# Covered Bonds and Wholesale Debt Runs

Draft - Comments welcome

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**Abstract**

We develop a model of covered bonds and study their impact on the fragility of banks funded with wholesale debt. Covered bonds are a collateralized debt instrument where a bank encumbers assets on its balance sheet to raise additional funding for profitable investment. However, covered bonds concentrate credit risk onto unencumbered assets, which makes wholesale debt runs more likely. A bank's asset encumbrance choice balances the former bank funding channel with the latter risk concentration channel. Our results support covered bond regulation that limits asset encumbrance.

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## 1. Introduction

Covered bonds are a major source of bank funding.[1] Covered bonds have existed in continental Europe since the eighteenth century, and have maintained a track record of no default over that period. They received renewed attention in advanced and emerging economies after the financial crisis of 2007–09. In the United States, where the markets for mortgage-backed securities faced severe disruptions during the crisis, Bernanke (2009) suggests that covered bonds are a secure and viable alternative for bank funding.

Notwithstanding their two-and-a-half century history, there is surprisingly little research on the costs and benefits of covered bonds. We close this gap by proposing a general-equilibrium model of covered bonds. We study the impact of covered bond issuance on a bank's balance sheet and identify two distinct channels. First, additional investment occurs because inexpensive collateralized funding is raised (*bank funding channel*). Second, issuing covered bonds asymmetrically shifts credit risk onto the bank's uncollateralized creditors (*risk concentration channel*).

In the model, a bank attracts wholesale funding to finance profitable investment. Such funding is subject to rollover risk before investment matures. A credit shock renders some of the bank's assets non-performing. We adopt the global games approach of Rochet and Vives (2004), where the rollover decision is delegated to professional fund managers and derive a unique equilibrium characterized by a critical shock size above which a wholesale debt run occurs.[2] Importantly, we link the ex-post incidence of a wholesale debt run to the ex-ante issuance of covered bonds.

[1]The total amount of covered bonds outstanding was EUR 2,672 trillion in 2011, according to the ECBC. Furthermore, new issuance in that year was nearly EUR 700 billion, increasing by 75% since 2003.

[2]The literature on global games was pioneered by Carlsson and van Damme (1993); see also Frankel et al. (2003) and Morris and Shin (2003). Bank runs and liquidity crises in global games have previously been studied by Goldstein and Pauzner (2005) and Rochet and Vives (2004).

Covered bond funding is raised by *encumbering*, or ring-fencing, existing assets on the bank's balance sheet. This pool of encumbered assets backs covered bonds, thereby allowing the bank to raise additional funding from safety-seeking creditors. The more assets are encumbered, the more covered bonds are issued to fund profitable investment. This beneficial effect of covered bonds – the bank funding channel – reduces the incidence of a wholesale debt run and increases the expected value of bank equity.

There is, however, a countervailing effect stems from the seniority of covered bonds. One of their constituting features is the *dynamic replenishment* of the pool of encumbered assets.[3] After the credit shock is realized, non-performing asset in this pool are replaced by performing asset outside of the pool. While protecting the collateral of covered bonds, dynamic replenishment concentrates credit losses on the unencumbered part of the bank's balance sheet. Covered bond holders become de facto senior creditors, superseding unsecured wholesale debt holders. The more assets are encumbered, the larger is the impact of credit losses on wholesale debt holders. This detrimental effect of covered bonds – the risk concentration channel – increases the incidence of a wholesale debt run.

What is the bank's optimal choice for the amount of assets to encumbrance and covered bonds issuance? First, covered bond issuance has diminishing returns to scale, since covered bond holders are more likely to suffer credit losses as more assets are encumbered. Second, the probability of a wholesale debt run increases in the level of asset encumbrance. We provide sufficient conditions for the existence of a unique interior level of asset encumbrance that globally maximizes bank equity value. An illustrative example is shown in Figure 2.

The bank, as residual claimant, internalizes the impact of its encumbrance choice. Therefore, a limit on asset encumbrance naturally arises and does not depend on diminishing returns to investment or distorting features such as bail-outs and deposit insurance. Furthermore, the bank's choice of asset encumbrance is affected by conditions in the whole-

[3] For a description of institutional features of covered bonds, see section 1.1.

sale funding market. If wholesale debt is more expensive, or the rollover behavior is less conservative, bankruptcy occurs for a larger range of credit shocks. Since the bank's equity is wiped out in this case, it issues fewer covered bonds, as lowering the amount of asset encumbrance reduces the risk of a debt run. We label this the *equity preservation effect*.

Our results inform a current regulatory debate on asset encumbrance and covered bond legislation in North America.[4] A hallmark of regulation in Canada and the United States is a strict limit imposed on the levels of asset encumbrance that support covered bond issuance. Our general-equilibrium result provides a possible justification for this regulation as attempting to mitigate the risk concentration channel on banks' balance sheets. However, this does not necessarily imply that limits to encumbrance therefore help forestall financial crises ex ante. Indeed, as the bank funding channel suggests, covered bonds provide a cheap and stable form of bank funding. To the extent that socially profitable investment is made, the overall effect of covered bond funding can be a higher value of unencumbered assets, which are available to meet withdrawals from wholesale debt holders.

Much of the existing academic literature on covered bonds focuses on legislative differences in market structures across countries.[5] On the empirical side, Carbò-Valverde et al. (2011) examines the extent to which covered bonds can substitute for mortgage-backed securities. More recently, Prokopczuk et al. (2013) investigate how market liquidity and asset quality influence the pricing of covered bonds.

