Pricing of Asian Temperature Risk

温度风险

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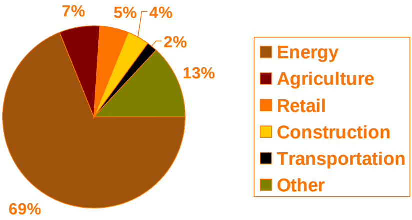
Weather

PricewaterhouseCoopers Survey 2005 releases the Top 5 sectors in need of financial instruments to hedge weather risk.



PwC survey 2005 for Weather Risk Management Association

Weather

- Influences our daily lives and choices
- Impact on corporate revenues and earnings
- Meteorological institutions: business activity is weather dependent
  - British Met Office: daily beer consumption gain 10% if temperature increases by 3° C
  - If temperature in Chicago is less than 0° C consumption of orange juice declines 10% on average

What are Weather Derivatives?

Hedge weather related risk exposures

- Payments based on weather related measurements
- Underlying: temperature, rainfall, wind, snow, frost

Chicago Mercantile Exchange (CME)

- Monthly/seasonal/weekly temperature Future/Option contracts
- 24 US, 6 Canadian, 9 European and 3 Asian-Pacific cities (Tokyo & Osaka since 2008 and Hiroshima since 2009)
- From 2.2 billion USD in 2004 to 15 billion USD through March 2009

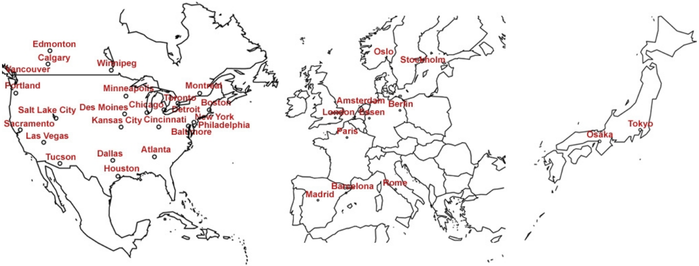


Figure 1: CME offers weather contracts on 43 cities throughout the world

Weather Derivatives

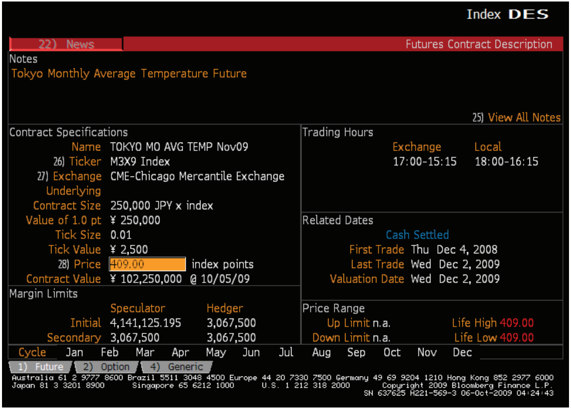


Figure 2: A WD table quoting prices future contracts. Source: Bloomberg

Types of Weather Derivatives

- CME products
  - HDD(τ_1,τ_2)=$\int_{\tau_1}^{\tau_2}\max(18^\circ\text{C}-T_t,0)dt$
  - CDD(τ_1,τ_2)=$\int_{\tau_1}^{\tau_2}\max(T_t-18^\circ\text{C},0)dt$
  - CAT(τ_1,τ_2)=$\int_{\tau_1}^{\tau_2} T_t dt$, where $T_t=\frac{T_{t,max}-T_{t,min}}{2}$
  - AAT(τ_1,τ_2)=$\int_{\tau_1}^{\tau_2}\widetilde{T}_t dt$, where $\widetilde{T}_t=\frac{1}{24}\int_1^{24} T_{t_i} dt_i$ and T_{t_i} denotes the temperature of hour t_i, (also referred to as C24AT index).
- HDD-CDD parity:
  CDD(τ_1,τ_2)-HDD(τ_1,τ_2)=CAT(τ_1,τ_2)-c($\tau_2-\tau_1$)

Weather Risk and Human Capital...

An investor organizes a conference on the 27-31 October 2009 in Kaohsiung. Since he knows there is another conference event that week, he estimates that every additional °C in excess of 135°C accumulated 24-hour average temperatures will reduce the number of participants in favor for the other conference and he will incur 2,500 JPY costs on human capital.

MPR Algorithm

Econometrics	Fin. Mathematics.
T_t	$CAR(3)$
$\downarrow$	$\downarrow$
$X_t = T_t - \Lambda_t$	$F_{CAT(t,\tau_1,\tau_2)}$
$\downarrow$	$\downarrow$
$X_{t+p} = a^\top X_t + \sigma_t \varepsilon_t$	MPR
$\downarrow$	
$\hat{\varepsilon}_t = \frac{\hat{X}_t}{\hat{\sigma}_t} \sim N(0,1)$	

Estimation of $\hat{\sigma}_t$: 2 Steps vs 1 Step

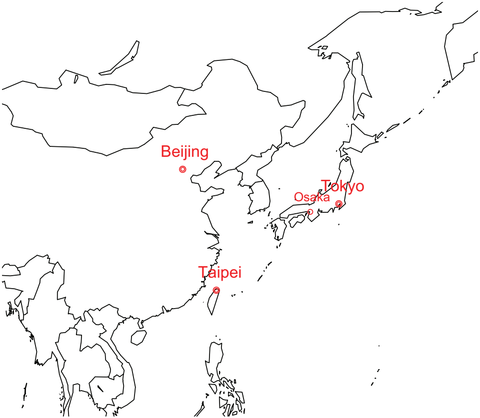
2 Steps	1 Step
Fourier Truncated Series (FTS)	Local linear Regression (LLR)
$+$	
$GARCH(p,q)$	
$\downarrow$	$\downarrow$
$\hat{\varepsilon}_t = \frac{\hat{X}_t}{\hat{\sigma}_{t,FTSG}\hat{\sigma}_{t,GARCH}} \sim N(0,1)$	$\hat{\varepsilon}_t = \frac{\hat{X}_t}{\hat{\sigma}_{t,LLR}} \sim N(0,1)$

Outline

1. Motivation ✓
2. Weather Dynamics
3. Fitting $\hat{\sigma}_t$: 1-2 Steps
4. Pricing
5. Conclusion

Weather Dynamics: Asian Data

Temperature Market (CME): Tokyo and Osaka



AAT Index

CME data on weather derivatives for 20081008-20090702:

	Trading Period		Measurement Period		Index	
Code	First-trade	Last-trade	τ_1	τ_2	CME[1]	AAT[2]
F9	20080203	20090202	20090101	20090131	200.2	181.0
G9	20080303	20090302	20090201	20090228	220.8	215.0
H9	20080403	20090402	20090301	20090331	301.9	298.0
J9	20080503	20100502	20090401	20090430	460.0	464.0
K9	20080603	20090602	20090501	20090531	592.0	621.0

Table 1: Osaka AAT contracts listed on CME. Source: Bloomberg. [1] prices of AAT Futures as listed on CME, [2] AAT index values computed from the historical temperature data.

