How to tame CDOs?

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Collateralized Debt Obligation

- **Synthetic investment**
- **Investor: interest income**
- **Reduction of statistical outliers**
- **Triggered the financial crisis**

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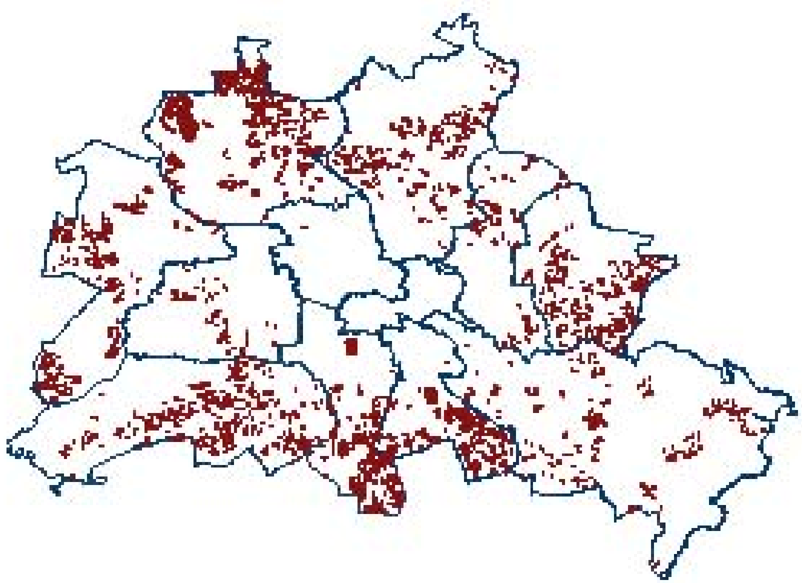
CDO construction

- Risk transfer
- Portfolio: fixed income assets
- Special purpose vehicle (SPV)
- NO residual risk for originator



Berlin

Single family detached homes

CDO (genetic) flaws

- Risk comes in tranches
- Substantial fees on issuance
- Failure of rating agencies
- Liquidity: mark to market risk

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Collateralized Debt Obligation

- CDOs are interesting risk transfer vehicles
- CDOs are potentially fulminating investments
- Deeper understanding needed

Canis lupus

Wolf

Vlk

Wilk

Collateralized Debt Obligation

- **CDOs are interesting risk transfer vehicles**
- **CDOs are potentially fulminating investments**
- **How to tame CDO (wolfs)?**

