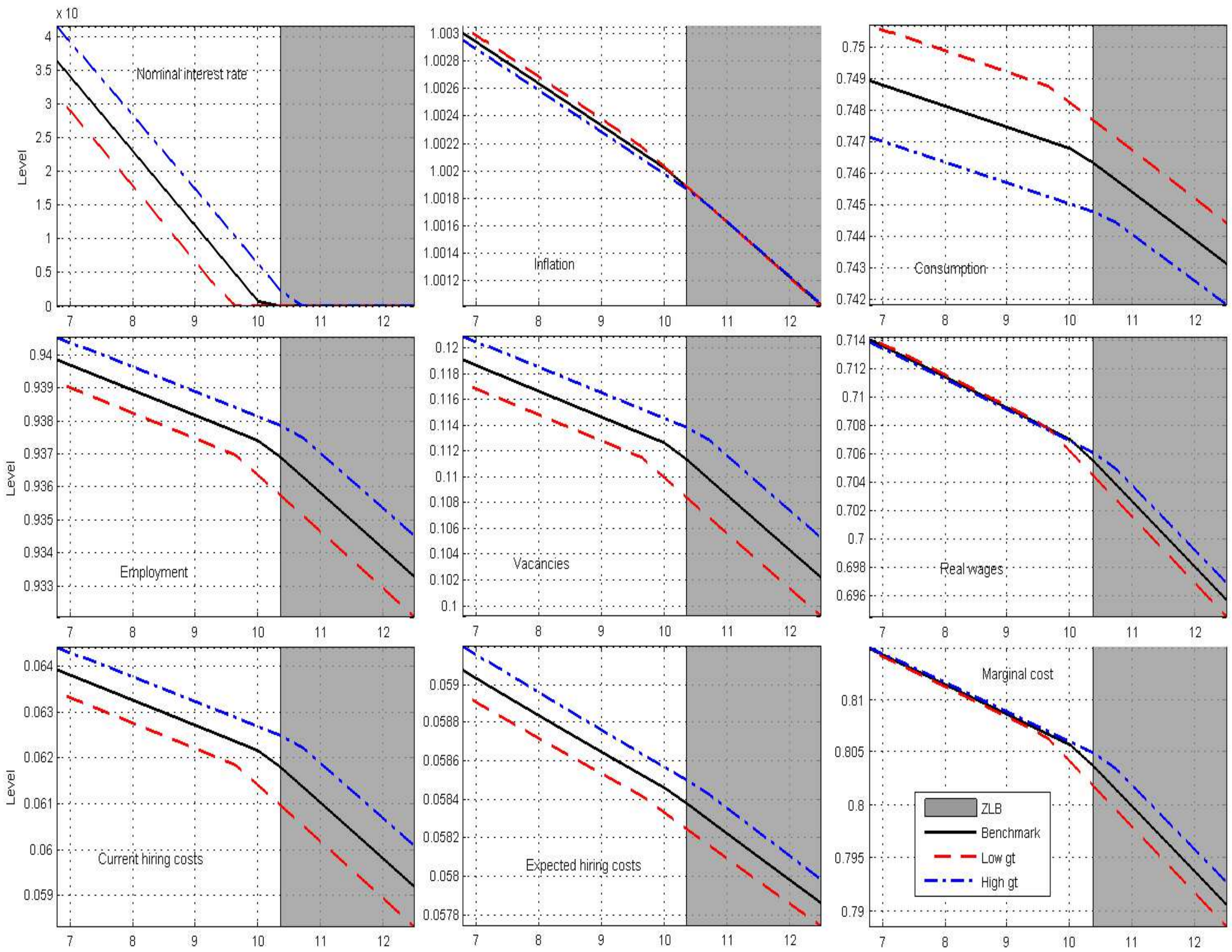

Figure 12: ***Policy rules - flexible labor market****The policy shock is equal to minus and plus five times its standard deviation.*