The theoretical literature on covered bonds and asset encumbrance is sparse but contains several recent contributions. Gai et al. (2013) investigate how secured debt and asset encumbrance influence the rollover risk of banks in a stylized global games set-up. The focus of the paper is on the effect of haircuts on rollover risk: higher haircuts increase the demand for asset encumbrance that, in turn, increases rollover risk. In a similar spirit, Eisenbach

[4] Haldane (2012) notes that, at high levels of encumbrance, the financial system is susceptible to pro-cyclical swings in the underlying value of banks' assets and prone to system-wide instability.

[5] See, for example, Schwarcz (2011) for a description of legal aspects of covered bonds. See Packer et al. (2007) for an overview of the covered bond market prior to the global financial crisis.

et al. (2014), investigate how higher haircuts for secured funding increases rollover risk, which is exogenously specified. Finally, Perotti and Matta (2014) investigate how short-term repo funding can affect unsecured short-term lending through the fire sales and haircuts channels. In contrast, our paper focuses on covered bonds and examines how their institutional features influence bank balance sheets. We propose a general-equilibrium model of covered bonds that accounts for the costs and benefits of asset encumbrance.

### *1.1. Institutional features of covered bonds*

Covered bonds are 'secured senior debt' typically used by banks. Covered bonds are secured by encumbering, or ring-fencing, a pool of high-quality assets – typically mortgages or public-sector loans – on the issuing bank's balance sheet. Unlike asset-backed securities, such as mortgage-backed securities, ring-fenced assets remain on the balance sheet and are placed within a bankruptcy-remote special purpose vehicle called the *cover pool*. As a consequence of holding these assets on balance sheet, regular capital requirements apply.

If the issuer experiences financial distress, covered bond holders have a preferential claim over the cover pool. Bankruptcy remoteness ensures that covered bond holders can always access their collateral. Moreover, the cover pool is *dynamic*, in the sense that a bank must replenish non-performing assets with performing ones of equal value and quality over the life of the bond in order to maintain the requisite collateralization. Covered bond holders are also protected by so-called *dual recourse*. If the value of the cover pool is insufficient to meet obligations in case of financial stress, covered bond holders have a claim of the shortfall on unencumbered assets, where this claim is equal in seniority to other debt holders. These institutional features, especially the dynamic ring fence, contribute to making covered bonds safe assets for investors and a cheap funding source for banks.

## 2. Model

The economy extends over three dates $t \in \{0,1,2\}$. It is populated by a banker, a unit mass of wholesale investors, and a large mass $\gamma \in (0,\infty)$ of long-term investors. There is universal risk-neutrality: the banker consumes at the final date, $U = C_2$, long-term investors consume at the initial or final date, $U = C_0 + C_2$, and wholesale investors consume at either date, $U = C_0 + C_1 + C_2$. At the initial date investors have a unit endowment, while the penniless banker has access to many profitable investment opportunities.

The banker attracts funding from investors at the initial date by offering a wholesale debt contract and a covered bond contract. As in Rochet and Vives (2004), wholesale debt requires a unit deposit at the initial date and can be withdrawn at the interim or final date. The face value of wholesale debt $D \in [1,\infty)$ is independent of the withdrawal date. The banker raises one unit of wholesale funding to invest the proceeds in high-quality assets (mortgages and government debt). Each unit of the asset yields a finite gross return $R > 1$ at the final date. Inefficient liquidation yields $\psi \in (0,1)$ of the final-date return, where $\psi R < 1$.[6]

Further funding is attracted at the initial date from long-term investors by issuing covered bonds. First, the banker ring-fences a fraction $\alpha \in [0,1]$ of assets, which is publicly observed. Second, the encumbered assets $\alpha$ are placed in a bankruptcy-remote vehicle, the cover pool, that remains on the bank's balance sheet. Third, these ring-fenced assets are valued marked-to-market and the final-date value of the cover pool is $CB \equiv \psi R\alpha < R\alpha$. This is known as over-collateralization. The banker raises $CB_0 \geq 0$ from issuing covered bonds with face value $CB$ and invests these proceeds in high-quality assets. Table 1 summarizes.

| | | |
|---|---|---|
| (cover pool) | $\alpha$ | $CB_0$ |
| (unencumbered assets) | $(1-\alpha)+CB_0$ | 1 |

Table 1: Balance sheet at $t = 0$

[6] This discount reflects the cost of physical liquidation, the relationship-specific knowledge of the lender lost when ownership is transferred (Diamond and Rajan (2001)), or an illiquidity discount due to fire sales (Shleifer and Vishny (1992)) or limited participation in asset markets (Allen and Gale (1994)).

A defining feature of covered bonds is the dynamic replenishment of the cover pool. The balance sheet suffers a shock $S \in \mathbb{R}$ at the final date, which is drawn from a continuous probability distribution function $f(S)$ with corresponding cumulative distribution function $F(S)$. The shock is bounded by assets on the balance sheet, so $S \leq R[1+CB_0]$. The banker, upon observing the realized shock $S$ at the interim date, must maintain the value of the cover pool at all dates. For example, the banker swap out any non-performing assets in the cover pool with performing unencumbered assets. While dynamic replenishment protects covered bond holders, the entire shock is concentrated on wholesale debt holders. Table 2 illustrates risk concentration on unencumbered assets after a small shock $S > 0$ and in the absence of wholesale debt runs.

| | | |
|---|---|---|
| (cover pool) | $R\alpha$ | $CB$ |
| (unencumbered assets) | $R[(1-\alpha)+CB_0]-S$ | $D$ |
| | | $E$ |