Canis lupus forma familiaris

Schäferhund

Německý ovčák

Owczarek niemiecki

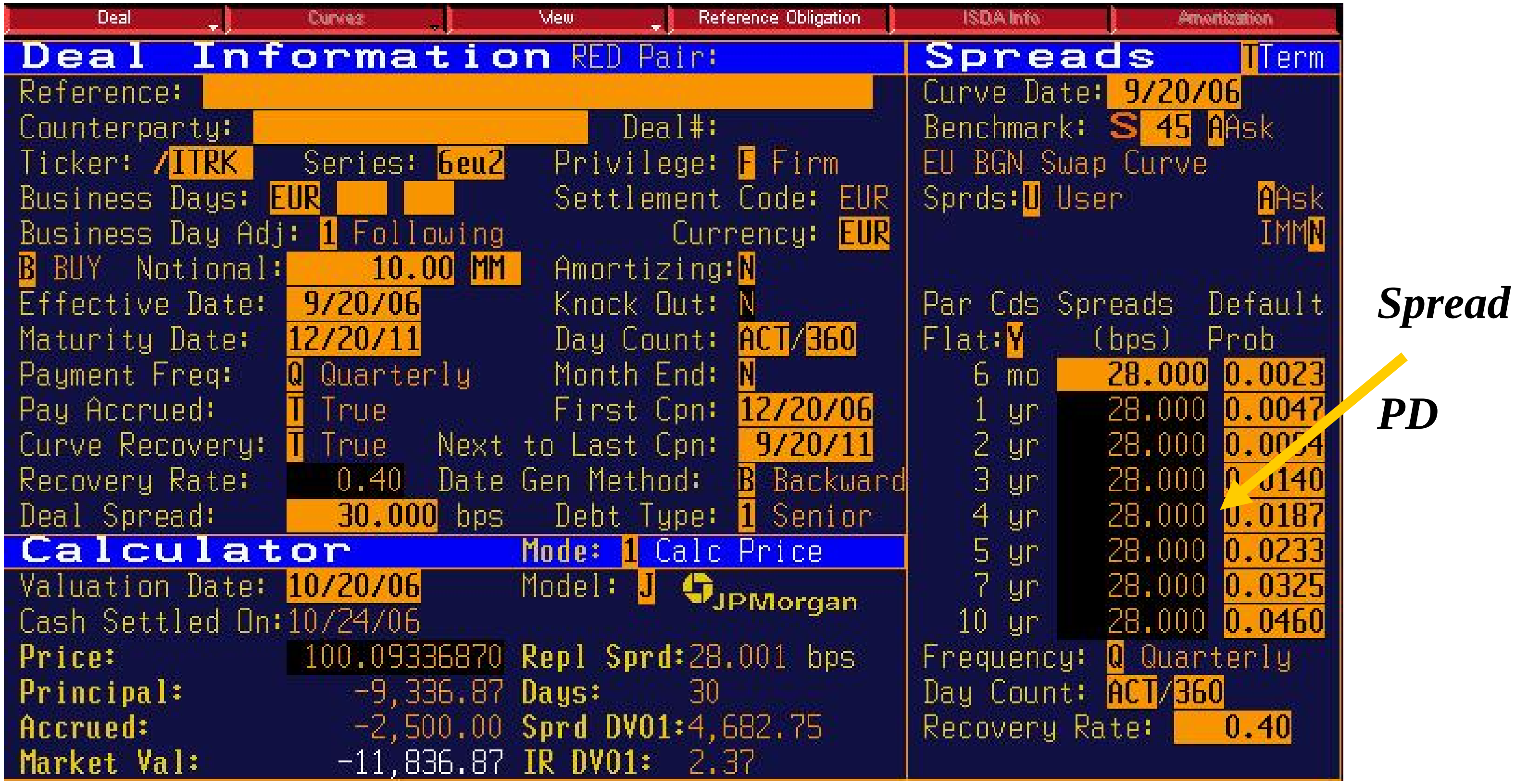
Outline

- Motivation ✓
- History
- Construction
- Pricing
- Rating
- Taming

History

- 1987 first CDO by Drexel Burnham Lambert
- 2000 Gaussian ONE factor copula model
- 2004 157G USD
- 2005 272G USD
- 2006 552G USD
- 2007 503G USD
- 2008 …