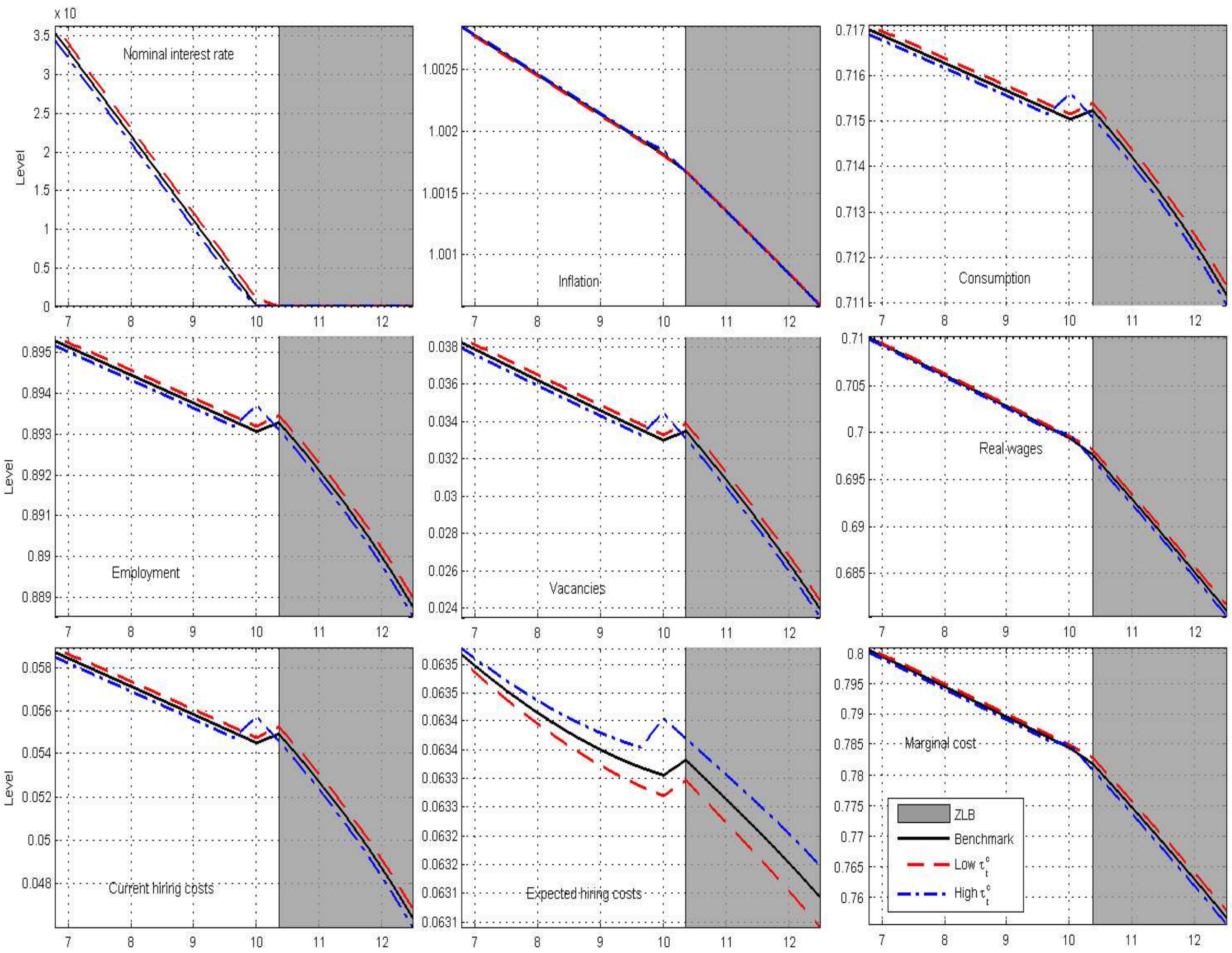

Figure 13: ***Policy rules - rigid labor market*** *The policy shock is equal to minus and plus five times its standard deviation.*