Table 2: Balance sheet at $t = 2$ (for a small shock and absent wholesale debt runs)

Bankruptcy occurs if the value of unencumbered assets is insufficient to repay wholesale debt. The bank is closed and each wholesale investor receives an equal share of liquidated unencumbered assets at the interim date:[7]

$$\min\left\{D\,,\,\psi\Big(R[(1-\alpha)+CB_0]-S\Big)\right\}. \tag{1}$$

The banker's equity value is zero in bankruptcy because of limited liability. Furthermore, the cover pool is liquidated to repay covered bond holder. If the banker is not bankrupt, then its equity is the value of investment net of debt payments:

$$E \equiv \max\left\{0\,,RCB_0-S-CB-D\right\}. \tag{2}$$

The banker maximizes the expected equity value.

[7]In bankruptcy, wholesale investors only access unencumbered assets. As Schwarcz (2011) suggests, wholesale investors have access to the excess value of the cover pool at the final date if and only if the bank is open.

*Rollover risk.* Following Rochet and Vives (2004), the rollover decision of wholesale investors is delegated to professional fund managers indexed by $i \in [0,1]$. Managers simultaneously decide whether to roll over funding at the interim date. If a proportion $\ell \in [0,1]$ refuses to roll over, the banker liquidates an amount $\ell D/\psi > \ell D$ to serve withdrawals. Consequently, bankruptcy occurs whenever:

$$R\left[\left((1-\alpha)+CB_0\right)\right]-S-\frac{\ell D}{\psi}<(1-\ell)D, \tag{3}$$

where the value of unencumbered assets is $R\left((1-\alpha)+CB_0\right)-S$ and the banker must serve $(1-\ell)D$ of withdrawals at the final date.

Rochet and Vives (2004) argue that the decision of fund managers is governed by their compensation.[8] In case of bankruptcy, the manager's relative compensation from rolling over is negative, $-c<0$. Otherwise, the relative compensation from rolling over is the benefit $b>0$. The conservativeness ratio $k \equiv c/(b+c) \in (0,1)$ summarizes the payoff parameters.

*Dominance regions.* Suppose all wholesale debt is rolled over, $\ell = 0$. Bankruptcy occurs whenever the shock is larger than a *bankruptcy threshold* $\overline{S}$:

$$\overline{S} \equiv R[(1-\alpha)+CB_0]-D. \tag{4}$$

It is a dominant strategy for fund managers not to roll over wholesale debt for $S>\overline{S}$.

Likewise, suppose no wholesale debt is rolled over, $\ell = 1$. Bankruptcy is avoided whenever the shock is smaller than a *liquidity threshold* $\underline{S}$:

$$\underline{S} \equiv R[(1-\alpha)+CB_0]-\frac{D}{\psi}<\overline{S}. \tag{5}$$

[8]This specification ensures global strategic complementarity in the rollover decisions of fund managers. Goldstein and Pauzner (2005) analyze a bank-run game with one-sided strategic complementarity.

It is a dominant strategy for fund managers to roll over wholesale debt for $S < \underline{S}$. The shock takes negative values with vanishing probability (formally, $f(S) = 0\ \forall S < 0$), so the upper and lower dominance regions are always well defined: $\underline{S} \geq -\frac{D}{\psi} > -\infty$ and $\overline{S} \leq R(1+\gamma) < \infty$ for all funding choices. Figure 1 shows the tripartite classification of the shock.

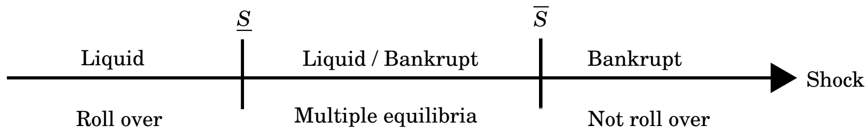


Figure 1: Tripartite classification of the shock

*Information.* There is incomplete information about the shock. At the interim date $S$ is drawn according to $f(S)$ and only observed by the banker. By contrast, each fund manager $i$ receives a noisy private signal:

$$x_i \equiv S + \epsilon_i. \tag{6}$$

The idiosyncratic noise terms $\epsilon_i$ are drawn from a continuous distribution $G$ with support over the interval $[-\epsilon, \epsilon]$, where $\epsilon > 0$. Idiosyncratic noise is independent of the shock and i.i.d. across fund managers. The realization of the shock is publicly observed at the final date. Figure 3 shows the timeline of the model.

| **Initial date** ($t = 0$) | **Interim date** ($t = 1$) | **Final date** ($t = 2$) |
|---|---|---|
| 1. Issue wholesale debt | 1. Banker learns shock $S$ | 1. Bank open if not bankrupt |
| 2. Investment | 2. Dynamic replenishment | 2. Debt payments $D$ and $CB$ |
| 3. Asset encumbrance $\alpha$ | 3. Noisy signals $x_i$ | 3. Residual is bank equity $E$ |
| 4. Issue covered bond $CB_0$ | 4. Wholesale debt withdrawal $\ell$ | |
| 5. Further investment | 5. Asset liquidation in bankruptcy | |

Table 3: Timeline of events.

## 3. Equilibrium

We solve the model by working backwards, starting with the rollover subgame between fund managers at the interim date. Each of these subgames is defined by the funding choices at the initial date: the face value of wholesale funding $D \in [1,\infty)$, the amount of covered bond funding $CB_0 \in [0,\gamma]$, and the proportion $\alpha \in [0,1]$ of encumbered assets. Proposition 1 summarizes the equilibrium in the rollover subgame.