Asian Temperature

Temperature: $T_t = X_t + \Lambda_t$

Seasonal function with trend: $\Lambda_t = a + bt + \sum_{i=1}^{p} c_i cos \left\{ \frac{2\pi(t-d_i)}{i \cdot 365} \right\}$

- $\hat{a}$: average temperature, $\hat{b}$: global Warming

City	Period	$\hat{a}$	$\hat{b}$	$\hat{c}_1$	$\hat{d}_1$
Tokyo	19730101-20081231	15.76	7.82e-05	10.35	-149.53
Osaka	19730101-20081231	15.54	1.28e-04	11.50	-150.54
Beijing	19730101-20081231	11.97	1.18e-04	14.91	-165.51
Taipei	19920101-20090806	23.21	1.68e-03	6.78	-154.02

Table 2: Seasonality estimates of daily average temperatures in Asia. All coefficients are nonzero at 1% significance level. Data source: Bloomberg

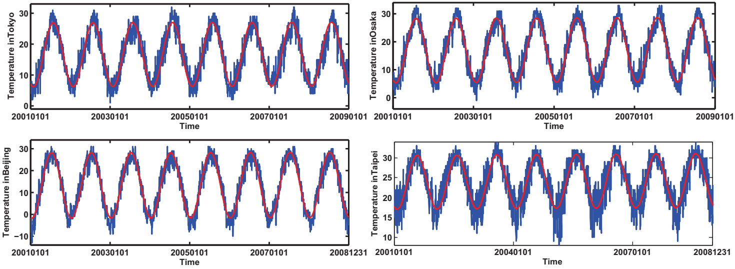


Figure 3: Seasonality effect and daily average temperatures for Tokyo Narito International Airport (upper left), Osaka Kansai International Airport (upper right), Beijing (lower left), Taipei (lower right).

Check Data

Date	Bloomberg	Japan Meteorological Agency (JMA)
20080921	13	23
20080918	14	24
20080705	16	26
20080628	13	23
20070906	16	26
20061004	12	22
19980214	5	13
19960114	18	8

Table 3: Tokyo: Check outliers with reference of JMA

Temporal Dependence

Remove seasonality: $X_t = T_t - \Lambda_t$

ADF-Test:
$(1-L)X = c_1 + \mu t + \tau LX + \alpha_1(1-L)LX + \ldots \alpha_p(1-L)L^p X + \varepsilon_t$

- Reject $H_0 : \tau = 0$, hence X_t is a stationary process $I(0)$

KPSS Test: $X_t = c + \mu t + k\sum_{i=1}^{t} \xi_i + \varepsilon_t$,

- Accept $H_0 : k = 0$ that the process is stationary.

City	$\hat{\tau}$(p-value)	$\hat{k}$(p-value)
Tokyo	-56.29(<0.01)	0.091(<0.1)
Osaka	-17.86(<0.01)	0.138(<0.1)
Beijing	-20.40(<0.01)	0.094(<0.1)
Taipei	-33.21(<0.01)	0.067(<0.1)

Table 4: Stationarity tests

PACF of X_t

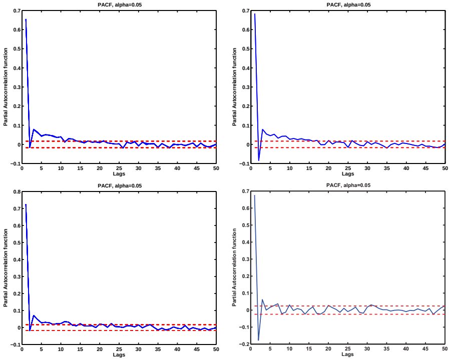


Figure 4: Partial autocorrelation function (PACF) for Tokyo (upper left), Osaka (upper right), Beijing (lower left), Taipei (lower right)

Moving Window for Stability: Tokyo

<table>
<tr><th>Year</th><th>every 3 years</th><th>every 6 years</th><th>every 9 years</th><th>every 12 years</th><th>every 18 years</th></tr>
<tr><td>73-75</td><td>AR(1)</td><td rowspan="2">AR(3)</td><td rowspan="3">AR(3)</td><td rowspan="4">AR(8)*</td><td rowspan="6">AR(9)*</td></tr>
<tr><td>76-78</td><td>AR(1)</td></tr>
<tr><td>79-81</td><td>AR(1)</td><td rowspan="2">AR(8)*</td></tr>
<tr><td>82-84</td><td>AR(8)*</td><td rowspan="3">AR(9)*</td></tr>
<tr><td>85-87</td><td>AR(1)</td><td rowspan="2">AR(3)</td><td rowspan="4">AR(3)</td></tr>
<tr><td>88-90</td><td>AR(1)</td></tr>
<tr><td>91-93</td><td>AR(1)</td><td rowspan="2">AR(3)</td><td rowspan="3">AR(3)</td><td rowspan="6">AR(3)</td></tr>
<tr><td>94-96</td><td>AR(1)</td></tr>
<tr><td>97-99</td><td>AR(1)</td><td rowspan="2">AR(1)</td><td rowspan="4">AR(3)</td></tr>
<tr><td>00-02</td><td>AR(1)</td><td rowspan="3">AR(3)</td></tr>
<tr><td>03-05</td><td>AR(3)</td><td rowspan="2">AR(3)</td></tr>
<tr><td>06-09</td><td>AR(1)</td></tr>
</table>