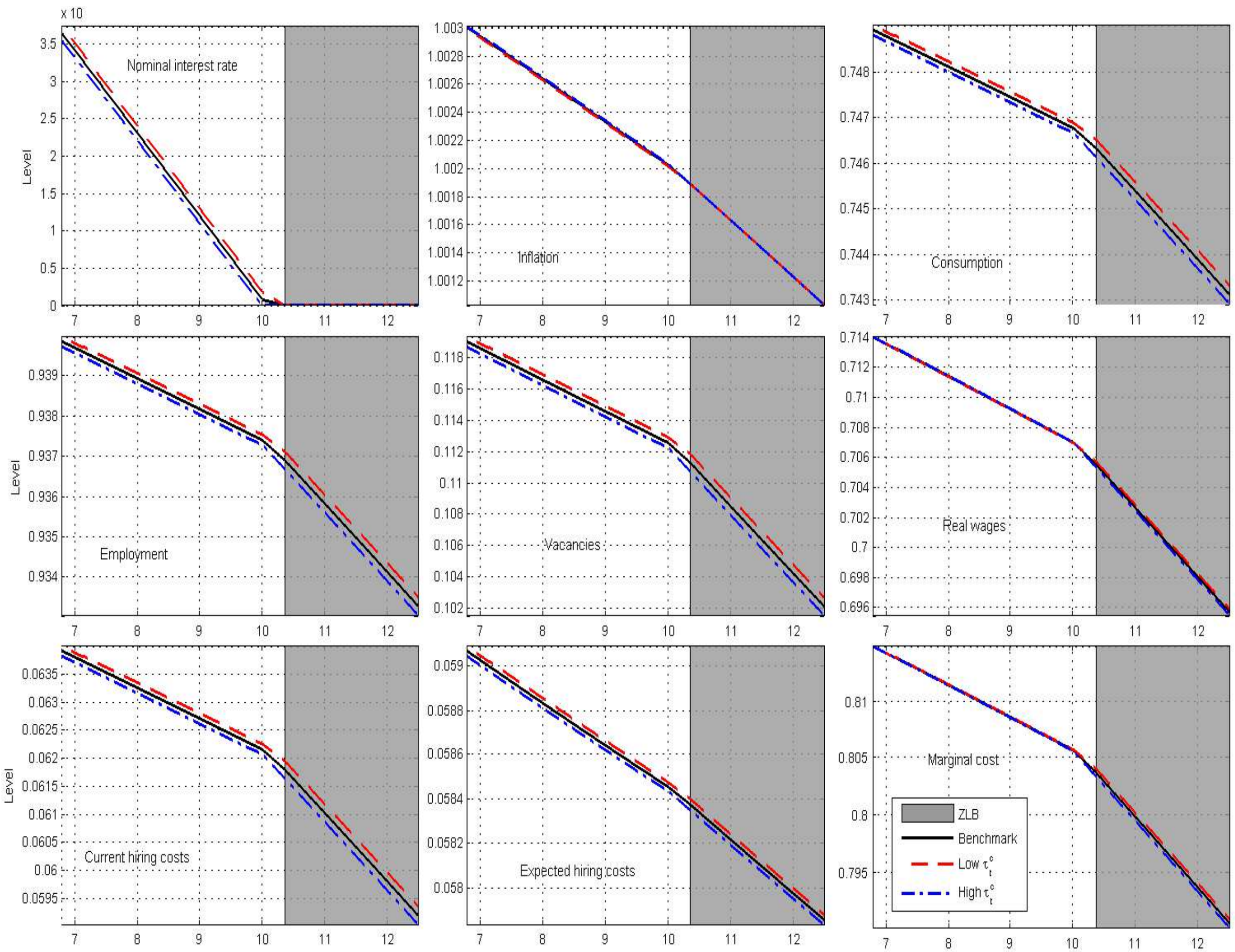

Figure 14: ***Policy rules - flexible labor market*** *The policy shock is equal to minus and plus five times its standard deviation.*