**Proposition 1. Uniqueness in rollover subgame** *If private noise vanishes, $\epsilon \to 0$, then there exists a unique Bayesian equilibrium in each rollover subgame. It is characterized by a threshold of the shock $S^*$ and a threshold of the private signal $x^*$. Fund manager $i$ rolls over debt if and only if $x_i < x^*$ and bankruptcy occurs if and only if $S > S^*$:*

$$S^* = R\left[(1-\alpha)+CB_0\right]-\kappa D \in \left(\underline{S},\overline{S}\right) \tag{7}$$

*where $\kappa \equiv 1+k\left(\frac{1}{\psi}-1\right) \in \left(1,\frac{1}{\psi}\right)$ and $x^* \to S^*$.*

**Proof.** See Appendix A. ■

Raising more funding from covered bond investors increases the amount of unencumbered assets at the interim date, reducing the probability of default on unsecured wholesale debt for any given shock. Consequently, fund managers have more incentives to roll over and a wholesale debt run occurs for a smaller range of shocks. In contrast, greater asset encumbrance reduces the amount of unencumbered assets for any given shock, which induces wholesale fund managers not to roll over funding for a larger range of shocks. Finally, a larger face value of wholesale debt means a smaller range of shocks for which all fund managers are repaid in full. This raises the incentive of fund managers not to roll over wholesale funding and debt runs occur for a larger range of shocks. Corollary 1 summarizes.

**Corollary 1. Funding choices and rollover risk** *The critical shock size, at which a wholesale debt run occurs, decreases in the level of asset encumbrance and the face value of wholesale debt, while it increases in the funding raised from covered bond investors:*

$$\frac{\partial S^*}{\partial \alpha} = -R < 0, \qquad \frac{\partial S^*}{\partial CB_0} = R > 0, \qquad \frac{\partial S^*}{\partial D} = -\kappa < 0. \tag{8}$$

Having established the equilibrium in all rollover subgames at the interim date, we now analyse the optimal choice of covered bond funding at the initial date. The banker chooses the level of asset encumbrance and the issuance of covered bonds to maximize the expected value of equity subject to the participation constraint of covered bond investors, taking the face value of wholesale funding as given.

In a first step, we derive the expected value of the banker's equity. For a small shock, $S < S^*$, the banker's equity is $E(S) = R(1 + CB_0) - S - CB - D$, while bankruptcy occurs for a large shock, $S > S^*$. This yields the expected value of equity:

$$\pi \equiv \int_{-\infty}^{S^*} E(S)\, dF(S) = F(S^*)[R(1 + CB_0) - CB - D] - \int_{-\infty}^{S^*} S\, dF(S) \tag{9}$$

Second, we analyze the maximum amount of covered bond funding a bank can attract for a certain amount of asset encumbrance, which we denote as $CB_0^*(\alpha)$. This amount is derived from the participation constraint of covered bond investors:

$$1 \leq F\left(\tilde{S}\right)\left(\frac{CB}{CB_0}\right) + \int_{\tilde{S}}^{\hat{S}} \psi \frac{R(1 + CB_0) - S}{CB_0}\, dF(S). \tag{10}$$

The outside option is consumption at the initial date that yields 1. Investing in covered bonds buys a claim to a fraction $1/CB_0$ of the face value $CB$ backed by the covered pool. Absent bankruptcy, $S \leq S^*$, long-term investors are repaid in full and receive $CB$. Even in bankruptcy, covered bond investors can liquidate the cover pool to receive $CB$ at the final date, provided the shock is not too large, $S^* < S \leq \tilde{S} \equiv R[(1 - \alpha) + CB_0]$. However, a large shocks, $\tilde{S} < S \leq \hat{S} \equiv R[1+CB_0]$, also wipes out part of the cover pool, so long-term investors only receive their fraction of its liquidation value $\psi[R(1+CB_0)-S]$. Proposition 2 states the amount of covered bond funding and its dependence on the level of asset encumbrance.

**Proposition 2. Bank funding channel** *Suppose that long-term investors are sufficiently abundant, $\gamma \geq \underline{\gamma} \equiv CB_0^*(1)$. For any given amount of asset encumbrance $\alpha \in [0,1]$, there exists a unique amount of covered bond funding $CB_0^*(\alpha) \in [0,\gamma]$ raised by the banker:*

$$CB_0^*(\alpha) = F\left(R[(1-\alpha)+CB_0^*(\alpha)]\right)\alpha\psi R + \psi\int_{R[(1-\alpha)+CB_0^*(\alpha)]}^{R[1+CB_0^*(\alpha)]}\Big(R(1+CB_0^*(\alpha))-S\Big)\,dF(S). \quad (11)$$

*While greater asset encumbrance increases the funding attracted from long-term investors with covered bonds:*

$$\frac{dCB_0^*(\alpha)}{d\alpha} \equiv \frac{F\Big(R[(1-\alpha)+CB_0^*(\alpha)]\Big)}{F\Big(R[(1-\alpha)+CB_0^*(\alpha)]\Big)+\frac{1}{\psi R}-1} \in (0,1), \quad (12)$$

*there are diminishing returns to scale:*

$$\frac{d^2CB_0^*(\alpha)}{d\alpha^2} = \frac{\psi R^2 \times f\left(R\left[1-\alpha+CB_0^*(\alpha)\right]\right)\times\left(\frac{dCB_0^*(\alpha)}{d\alpha}-1\right)\times\left(1-\psi R\right)}{\left[1-\psi R\left(1-F\left(R\left[1-\alpha+CB_0^*(\alpha)\right]\right)\right)\right]^2} < 0. \quad (13)$$

**Proof.** See Appendix B. ■

Proposition 2 states that as the banker encumbers more assets, it can attract more funding from long-term investors. This leads to an expansion of the balance sheet, more profitable investment, and greater expected equity value. However, this channel is characterized by a diminishing returns to scale. For a given $CB_0^*$, greater asset encumbrance increases the range of shocks where covered bond holders suffer a loss. This negative influence reduces the positive effect of balance sheet expansion, yielding diminishing returns.