Table 5: Tokyo Moving window for AR * denotes instability

Moving Window for Stability: Osaka

<table>
<tr><th>Year</th><th>every
3 years</th><th>every
6 years</th><th>every
9 years</th><th>every
12 years</th><th>every
18 years</th></tr>
<tr><td>73-75</td><td>AR(1)</td><td rowspan="2">AR(3)</td><td rowspan="3">AR(3)</td><td rowspan="4">AR(3)</td><td rowspan="6">AR(6)*</td></tr>
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<tr><td>79-81</td><td>AR(3)</td><td rowspan="2">AR(3)</td></tr>
<tr><td>82-84</td><td>AR(2)</td><td rowspan="3">AR(3)</td></tr>
<tr><td>85-87</td><td>AR(3)</td><td rowspan="2">AR(3)</td><td rowspan="4">AR(6)*</td></tr>
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<tr><td>94-96</td><td>AR(1)</td></tr>
<tr><td>97-99</td><td>AR(2)</td><td rowspan="2">AR(2)</td><td rowspan="4">AR(7)*</td></tr>
<tr><td>00-02</td><td>AR(1)</td><td rowspan="3">AR(3)</td></tr>
<tr><td>03-05</td><td>AR(3)</td><td rowspan="2">AR(3)</td></tr>
<tr><td>06-09</td><td>AR(1)</td></tr>
</table>

Table 6: Osaka Moving window for AR * denotes instability

AR(p): $X_{t+p} = \sum_{i=1}^{p} \beta_i X_{t+p-i} + \sigma_t \varepsilon_t$

City	Tokyo(p=3)	Osaka(p=3)	Beijing(p=3)	Taipei(p=3)
β_1	0.668	0.748	0.741	0.808
β_2	-0.069	-0.143	-0.071	-0.228
β_3	0.079	-0.079	0.071	0.063

Table 7: Coefficients of AR(p) , Model selection: AIC

The long memory diagnosis can be replicated by a short memory process with structural breaks!

(Squared) Residuals: China - Taiwan

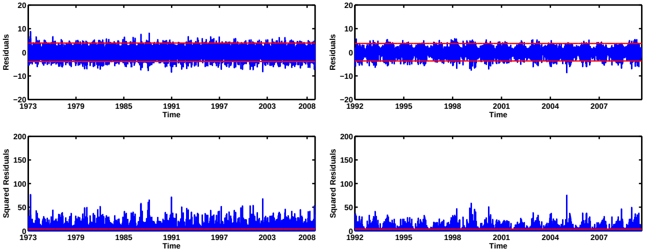


Figure 5: Residuals $\hat{\varepsilon}_t$ (up) and squared residuals $\hat{\varepsilon}_t^2$ (down) of the AR(p) (Beijing (left), Taipei (right)). No rejection of H_0 that the residuals are uncorrelated at 0% significance level, according to the modified Li-McLeod Portmanteau test

(Squared) Residuals: Japan

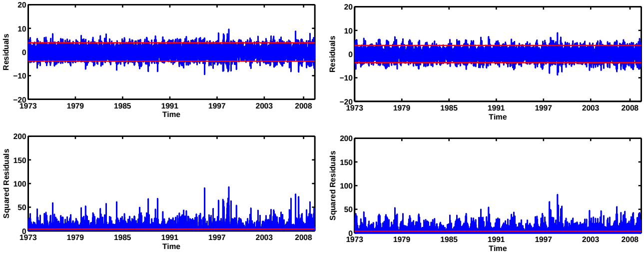


Figure 6: Residuals $\hat{\varepsilon}_t$ (up) and squared residuals $\hat{\varepsilon}_t^2$ (down) of the AR(p) (Tokyo (left), Osaka (right)) during 19730101-20081231. No rejection of H_0 that the residuals are uncorrelated at 0% significance level, according to the modified Li-McLeod Portmanteau test

Seasonal Volatility: China - Taiwan

Close to zero ACF for residuals and highly seasonal ACF for squared residuals of AR(p)

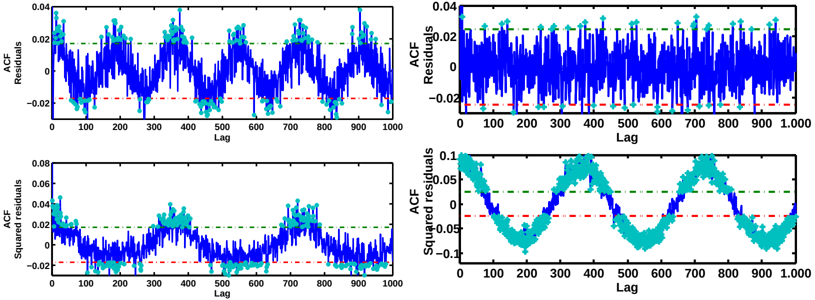


Figure 7: ACF for residuals $\hat{\varepsilon}_t$ (up) and squared residuals $\hat{\varepsilon}_t^2$ (down) of the AR(p) for Beijing (left), Taipei (right).

Seasonal Volatility: Japan

Close to zero ACF for residuals and highly seasonal ACF for squared residuals of AR(p)

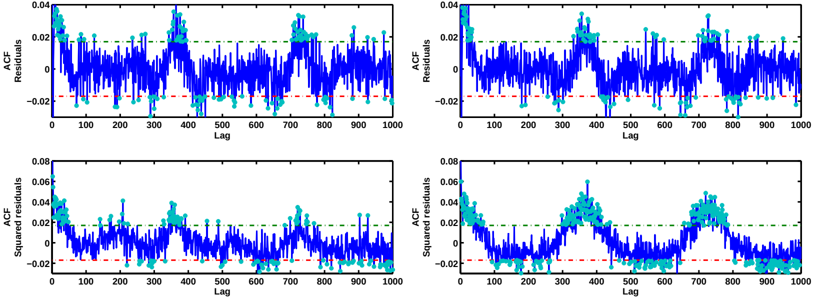


Figure 8: ACF for residuals $\hat{\varepsilon}_t$ (up) and squared residuals $\hat{\varepsilon}_t^2$ (down) of the AR(p) for Tokyo (left), Osaka (right)

Calibration of Seasonal Variance: σ_t^2

Calibration of daily variances of residuals AR(3) for 36 years:

- 2 Steps: Fourier truncated series + GARCH(p,q) $\hat{\sigma}^2_{t,FTSG}$

$$
\begin{aligned}
\sigma_t^2 &= c_1 + \sum_{i=1}^{16} \left\{ c_{2i} \cos\left(\frac{2i\pi t}{365}\right) + c_{2i+1} \sin\left(\frac{2i\pi t}{365}\right) \right\} \\
&+ \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (1)
\end{aligned}
$$

- 1 Step: Local linear Regression (LLR) $\hat{\sigma}^2_{t,LLR}$, $Y_i = \hat{\varepsilon}^2_{t_i}$

$$
\min_{a,b} \sum_{i=1}^{n} \{Y_i - a(t) - b(t)(t_i - t)\}^2 K\left(\frac{t_i - t}{h}\right) \qquad (2)
$$

Calibration of Seasonal Variance: σ_t^2

Calibration of daily variances of residuals AR(3) for 36 years:

	$\hat{c}_1$	$\hat{c}_2$	$\hat{c}_3$	$\hat{c}_4$	$\hat{c}_5$	$\hat{c}_6$	$\hat{c}_7$	α	β
Tokyo	3.91	-0.08	0.74	-0.70	-0.37	-0.13	-0.14	0.09	0.50
Osaka	3.40	0.76	0.81	-0.58	-0.29	-0.17	-0.07	0.04	0.52
Beijing	3.95	0.70	0.82	-0.26	-0.50	-0.20	-0.17	0.03	0.33
Taipei	3.54	1.49	1.62	-0.41	-0.19	0.03	-0.18	0.06	0.33

Table 8: First 7 Coefficients of σ_t^2 and $GARCH(p = 1, q = 1)$. The coefficients in black are significant at 1% level.

Seasonal Variance $\hat{\sigma}^2_{t,FTSG}$ and $\hat{\sigma}^2_{t,LLR}$: China - Taiwan

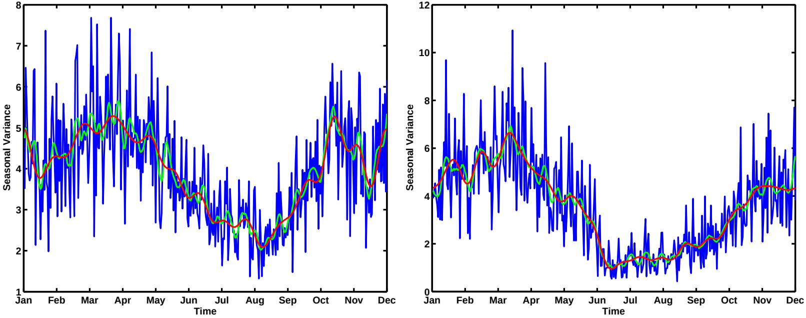


Figure 9: Daily empirical variance, $\hat{\sigma}^2_{t,FTSG}$, $\hat{\sigma}^2_{t,LLR}$ using Epanechnikov Kernel and bandwidth $h = 4.49$ for Beijing (left), Taipei (right).

Seasonal Variance $\hat{\sigma}^2_{t,FTSG}$ and $\hat{\sigma}^2_{t,LLR}$: Japan

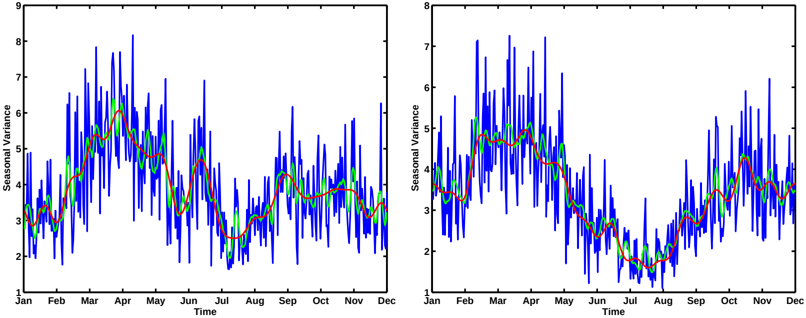


Figure 10: DDaily empirical variance, $\hat{\sigma}^2_{t,FTSG}$, $\hat{\sigma}^2_{t,LLR}$ using Epanechnikov Kernel and bandwidth $h = 3.79$ for Tokyo (left), Osaka (right).

ACF of (Squared) Residuals after Correcting Seasonal Volatility: China - Taiwan

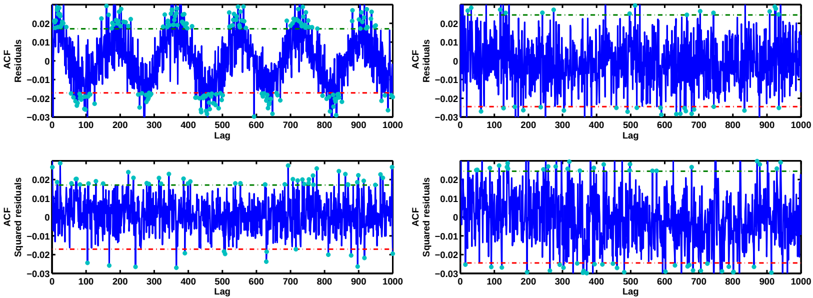


Figure 11: (Down) Up: ACF for temperature (squared) residuals $\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$ for Beijing (left), Taipei (right)

ACF of (Squared) Residuals after Correcting Seasonal Volatility: Japan

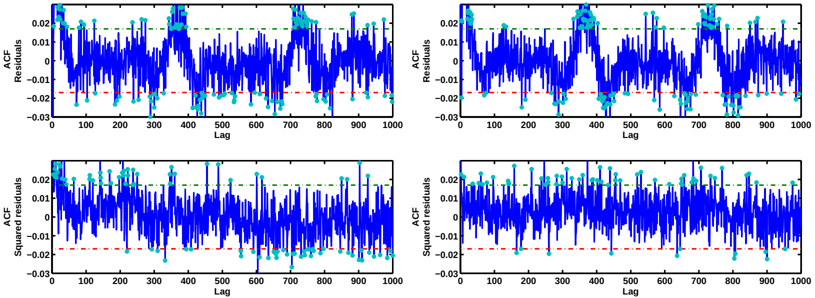


Figure 12: (Down) Up: ACF for temperature (squared) residuals $\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$ for Tokyo (left), Osaka (right)