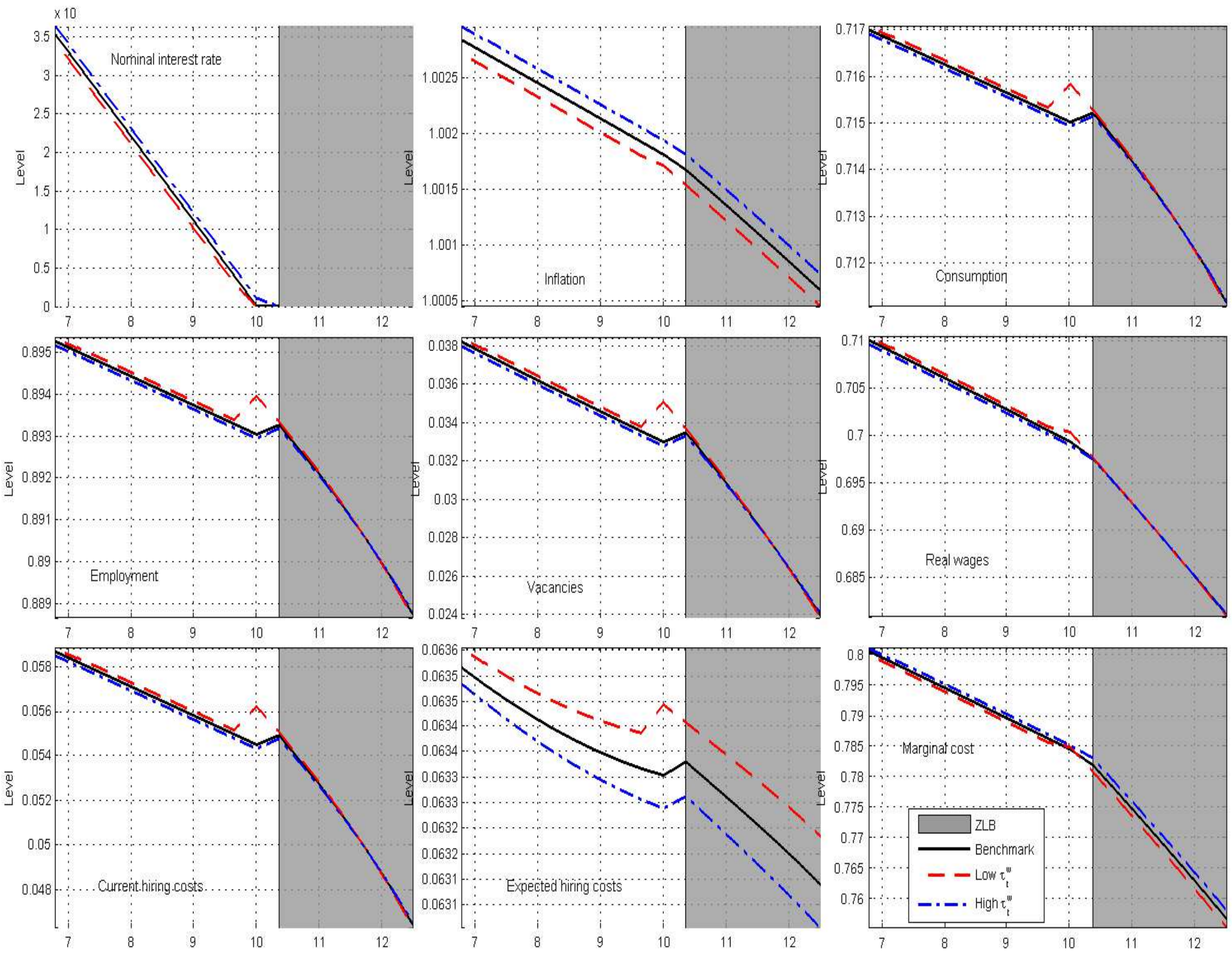

Figure 15: ***Policy rules - rigid labor market*** *The policy shock is equal to minus and plus five times its standard deviation.*