**Proposition 3. Risk concentration channel** *The total effect of asset encumbrance on the critical shock size $S^*(\alpha, CB_0^*(\alpha))$ is negative:*

$$\frac{dS^*}{d\alpha} = \underbrace{\frac{\partial S^*}{\partial \alpha}}_{\textit{Risk}} + \underbrace{\frac{\partial S^*}{\partial CB_0^*}\frac{dCB_0^*}{d\alpha}}_{\textit{Funding}} = R\left(\frac{dCB_0^*}{d\alpha} - 1\right) < 0. \tag{14}$$

**Proof.** See Appendix C. ■

In contrast to the bank funding channel, the risk concentration channel of Proposition 3 betrays the negative influence of asset encumbrance. By concentrating the shock on wholesale debt holders, greater asset encumbrance leads to higher instances of wholesale bank runs. This is true even after accounting for the indirect beneficial influence of enhanced bank funding, and thus greater investment, on the critical shock size.

The banker encumbers assets to maximize its expected equity value. In doing so, it considers the effects of asset encumbrance on risk concentration and bank funding:

$$\max_{\alpha \in [0,1]} \pi(\alpha, CB_0^*(\alpha)) \tag{15}$$

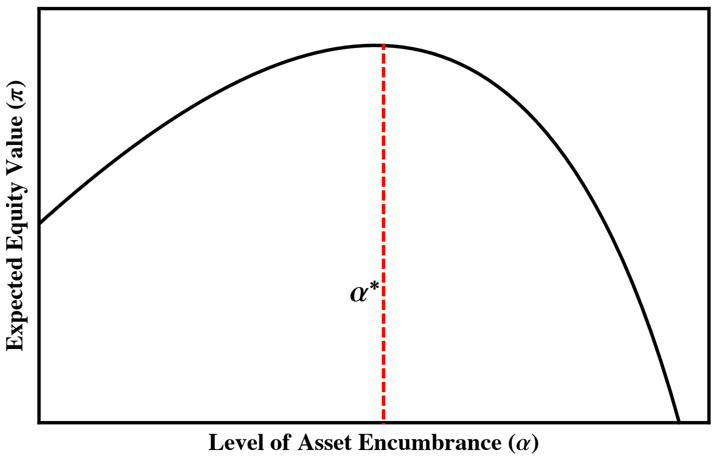


Figure 2: Expected equity value as a function of the level of asset encumbrance. Additional parameters were $R = 3$, $\psi = 0.2$, $k = 0.025$, and $D = 1.1$, where the shock was exponentially distributed with intensity $\lambda = 1.1$.

Figure 2 illustrates how the banker's expected equity value $\pi(\alpha, CB_0^*(\alpha))$ changes with asset encumbrance for an exponential distribution of the credit shock. The objective function is strictly concave and has a global maximum at the optimal interior level of asset encumbrance. In the following proposition, we provide sufficient conditions for a general class of shock distributions that yield optimal and positive levels of asset encumbrance.

**Proposition 4. Optimal asset encumbrance** *If long-term investors are sufficiently abundant, $\gamma \geq \underline{\gamma}$, and the balance sheet shock satisfies $f'(S) \leq 0$, then there exists a unique choice of asset encumbrance $\alpha^* \in [0,1]$, and a corresponding amount of covered bond funding $CB_0^*(\alpha^*)$. If $R(1-\psi)F(R) < 1 - R\psi$, then no assets are encumbered, $\alpha^* = 0$. If $R(1-\psi)F\left(R\underline{\gamma}\right) \geq 1 - R\psi$, an interior solution of encumbrance is optimal, $\alpha^* \in (0,1)$, which is implicitly defined by:*

$$F(S^*(\alpha^*))\left(\frac{dCB_0^*(\alpha^*)}{d\alpha} - \psi\right) + f(S^*(\alpha^*))[(\kappa-1)D + \alpha^*(1-\psi)R]\left(\frac{dCB_0^*(\alpha^*)}{d\alpha} - 1\right) = 0. \qquad (16)$$

**Proof.** See Appendix C. ■

In what follows, we focus on the interior solution, $\alpha^* \in (0,1)$, and how this is influenced by the wholesale debt market. As wholesale debt becomes more expensive (higher $D$), this decreases the critical shock size, $\partial S^*/\partial D < 0$, and thereby increases the likelihood of a run (Corollary 1). We label this the **coordination effect**. However, since inefficient liquidation of investment occurs in the event of bankruptcy and all equity value is wiped out, the banker wishes to avoid these outcomes. Hence, more expensive wholesale funding leads to fewer asset encumbrance. We label this the **equity preservation effect**.