History

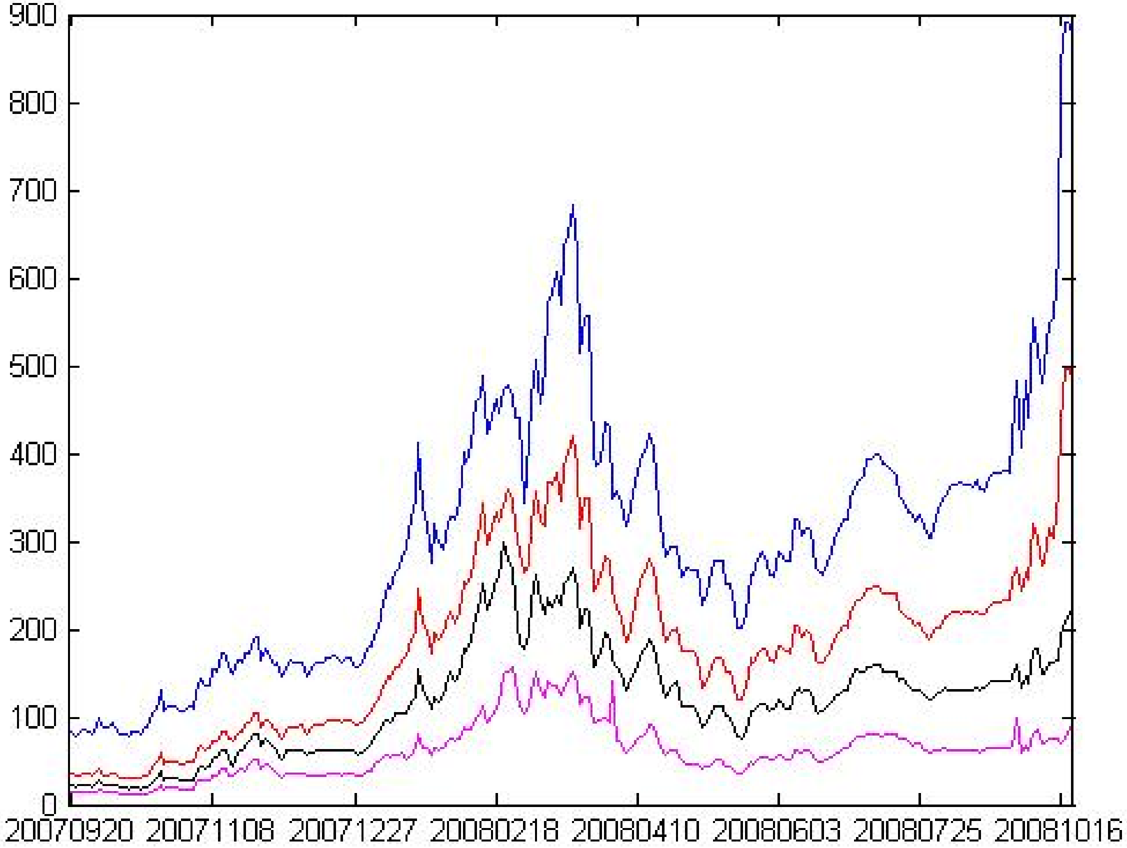


Bloomberg, ITRAXX Europe, series 6EU2 with maturity 5 years.

History

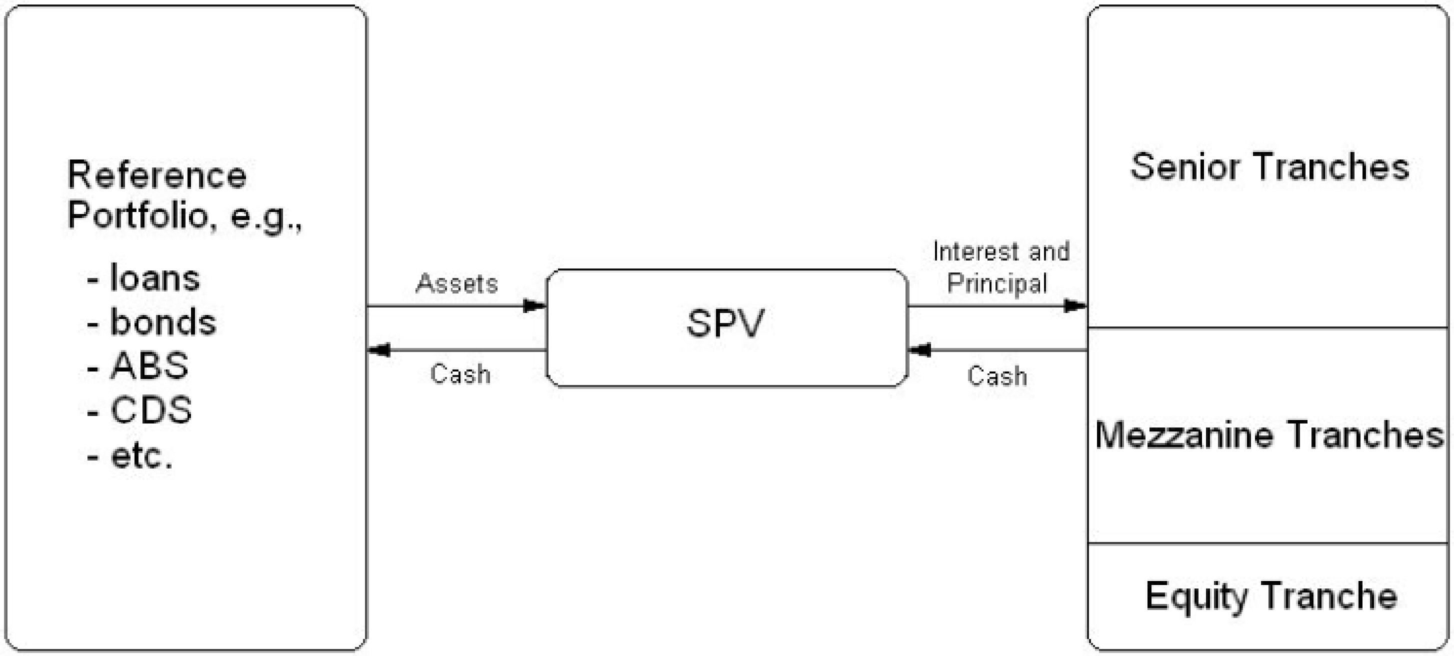
- A static portfolio of 125 equally weighted CDS on European entities
- New series of iTraxx Europe issued every 6 months (March and September) and the underlying reference entities are reconstituted
- Sectors: Consumer (30), Financial (25), TMT (20), Industrials (20), Energy (20), Auto (10)
- Maturities: 5Y, 7Y, 10Y