Residuals $\left(\frac{\hat{\varepsilon}_t}{\hat{\sigma}_t}\right)$ become normal

City		$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTS}}$	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTSG}}$	$\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$
Tokyo	Jarque Bera	**6.49**	**5.30**	**4.68**
	Kurtosis	3.59	3.53	3.49
	Skewness	-0.14	-0.13	-0.13
Osaka	Jarque Bera	**7.25**	**6.35**	**6.25**
	Kurtosis	3.12	3.09	3.04
	Skewness	-0.34	-0.33	-0.32
Beijing	Jarque Bera	**8.03**	**7.67**	**6.98**
	Kurtosis	3.41	3.38	3.35
	Skewness	-0.30	-0.30	-0.29
Taipei	Jarque Bera	**12.47**	**11.57**	**11.00**
	Kurtosis	3.46	3.39	3.34
	Skewness	-0.39	-0.39	-0.39

Table 9: Skewness, kurtosis and values of Jarque Bera test statistics (365 days). Critical value at 5% significance level is 5.99, at 1% is -9.21.

Residuals ($\frac{\hat{\varepsilon}_t}{\hat{\sigma}_t}$) become normal:

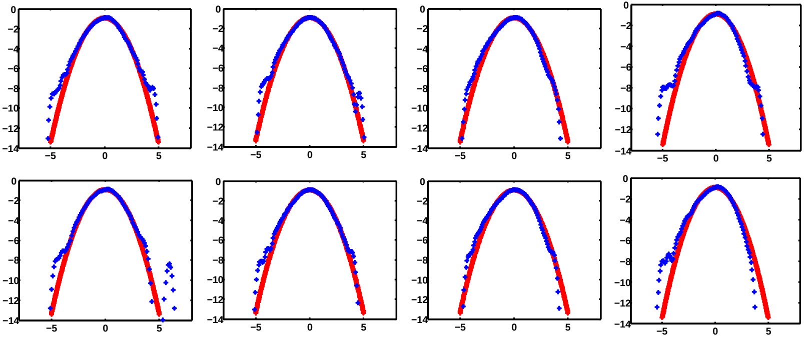


Figure 13: Log of Kernel smoothing density estimate vs Log of Normal Kernel for $\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,LLR}}$ (upper) and $\frac{\hat{\varepsilon}_t}{\hat{\sigma}_{t,FTSG}}$ (lower) of Tokyo (left), Osaka (left middle), Beijing (right middle), Taipei (right)

Temperature Dynamics

Temperature time series:

$$T_t = \Lambda_t + X_t$$

with seasonal function Λ_t. X_t can be seen as a discretization of a continuous-time process AR(p) (CAR(p)).

This stochastic model allows CAR(p) futures/options pricing.

Stochastic Pricing

Ornstein-Uhlenbeck process $\mathbf{X}_t \in \mathbb{R}^p$:

$$d\mathbf{X}_t = A\mathbf{X}_t dt + \mathbf{e}_p \sigma_t dB_t$$

$\mathbf{e}_k$: kth unit vector in $\mathbb{R}^p$ for $k = 1, ...p$, $\sigma_t > 0$, A: $(p \times p)$-matrix

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & \dots & \dots & 0 & 1 \\ -\alpha_p & -\alpha_{p-1} & \dots & & -\alpha_1 \end{pmatrix}$$

Stationarity condition

Solution of $\mathbf{X}_t = \mathbf{x} \in \mathbb{R}^p$, $s \geq t \geq 0$:

$$\mathbf{X}_s = \exp\{A(s-t)\}\mathbf{x} + \int_t^s \exp\{A(s-u)\}\mathbf{e}_p \sigma_u dB_u$$

is stationarity as long as all the eigenvalues $\lambda_1, \ldots, \lambda_p$ of A have negative real parts, i.e. the variance matrix:

$$\int_0^t \sigma_{t-s}^2 \exp\{A(s)\}\mathbf{e}_p \mathbf{e}_p^\top \exp\left\{A^\top(s)\right\} ds$$

converges as $t \to \infty$.

$\mathbf{X}_t$ can be written as a Continuous-time AR(p) (CAR(p)):

For $p = 1$,

$$dX_{1t} = -\alpha_1 X_{1t} dt + \sigma_t dB_t$$

For $p = 2$,

$$\begin{aligned} X_{1(t+2)} &\approx (2-\alpha_1) X_{1(t+1)} \\ &+ (\alpha_1 - \alpha_2 - 1) X_{1t} + \sigma_t (B_{t-1} - B_t) \end{aligned}$$

For $p = 3$,

$$\begin{aligned} X_{1(t+3)} &\approx (3-\alpha_1) X_{1(t+2)} + (2\alpha_1 - \alpha_2 - 3) X_{1(t+1)} \\ &+ (-\alpha_1 + \alpha_2 - \alpha_3 + 1) X_{1t} + \sigma_t (B_{t-1} - B_t) \end{aligned}$$

AR(3) → CAR(3)

	Coefficient	Tokyo(p=3)	Osaka(p=3)	Beijing(p=3)	Taipei(p=3)
AR	β_1	0.668	0.748	0.741	0.808
	β_2	-0.069	-0.143	-0.071	-0.228
	β_3	-0.079	-0.079	0.071	0.063
CAR	α_1	-2.332	-2.252	-2.259	-2.192
	α_2	1.733	-1.647	-1.589	-1.612
	α_3	-0.480	-0.474	-0.259	-0.357
Eigenvalues	λ_1	-1.257	-1.221	-0.231	-0.396
	$\lambda_{2,3}$	-0.537	-0.515	-1.013	-0.898

Table 10: Coefficients of (C)AR(p) (Berlin (p=3)), Model selection: AIC. real parts of eigenvalues of A are negative.