**Proposition 5. Equity preservation effect** *More expensive wholesale debt reduces the optimal amount of asset encumbrance:*

$$\frac{d\alpha^*}{dD} < 0. \qquad (17)$$

*The net effect of more expensive wholesale debt on the rollover decisions of fund managers is:*

$$\frac{dS^*}{dD} = \underbrace{\frac{\partial S^*}{\partial D}}_{\textit{Coordination}} + \underbrace{\frac{dS^*}{d\alpha^*}\frac{d\alpha^*}{dD}}_{\textit{Equity Preservation}} \lesseqgtr 0 \tag{18}$$

**Proof.** See Appendix D. ■

Identical results are derived were we to vary the conservativeness ratio, $k$, which together with the face value of wholesale debt, drives the decisions of fund managers to rollover debt at the interim date. Both more expensive wholesale debt and less conservative fund managers lead to more coordination failure and thus a wholesale debt run for a larger range of credit shocks. For the banker to preserve his equity, it issues fewer covered bonds since the lower amount of asset encumbrance will reduce the risk of a wholesale debt crisis.

## 4. Discussion

### *4.1. Moral hazard*

In our model, the banker observes the realized credit shock $S$ at the interim date and replenishes the cover pool accordingly. Could the banker misreport the credit shock to increase the bank's equity value? We argue that two features of the model prevent such an outcome. First, the threat of bankruptcy is a deterrent to the banker misreporting the credit shock. In the event of bankruptcy, the banker is stripped of all assets and the equity value is zero. In the limit of vanishing private noise, fund managers have precise information regarding the credit shock. While not formally modeled, the demandable debt controlled by the fund managers therefore disciplines the banker (Calomiris and Kahn (1991); Diamond and Rajan (2001)).[9] Second, the over-collateralization of the cover pool contains the banker's

[9] Other reasons for demandable debt to arise endogenously are the presence of idiosyncratic liquidity risk (e.g., Diamond and Dybvig (1983)) and demand for absolutely safe claims (e.g., Caballero and Farhi (2013); Gennaioli et al. (2013); Ahnert and Perotti (2014)).

incentives to misreport, as it would lose its claim on the value of the cover pool at maturity.

*4.2. Asset heterogeneity*

Covered bonds are typically backed by a mixture of public debt and mortgages. To the extent that the risk profile of these asset classes differs, the replenishment of the cover pool after a credit shock also changes the composition of unencumbered assets and its risk profile. If low risk assets are swapped into the cover pool first, the risk concentration on wholesale debt holders is exacerbated. Thus, asset heterogeneity is a potential source of amplification of wholesale debt runs. However, our current setup with a homogeneous asset already captures the asymmetric credit risk transfer of credit risk that comes with the replenishment of the cover pool.

*4.3. Competition for funding*

To distill some main insights of asset encumbrance in the context of bank funding, our approach assumes a monopolistic banker. Under other market structures, the equilibrium amount of covered bond funding raised by encumbering assets may be lower. While having a quantitatively effect, our main insights remain unchanged. Encumbering assets allows banks to raise covered bond funding that can be invested in profitable projects. The optimal amount of asset encumbrance balances the risk concentration channel with the bank funding channel. Since a smaller amount of covered bond funding raised lowers the bank funding channel, competition for bank funding reduces the optimal amount of asset encumbrance.

*4.4. Cash holdings and bank portfolio choice*

Another extension concerns the banker's optimal asset portfolio choice at the initial date. Instead of investing the proceeds from covered bond funding into the risky asset, the banker could instead invest a fraction in liquid assets. Since such assets are not subject to fire sales, they help deter runs by wholesale investors. This constitutes the benefit of ex-ante liquidity holdings. However, liquid assets have a lower expected return than long-term investment, which constitutes the opportunity cost of ex-ante liquidity holdings. In the context of rollover risk without asset encumbrance, Ahnert (2014) analyzes the optimal trade-off of preventing fire sales at the cost of a low expected return.

### *4.5. Bail-outs and government guarantees*

## 5. Conclusion

Following the collapse of Lehman Brothers in September 2008, and the freezing up of unsecured debt markets, banks have increasingly looked to secured debt such as covered bonds to meet funding requirements. Our paper contributes to an understanding of how these markets can affect financial stability. Specifically, we develop a model of covered bonds and study their impact on the fragility of banks funded with wholesale debt. We decompose the influence of covered bonds into two channels: risk concentration and bank funding. The banker in our model fully internalizes these channels in setting the optimal level of asset encumbrance and covered bond funding.

While illustrative, our results support the view taken by policymakers in Canada and the United States on having limits to asset encumbrance for covered bonds. From a macro-prudential perspective as well, a limit on asset encumbrance will limit negative externalities – such as fire sales and systemic risk – that may follow bankruptcy of a financial institution. Further analysis on the systemic implications of covered bonds is left for future work.