History



Time series of iTraxx spreads, Series 7, Maturity: 5 years, 21.03.2007-22.01.2008

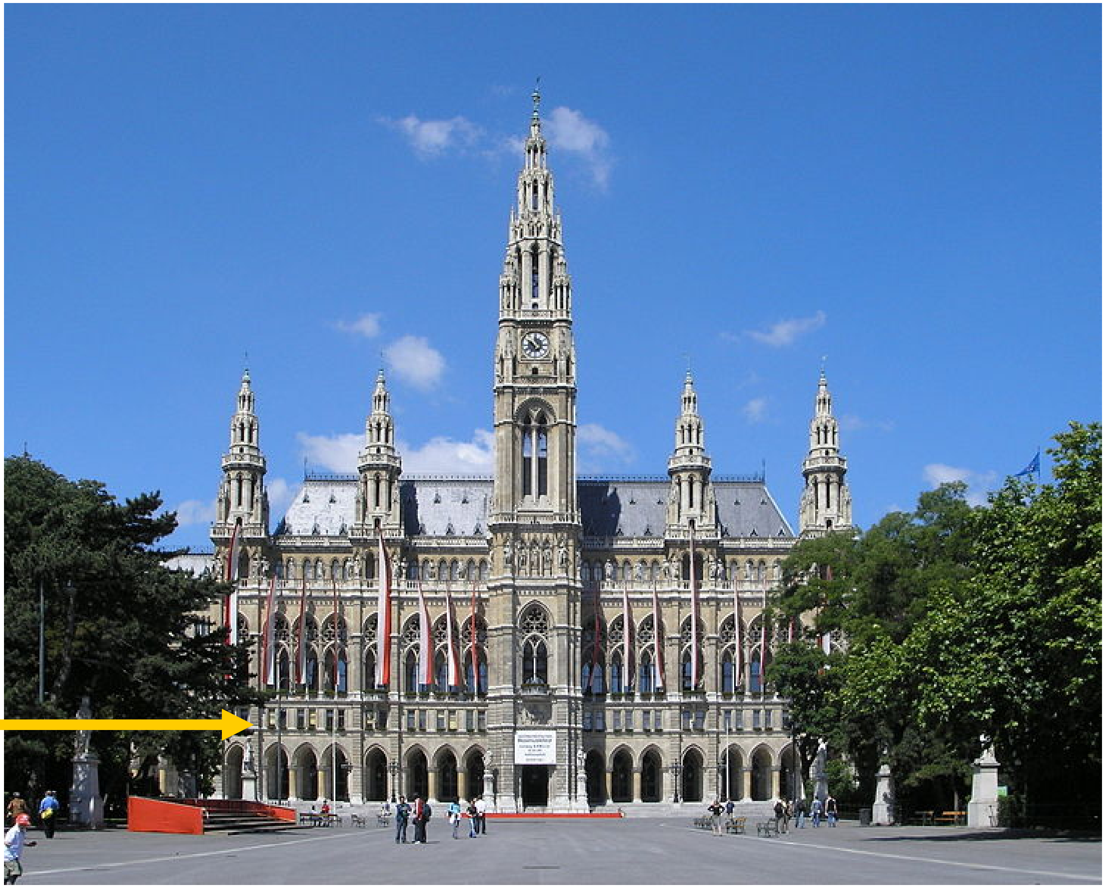
Construction



CDO Transaction, Tranches

Construction

Mezzanine



Vienna City Hall

Construction

Tranche number	Tranche name	Attachment points (%) Lower l	Upper u
1	Equity	0	3
2	Mezzanine Junior	3	6
3	Mezzanine	6	9
4	Senior	9	12
5	Super Senior	12	22
6	Super Super Senior	22	100

Attachment points, ITRAXX, CDO tranche structure

Construction

Example

Suppose the equity tranche investor receives 500bp annually for protecting the first 3% of losses on a 10 million EUR pool.