Temperature Futures Price

$\exists$ Q_θ pricing so that:

$$F_{(t,\tau_1,\tau_2)} = \mathsf{E}^{Q_\theta}\left[Y|\mathcal{F}_t\right] \tag{3}$$

where Y equals the payoff of the temperature index and by Girsanov theorem:

$$B_t^\theta = B_t - \int_0^t \theta_u du$$

is a Brownian motion for $t \leq \tau_{\max}$. θ: a real valued, bounded and piecewise continuous function (market price of risk)

Temperature Dynamics under Q_θ

Under Q_θ:

$$d\mathbf{X}_t = (A\mathbf{X}_t + \mathbf{e}_p \sigma_t \theta_t)dt + \mathbf{e}_p \sigma_t dB_t^\theta \tag{4}$$

with explicit dynamics, for $s \geq t \geq 0$:

$$\begin{aligned} \mathbf{X}_s &= \exp\{A(s-t)\}\mathbf{x} + \int_t^s \exp\{A(s-u)\}\mathbf{e}_p \sigma_u \theta_u du \\ &\quad + \int_t^s \exp\{A(s-u)\}\mathbf{e}_p \sigma_u dB_u^\theta \end{aligned} \tag{5}$$

AAT Futures

For $0 \leq t \leq \tau_1 < \tau_2$:

$$\begin{aligned} F_{AAT(t,\tau_1,\tau_2)} &= \mathrm{E}^{Q_\theta}\left[\int_{\tau_1}^{\tau_2} T_s ds | \mathcal{F}_t\right] \\ &= \int_{\tau_1}^{\tau_2} \Lambda_u du + \mathbf{a}_{t,\tau_1,\tau_2}\mathbf{X}_t + \int_t^{\tau_1} \theta_u \sigma_u \mathbf{a}_{t,\tau_1,\tau_2}\mathbf{e}_p du \\ &+ \int_{\tau_1}^{\tau_2} \theta_u \sigma_u \mathbf{e}_1^\top A^{-1}\left[\exp\{A(\tau_2 - u)\} - I_p\right]\mathbf{e}_p du \end{aligned} \quad (6)$$

with $\mathbf{a}_{t,\tau_1,\tau_2} = \mathbf{e}_1^\top A^{-1}\left[\exp\{A(\tau_2 - t)\} - \exp\{A(\tau_1 - t)\}\right]$, $I_p : p \times p$ identity matrix

Benth et al. (2007)

Constant MPR θ_t^i

$\hat{\theta}_t^i$ - constant for each contract i, $i = 1, 2 \ldots 7$ obtained as a solution to:

$$
\begin{aligned}
F_{AAT(t,\tau_1^i,\tau_2^i)} &\stackrel{!}{=} \int_{\tau_1^i}^{\tau_2^i} \hat{\Lambda}_u du - \hat{\mathbf{a}}_{t,\tau_1^i,\tau_2^i} \widehat{\mathbf{X}}_t - \theta_t^i \left\{ \int_t^{\tau_1^i} \hat{\sigma}_u \hat{\mathbf{a}}_{t,\tau_1^i,\tau_2^i} \mathbf{e}_p du \right. \\
&+ \left. \int_{\tau_1^i}^{\tau_2^i} \hat{\sigma}_u \mathbf{e}_1^\top A^{-1} \left[\exp\left\{ A(\tau_2^i - u) \right\} - I_p \right] \mathbf{e}_p du \right\}
\end{aligned}
$$

MPR General Case

$$
\begin{aligned}
\arg\min_{\gamma_k} \Sigma_{i=1}^{7} \quad \Bigg(\quad & F_{AAT(t,\tau_1^i,\tau_2^i)} - \int_{\tau_1^i}^{\tau_2^i} \hat{\Lambda}_u du - \hat{\mathbf{a}}_{t,\tau_1^i,\tau_2^i} \widehat{\mathbf{X}}_t \\
- \quad & \int_t^{\tau_1^i} \sum_{k=1}^{K} \gamma_k h_k(u_i) \hat{\sigma}_{u_i} \hat{\mathbf{a}}_{t,\tau_1^i,\tau_2^i} \mathbf{e}_p du_i \\
- \quad & \int_{\tau_1^i}^{\tau_2^i} \sum_{k=1}^{K} \gamma_k h_k(u_i) \hat{\sigma}_{u_i} \mathbf{e}_1^\top A^{-1} \left[\exp\left\{ A(\tau_2^i - u_i) \right\} \right. \\
- \quad & \left. I_p \right] \mathbf{e}_p du_i \Bigg)^2
\end{aligned}
\tag{7}
$$

where $h_k(u_i)$ is a vector of known basis functions, γ_k defines the coefficients. MPR changes in sign!!

MPR with Splines

$\hat{\theta}_t^{spl}$ – MPR as a solution to the minimization problem defined in (7), with $h_k(u_i)$ a B-spline basis $B_{i,p,\tau}$ of order p, $i = 1, 2, \ldots, n - p - 2$ and a knot sequence $\tau = (\tau_0, \ldots, \tau_{n-1})$.

$$
\begin{aligned}
B_{i,0,\tau}(u) &= \begin{cases} 1, & u \in [\tau_i, \tau_{i+1}] \\ 0, & \text{else} \end{cases}, \\
B_{i,p,\tau}(u) &= \frac{u - \tau_i}{\tau_{i+p} - \tau_i} B_{i,p-1,\tau}(u) + \frac{\tau_{i+p+1} - u}{\tau_{i+p+1} - \tau_{i+1}} B_{i+1,p-1,\tau}(u).
\end{aligned}
$$

To compute $\hat{\theta}_t^{spl}$ use order p =3 of B-splines with 7 knots corresponding to the number of traded contracts.

Tokyo & Osaka AAT Future Prices

City	Code	$F_{AATBloomberg}$	$F_{AAT,\hat{\theta}_t^0}$	$F_{AAT,\hat{\theta}_t^i}$	$F_{AAT,\hat{\theta}_t^{spl}}$
Tokyo	J9	450.000	452.125	450.000	461.213
	K9	592.000	630.895	592.000	640.744
Osaka	J9	460.000	456.498	460.000	-
	K9	627.000	663.823	627.000	-

Table 11: Tokyo & Osaka AAT future prices estimates on 20090130 from different MPR calibration methods: $F_{AAT,\hat{\theta}_t^0}$ with zero MPR, $F_{AAT,\hat{\theta}_t^i}$ with constant MPR, $F_{AAT,\hat{\theta}_t^{spl}}$ with spline MPR.