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## Appendix A. Proof of Proposition 1

The limit of vanishing private noise, $\epsilon \to 0$, is sufficient to establish the existence of a unique Bayesian equilibrium in each rollover subgame, which is in threshold strategies.[10] Each fund manager $i$ uses a threshold strategy, whereby wholesale debt is rolled over if and only if the private signal suggests that the shock is small, $x_i < x^*$. Hence, for a given realization $S \in [\underline{S}, \overline{S}]$, the proportion of fund managers who do not roll over debt is:

$$\ell\left(S, x^*\right) = \text{Prob}\left(x_i > x^* | S\right) = \text{Prob}\left(\epsilon_i > x^* - S\right) = 1 - G\left(x^* - S\right).$$

The critical mass condition states that bankruptcy occurs when the shock reaches a threshold $S^*$, where the proportion of wholesale debt that is not rolled over is evaluated at $S^*$:

$$R\left[(1-\alpha) + CB_0\right] - S^* - \ell\left(S^*, x^*\right)\frac{D}{\psi} = \left(1 - \ell\left(S^*, x^*\right)\right)D \tag{A.1}$$

the posterior distribution of the shock conditional on the private signal is derived using Bayes' rule. The indifference condition states that the fund manager who receives the critical signal $x_i = x^*$ is indifferent between rolling over and not rolling over wholesale debt:

$$k = \Pr\left(S < S^* | x_i = x^*\right). \tag{A.2}$$

Using the definition of the private signal $x_j = S + \epsilon_j$ of the indifferent fund manager, we can state the conditional probability as follows:

[10]Morris and Shin (2003) show that only threshold strategies survive the iterated deletion of strictly dominated strategies. See also Frankel et al. (2003).

$$1-k = \Pr\left(S \geq S^* | x_i = x^*\right) \tag{A.3}$$

$$= \Pr\left(S \geq S^* | x_i = x^* = S + \epsilon_j\right) \tag{A.4}$$

$$= \Pr\left(x^* - \epsilon_j \geq S^*\right) \tag{A.5}$$

$$= \Pr\left(\epsilon_j \leq x^* - S^*\right) \tag{A.6}$$

$$= G\left(x^* - S^*\right) \tag{A.7}$$

Therefore, the indifference condition implies that $x^* - S^* = G^{-1}\left(1-k\right)$. Inserting the indifference condition into $\ell\left(S^*, x^*\right)$, the proportion of fund managers who do not roll over when the shock is at the critical level $S^*$ is perceived by the threshold fund manager to be:

$$\ell\left(S^*, x_i = x^*\right) = 1 - G\left(x^* - S^*\right) = 1 - G\left(G^{-1}\left(1-k\right)\right) = k. \tag{A.8}$$

Therefore, the threshold of the shock is $S^* = R\left[(1-\alpha) + CB_0\right] - \kappa D$. If private noise vanishes, the signal threshold also converges to this value. □

## Appendix B. Proof of Proposition 2

The derivative of the banker's expected profit with respect to the funding raised from covered bonds issuance is:

$$\frac{\partial \pi}{\partial CB_0} = RF(S^*) + Rf(S^*)\left[(\kappa - 1)D + (1-\psi)\alpha R\right] > 0 \tag{B.1}$$

for all feasible values of $\alpha$, $CB_0$, and $D$. Therefore, the banker raises as much funding from covered bond investors as possible.

The participation constraint of long-term investors can be written as:

$$CB_0 \leq F\left(\tilde{S}\right)CB + \int_{\tilde{S}}^{\hat{S}} g(S)\,dF(S), \tag{B.2}$$

where $g(S) = \psi[R(1+CB_0) - S]$, which implies that $g\left(\tilde{S}\right) = CB$ and $g\left(\hat{S}\right) = 0$. Taking the derivatives with respect to the funding from covered bond investors, the left-hand side has a unit slope, while the right-hand side's slope is (by Leibniz rule):

$$\begin{aligned}\frac{dRHS}{dCB_0} &= f\left(\tilde{S}\right)CB\frac{d\tilde{S}}{dCB_0} - g\left(\tilde{S}\right)f\left(\tilde{S}\right)\frac{d\tilde{S}}{dCB_0} - g\left(\hat{S}\right)f\left(\hat{S}\right)\frac{d\hat{S}}{dCB_0} + \psi R\left(F\left(\hat{S}\right) - F\left(\tilde{S}\right)\right)\\ &= \psi R\left(1 - F\left(\tilde{S}\right)\right) \geq 0 \end{aligned} \tag{B.3}$$

since $F(\hat{S}) = 1$. Moreover, since $\psi R < 1$, we have that $\frac{dRHS}{dCB_0} < 1$. It thus follows that as $CB_0$ increases, the lefthand side of the participation constraint increases faster than the right-hand side. Hence, if long-term investors are sufficiently abundant, there exists a unique solution $CB_0^*(\alpha)$ given by the binding participation constraint:

$$CB_0^*(\alpha) = F\left(R[(1-\alpha) + CB_0^*(\alpha)]\right)\alpha\psi R + \psi\int_{R[(1-\alpha)+CB_0^*(\alpha)]}^{R[1+CB_0^*(\alpha)]}\left(R(1+CB_0^*(\alpha)) - S\right)dF(S), \tag{B.4}$$

Naturally, we have that $CB_0^*(0) = 0$. Finally, observing that the righthand side of the participation constraint increases in the level of asset encumbrance $\alpha$:

$$\frac{RHS}{d\alpha} = \psi RF\left(\tilde{S}\right) > 0 \tag{B.5}$$

it follows that the greater asset encumbrance leads to the banker raising more funding by issuing covered bonds:

$$\frac{dCB_0^*(\alpha)}{d\alpha} \equiv \frac{F\Big(R[(1-\alpha) + CB_0^*(\alpha)]\Big)}{\frac{1}{\psi R} - 1 + F\Big(R[(1-\alpha) + CB_0^*(\alpha)]\Big)} > 0 \tag{B.6}$$

For long-term investors to be sufficiently abundant, we assume that $\gamma \geq \underline{\gamma} \equiv CB_0^*(1)$, where the lower bound is implicitly and uniquely defined by:

$$\underline{\gamma} = \psi R F\left(R\underline{\gamma}\right) + \psi \int_{R\underline{\gamma}}^{R[1+\underline{\gamma}]} \left(R(1+\underline{\gamma}) - S\right) dF(S). \tag{B.7}$$