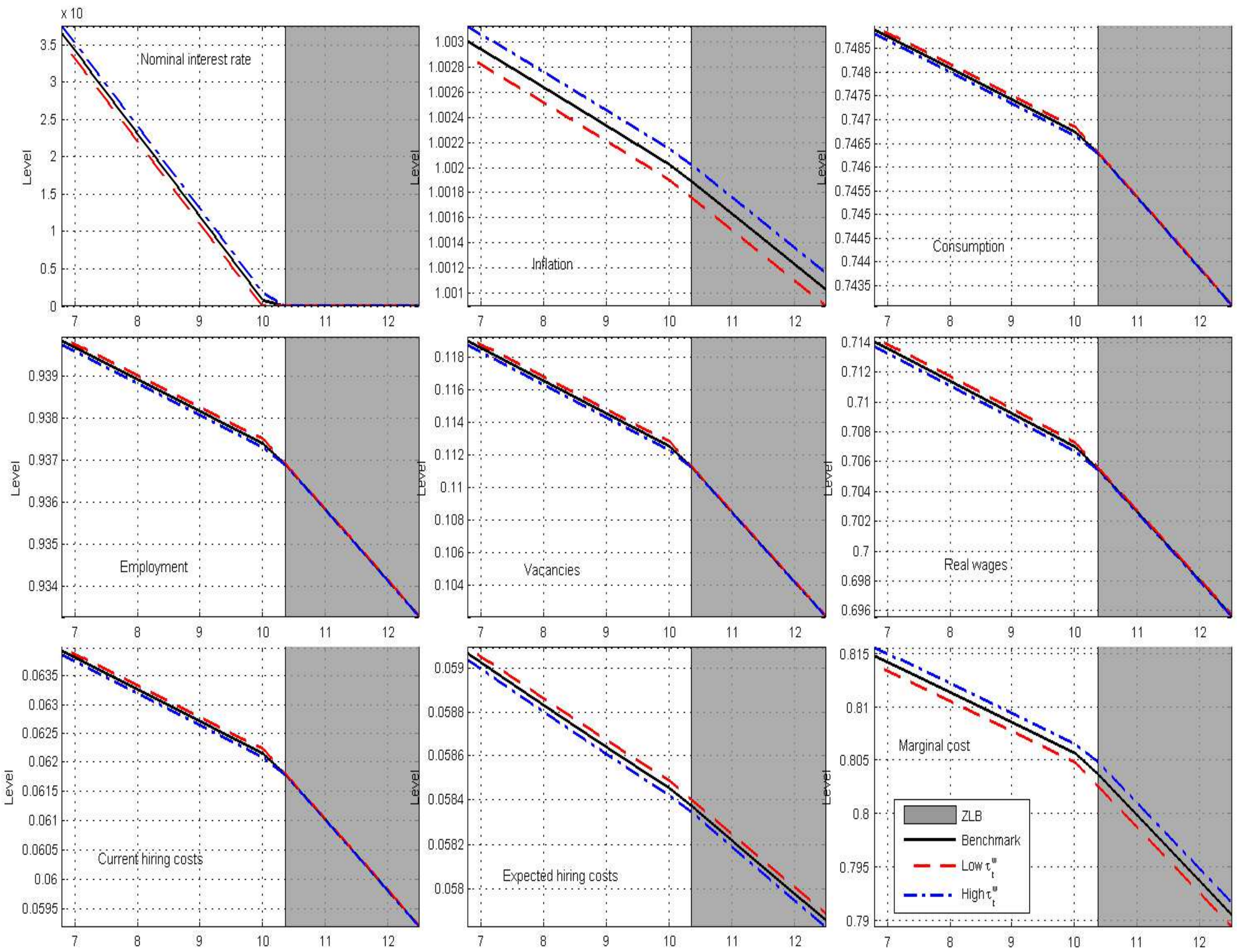

Figure 16: ***Policy rules - flexible labor market*** *The policy shock is equal to minus and plus five times its standard deviation.*