Parametrization of Constant MPR θ_t^i

- Average MPR over trading period – parameter depending on the risk source over the measurement period – temperature variation.
- Average MPR – average of the calibrated θ_t^i:

$$\hat{\theta}^i_{\tau_1,\tau_2} = \frac{1}{\tau_2 - t_{\tau_1,\tau_2}} \sum_{t=t_{\tau_1,\tau_2}}^{\tau_2} \hat{\theta}^i_t,$$

t_{τ_1,τ_2} is the first trading day of the measurement period $[\tau_1, \tau_2]$.

- Variation in period $[\tau_1, \tau_2]$:

$$\hat{\sigma}^2_{\tau_1,\tau_2} = \frac{1}{\tau_2 - \tau_1} \sum_{t=\tau_1}^{\tau_2} \hat{\sigma}^2_t.$$

- Regress $\hat{\theta}^i_{\tau_1,\tau_2}$ on $\hat{\sigma}^2_{\tau_1,\tau_2}$ to parametrize the dependence.

Parametrization of θ_t^i: Tokyo

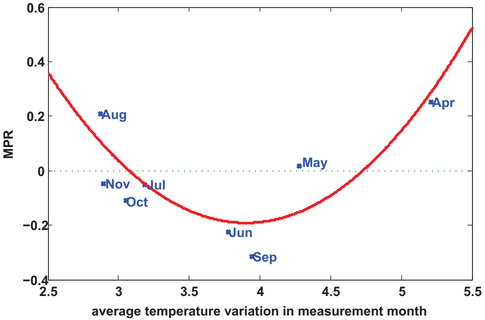


Figure 14: Calibrated MPR and Monthly Temperature Variation of AAT Tokyo Futures from November 2008 to November 2009 (prices for 8 contracts were available). MPR here is a nonmonotone quadratic function of $\hat{\sigma}^2_{\tau_1,\tau_2}$.

Parametrization of θ_t^i: Tokyo

Parameters	$\hat{\theta}_{\tau_1,\tau_2} = a + b\hat{\sigma}^2_{\tau_1,\tau_2} + c\hat{\sigma}^4_{\tau_1,\tau_2}$
a	4.08
b	-2.19
c	0.28
R^2_{adj}	0.71

Table 12: Parametrization of MPR for AAT Tokyo Futures.

Parametrization of θ_t^i: What is the Message?

- Example of Tokyo shows that even simple parametrization for $\hat{\theta}_{\tau_1,\tau_2}$ is possible.
- Infer MPR for regions without weather derivative markets knowing the formal dependence of MPR on seasonal variation.
- Uncertainty about spatial characteristics of MPR – parametrization using the closest location with organized weather derivative market.

Analysis of Weather Dynamics in Kaohsiung

Analysis of Weather Dynamics in Kaohsiung

Analysis of Weather Dynamics in 高雄市

1. Seasonal function with trend:

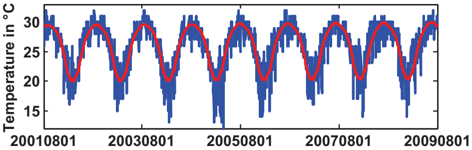
$$\begin{aligned}\hat{\Lambda}_t &= 24.4 + 16 \cdot 10^{-5} t + \sum_{i=1}^{3} \hat{c}_i \cdot \cos\left\{\frac{2\pi i(t - \hat{d}_i)}{365}\right\} \\ &+ \mathcal{I}(t \in \omega) \cdot \sum_{i=4}^{6} \hat{c}_i \cdot \cos\left\{\frac{2\pi (i-4)(t - \hat{d}_i)}{365}\right\},\end{aligned}$$

with $\mathcal{I}(t \in \omega)$ an indicator function taking value 1 for December, January and February and value zero else.

i	1	2	3	4	5	6
$\hat{c}_i$	5.11	-1.34	-0.39	0.61	0.56	0.34
$\hat{d}_i$	-162.64	19.56	16.72	28.86	16.63	21.84

Analysis of Weather Dynamics in 高雄市

1. Seasonal function with trend:

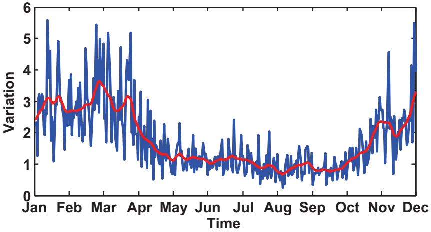


2. AR(p) process, by AIC p=3

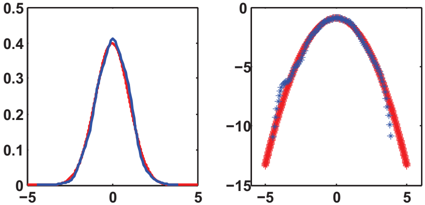
$$\hat{\beta}_1 = \ 0.77, \quad \hat{\beta}_2 = -0.12, \quad \hat{\beta}_3 = \ 0.04.$$

CAR(p) $\quad \hat{\alpha}_1 = -2.24, \quad \hat{\alpha}_2 = -1.59, \quad \hat{\alpha}_3 = -0.31.$

3. Seasonal volatility: Local Linear Regression (LLR)



4. Normality of residuals: kurtosis=3.32, skewness=-0.22, JB=4.41.



AAT Future Contracts for Kaohsiung

For $0 \leq t \leq \tau_1 < \tau_2$:

$$\begin{aligned} \widehat{F}_{AAT(t,\tau_1,\tau_2)} &= \int_{\tau_1}^{\tau_2} \hat{\Lambda}_u du + \hat{\mathbf{a}}_{t,\tau_1,\tau_2} \widehat{\mathbf{X}}_t + \int_t^{\tau_1} \hat{\theta}_{\tau_1,\tau_2} \hat{\sigma}_u \hat{\mathbf{a}}_{t,\tau_1,\tau_2} \mathbf{e}_p du \\ &+ \int_{\tau_1}^{\tau_2} \hat{\theta}_{\tau_1,\tau_2} \hat{\sigma}_u \mathbf{e}_1^\top A^{-1} \left[\exp\left\{A(\tau_2 - u)\right\} - I_p\right] \mathbf{e}_p du \quad (8) \end{aligned}$$

where $\hat{\theta}_{\tau_1,\tau_2} = 4.08 - 2.19 \cdot \hat{\sigma}^2_{\tau_1,\tau_2} + 0.28 \cdot \hat{\sigma}^4_{\tau_1,\tau_2}$. In this case $\hat{\sigma}^2_{\tau_1,\tau_2}$=1.10 $\rightarrow$ $\hat{\theta}_{\tau_1,\tau_2}$=2.01, and $\widehat{F}_{AAT(20090901,20091027,20091031)}$=139.60.