**Appendix C. Proof of Proposition 4**

Taking the Total derivative of the threshold $S^*(\alpha, CB_0^*(\alpha))$ with respect to the level of asset encumbrance yields:

$$\frac{dS^*(\alpha)}{d\alpha} = R\left(-1 + \frac{dCB_0^*(\alpha)}{d\alpha}\right) = R\left[-1 + \frac{F\left(R[(1-\alpha)+CB_0^*(\alpha)]\right)}{\frac{1}{\psi R} - F\left(R[1+CB_0^*(\alpha)]\right) + F\left(R[(1-\alpha)+CB_0^*(\alpha)]\right)}\right] \tag{C.1}$$

$$= R\frac{1-\frac{1}{\psi R}}{\frac{1}{\psi R} - 1 + F\left(R[(1-\alpha)+CB_0^*(\alpha)]\right)} < 0 \tag{C.2}$$

Taking the risk concentration channel – $dS^*/d\alpha < 0$ – and bank funding channel – $dCB_0^*/d\alpha > 0$ – into account, the banker chooses $\alpha \in [0,1]$ to maximize his expected equity value. This yields the following first-order condition:

$$F(S^*)\left(\frac{dCB_0^*(\alpha)}{d\alpha} - \psi\right) + f(S^*)[(\kappa-1)D + \alpha^*(1-\psi)R]\left(\frac{dCB_0^*(\alpha)}{d\alpha} - 1\right) = 0. \tag{C.3}$$

There are two possible solutions. First, $\alpha^* = 0$ if $\frac{dCB_0^*(0)}{d\alpha} \leq \psi$. Since $\frac{dCB_0^*(0)}{d\alpha} < 1$, the inequality is toughest to hold for $\alpha = 0$. This yields the sufficient condition is $F(R) \leq \frac{1-R\psi}{R(1-\psi)} \in (0,1)$. Second, an interior solution $\alpha^* \in (0,1)$ requires that $\frac{dCB_0^*(0)}{d\alpha} > \psi$. Since $\frac{dCB_0^*(0)}{d\alpha} < 1$ for $\psi R < 1$, the inequality is toughest to hold for $\alpha = 1$, which yields the sufficient condition is $F(R\underline{\gamma}) > \frac{1-R\psi}{R(1-\psi)} \in (0,1)$.

Concentrating on the interior solution, the next step is to show that this is indeed a maximum of the banker's expected equity value. The second derivative of the equity value with respect to $\alpha$ is:

$$\begin{aligned}\frac{1}{R}\frac{d^2\pi}{d\alpha^2} &= \frac{d^2CB_0^*}{d\alpha^2}\left\{F(S^*)+f(S^*)\left[\alpha^*R(1-\psi)+D(\kappa-1)\right]\right\} \\ &+ \left(\frac{dCB_0^*}{d\alpha}-1\right)\left\{f(S^*)\left[\frac{dS^*}{d\alpha}+R(1-\psi)\right]+f'(S^*)\frac{dS^*}{d\alpha}\left[\alpha^*R(1-\psi)+D(\kappa-1)\right]\right\} \\ &+ \frac{dS^*}{d\alpha}f(S^*)(1-\psi).\end{aligned}$$

The third term is strictly negative since $dS^*/d\alpha < 0$. For the second term, since $dCB_0^*/d\alpha < 1$, it too is negative if the expression in the parenthesis is positive. Since $f' \leq 0$, a sufficient condition is $dCB_0^*/d\alpha - 1 + R(1-\psi) > 0$. This, however, is guaranteed by the sufficient condition on the shock distribution for an interior $\alpha^* \in (0,1)$ solution. Finally, the amount of covered bond funding raised as more assets are encumbered satisfied decreasing returns to scale, the banker's expected equity value function is strictly concave, and the choice $\alpha^*$ is the global maximum.

## Appendix D. Proof of Proposition 5

The total effect of changes in the face value of wholesale debt comprise the direct effect $\frac{\partial S^*}{\partial D} = -\kappa$ and the indirect effect via the changes of the optimal asset encumbrance choice:

$$\frac{dS^*}{dD} = \frac{\partial S^*}{\partial D} + \frac{dS^*}{d\alpha^*}\frac{d\alpha^*}{dD} \lesseqgtr 0 \tag{D.1}$$

Consider the sufficient conditions for the interior optimum, so $\alpha^* \in (0,1)$. From Appendix C it follows that $\frac{dS^*}{d\alpha^*} < 0$. For the derivative of the optimal level of encumbrance with respect to the face value of debt, it follows from the implicit function theorem that

$$\frac{d\alpha^*}{dD} = -\frac{\frac{d^2\pi}{d\alpha dD}}{\frac{d^2\pi}{d\alpha^2}}. \tag{D.2}$$

The denominator is the curvature of the expected equity value, which from Proposition 3 we know to be strictly concave, i.e., $\frac{d^2\pi}{d\alpha^2} < 0$. For the numerator, we obtain:

$$\frac{d^2\pi}{d\alpha dD} = f(S^*(\alpha^*))(\kappa - 1)\left(\frac{dCB_0^*(\alpha^*)}{d\alpha} - 1\right) < 0\,, \tag{D.3}$$

implying that $d\alpha^* / dD < 0$. Thus, the direct and indirect effects oppose each other, leading to an ambiguous result for the sign of $dS^*/dD$.