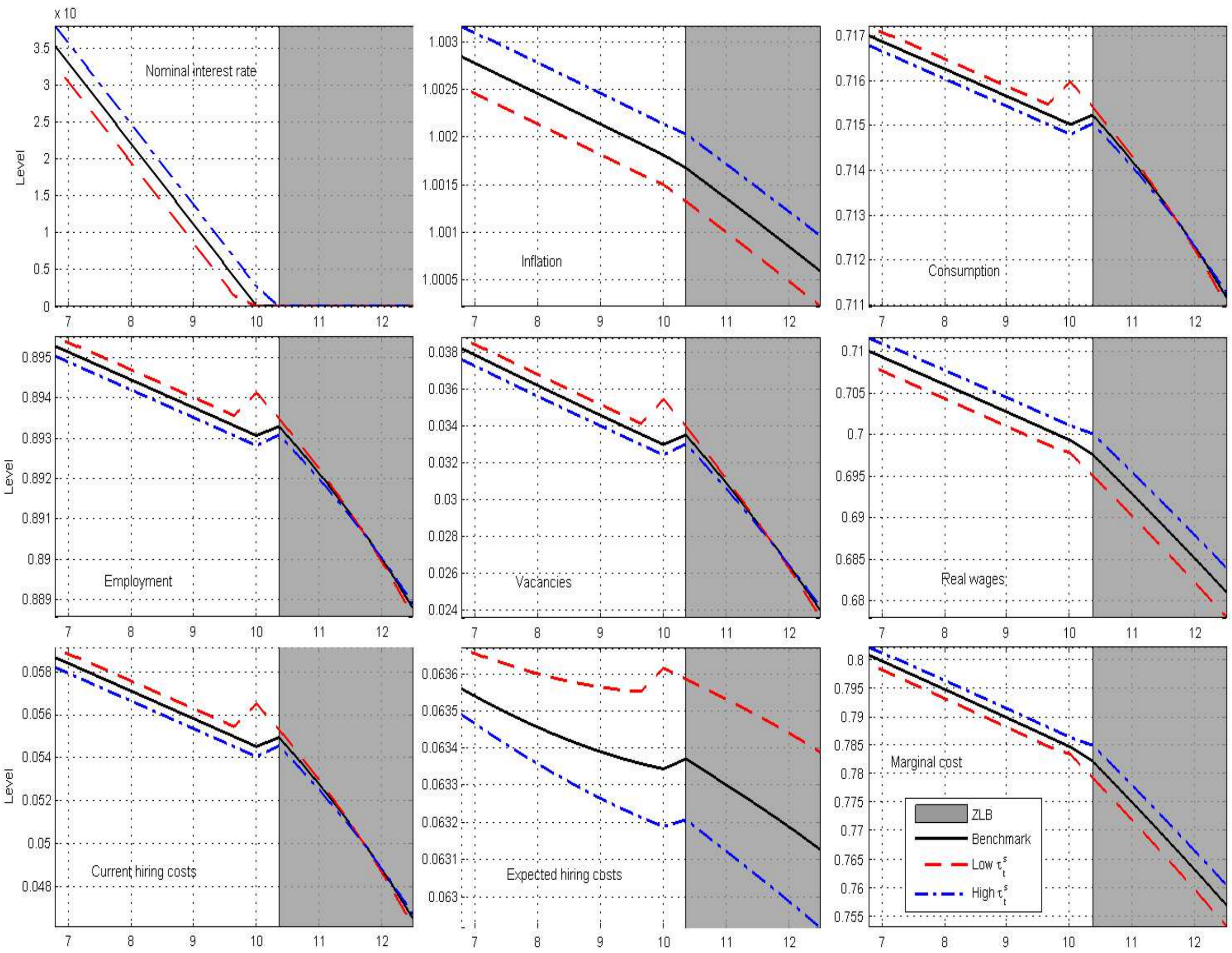

Figure 17: ***Policy rules - rigid labor market*** *The policy shock is equal to minus and plus five times its standard deviation.*