AAT Call Option

written on an AAT future with strike K at exercise time $\tau < \tau_1$ during period $[\tau_1, \tau_2]$.

$$\begin{aligned} C_{AAT(t,\tau,\tau_1,\tau_2)} &= \exp\{-r(\tau - t)\} \\ &\times \Big[\big(F_{AAT(t,\tau_1,\tau_2)} - K\big) \Phi\{d(t,\tau,\tau_1,\tau_2)\} \\ &+ \int_t^\tau \Sigma^2_{AAT(s,\tau_1,\tau_2)} ds \varphi\{d(t,\tau,\tau_1,\tau_2)\} \Big], \end{aligned}$$

$$d(t,\tau,\tau_1,\tau_2) = \frac{F_{AAT(t,\tau_1,\tau_2)} - K}{\sqrt{\int_t^\tau \Sigma^2_{AAT(s,\tau_1,\tau_2)} ds}},$$

$$\Sigma^2_{AAT(s,\tau_1,\tau_2)} = \sigma_t \mathbf{a}_{t,\tau_1,\tau_2} \mathbf{e}_p,$$

Φ and φ denote standard normal cdf and pfd respectively.

Weather Risk and Human Capital...

An investor organizes a conference on the 27-31 October 2009 in Kaohsiung. Since he knows there is another conference event that week, he estimates that every additional °C in excess of 135°C cumulated 24-hour average temperatures will reduce the number of participants in favor for the other conference and he will incur 2,500 JPY costs on human capital.

Weather Risk and Human Capital...

An investor organizes a conference on the 27-31 October 2009 in Kaohsiung. Since he knows there is another conference event that week, he estimates that every additional °C in excess of 135°C cumulated 24-hour average temperatures will reduce the number of participants in favor for the other conference and he will incur 2,500 JPY costs on human capital.

Example: the Human Capital Problem

Derivative	Parameters
index	AAT
r	4%
t	1. September 2009
measurement period	27-31. October 2009
strike	135°C
tick value	1°C=2,500 JPY
$\widehat{F}_{AAT(20090901,20091027,20091031)}$	139.60
$\widehat{C}_{AAT(20090901,20090908,20091027,20091031)}$	3.49
$\widehat{C}_{AAT(20090901,20090915,20091027,20091031)}$	2.64
$\widehat{C}_{AAT(20090901,20090922,20091027,20091031)}$	2.00
$\widehat{C}_{AAT(20090901,20090929,20091027,20091031)}$	1.51

Table 13: Call Options on AAT Future.

Hedging strategy for CAT call option

Delta of the call option:

$$\frac{\partial C_{AAT(t,\tau,\tau_1,\tau_2)}}{\partial F_{AAT(t,\tau_1,\tau_2)}} = \Phi\left\{d\left(t,\tau,\tau_1,\tau_2\right)\right\} \tag{9}$$

Hold: close to zero CAT futures when the option is far out of the money, otherwise close to 1.

Outlook

- Financial mathematics can be applied to Beijing, Taipei and Kaohsiung
- new solutions to abolish the remaining seasonality in the data
- $\hat{\theta}_t$ for CDD/HDD temperature futures/options: pricing of other exotic options
- long term (interannual) variability of parameters - capture volatility due to climate changes and urbanization.

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Pricing of Asian Temperature Risk

温度风险

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Appendix A

Li-McLeod Portmanteau Test– modified Portmanteau test statistic Q_L to check the uncorrelatedness of the residuals:

$$Q_L = n \sum_{k=1}^{L} r_k^2(\hat{\varepsilon}) + \frac{L(L+1)}{2n},$$

where r_k, $k = 1, \ldots, L$ are values of residuals ACF up to the first L lags and n is the sample size. Then,

$$Q_L \sim \chi^2_{(L-p-q)}$$

Q_L is χ^2distributed on $(L - p - q)$ degrees of freedom where p,q denote AR and MA order respectively and L is a given value of considered lags.

Appendix B

Proof **CAR(3)** $\approx$ **AR(3)**
Let

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_3 & -\alpha_2 & -\alpha_1 \end{pmatrix}$$

- use $B_{t+1} - B_t = \varepsilon_t$
- substitute iteratively into X_1 dynamics:

Appendix B

$$
\begin{aligned}
X_{1(t+1)} - X_{1(t)} &= X_{2(t)}dt + \sigma_t \varepsilon_t \\
X_{2(t+1)} - X_{2(t)} &= X_{3(t)}dt + \sigma_t \varepsilon_t \\
X_{3(t+1)} - X_{3(t)} &= -\alpha_1 X_{1(t)}dt - \alpha_2 X_{2(t)}dt - \alpha_3 X_{3(t)}dt + \sigma_t \varepsilon_t \\
X_{1(t+2)} - X_{1(t+1)} &= X_{2(t+1)}dt + \sigma_{t+1} \varepsilon_{t+1} \\
X_{2(t+2)} - X_{2(t+1)} &= X_{3(t+1)}dt + \sigma_{t+1} \varepsilon_{t+1} \\
X_{3(t+2)} - X_{3(t+1)} &= -\alpha_1 X_{1(t+1)}dt - \alpha_2 X_{2(t+1)}dt \\
&\quad -\alpha_3 X_{3(t+1)}dt + \sigma_{t+1} \varepsilon_{t+1} \\
X_{1(t+3)} - X_{1(t+2)} &= X_{2(t+2)}dt + \sigma_{t+2} \varepsilon_{t+2} \\
X_{2(t+3)} - X_{2(t+2)} &= X_{3(t+2)}dt + \sigma_{t+2} \varepsilon_{t+2} \\
X_{3(t+3)} - X_{3(t+2)} &= -\alpha_1 X_{1(t+2)}dt - \alpha_2 X_{2(t+2)}dt \\
&\quad -\alpha_3 X_{3(t+2)}dt + \sigma_{t+2} \varepsilon_{t+2}
\end{aligned}
$$