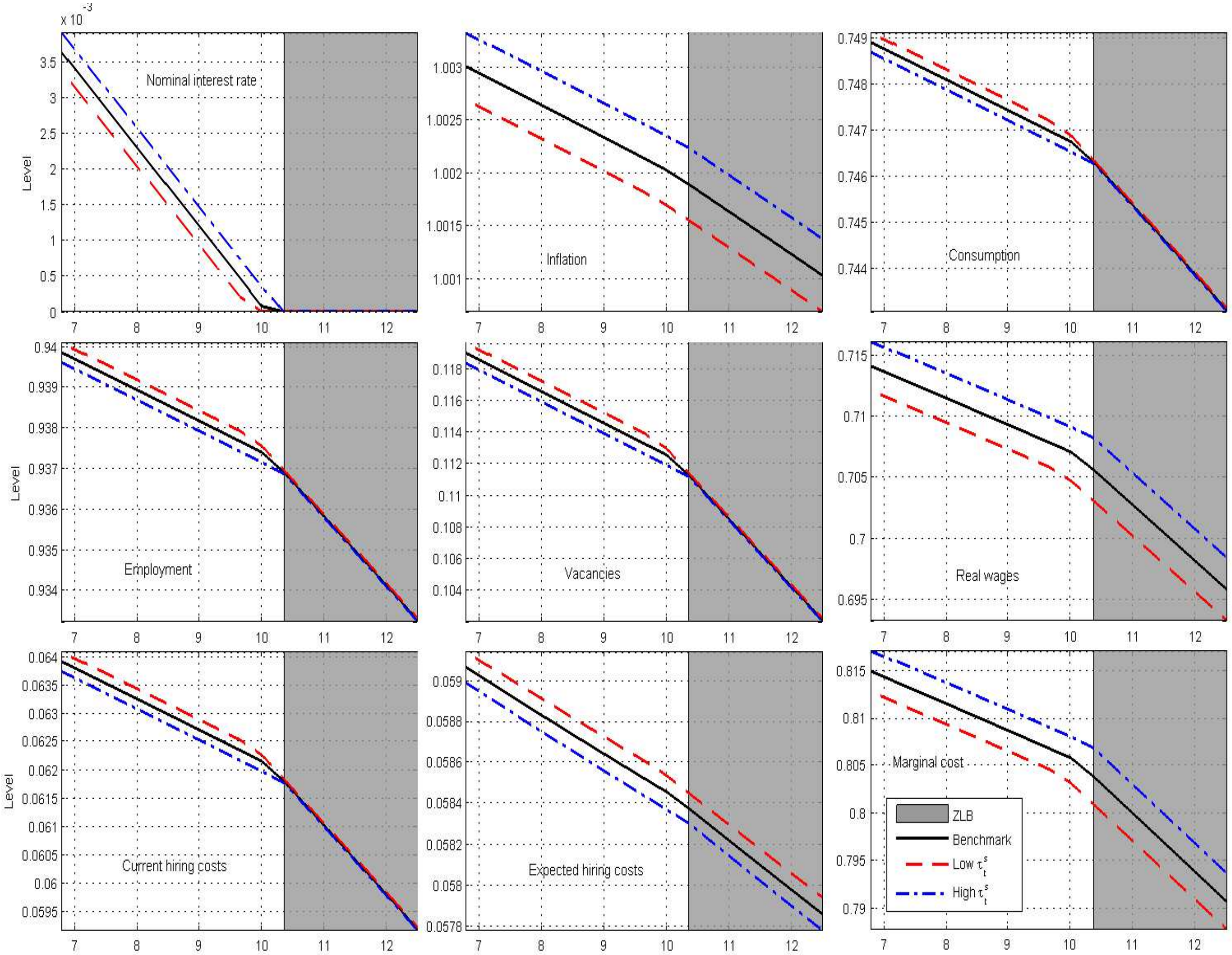

Figure 18: ***Policy rules - flexible labor market*** *The policy shock is equal to minus and plus five times its standard deviation.*