Possible scenarios:

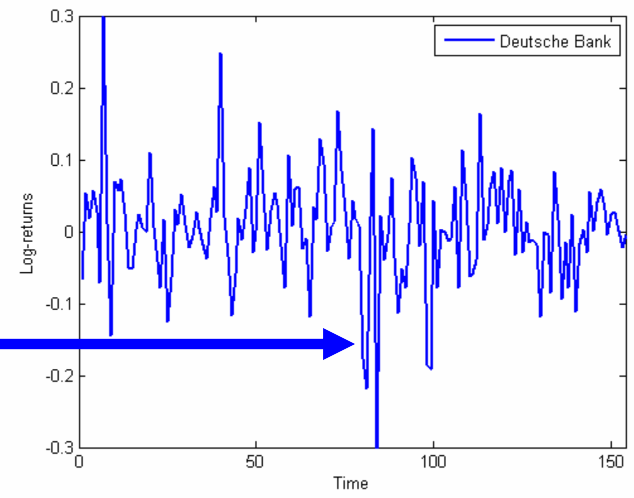
- no losses have occurred, then the investor is protecting the full $300,000\mathrm{EUR}$ and is paid 500bp on this amount,
- the losses of $100,000\mathrm{EUR}$ occurred, then the premium is paid on remaining $200,000\mathrm{EUR}$ that the investor is protecting.

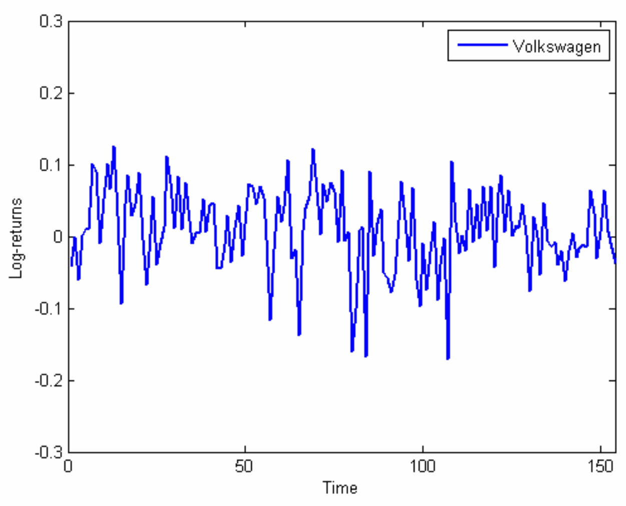
Construction

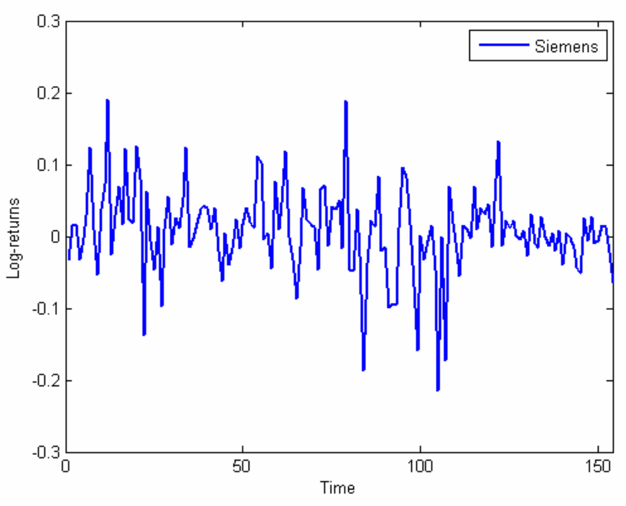
Risk factor i

=

Economy + Company i

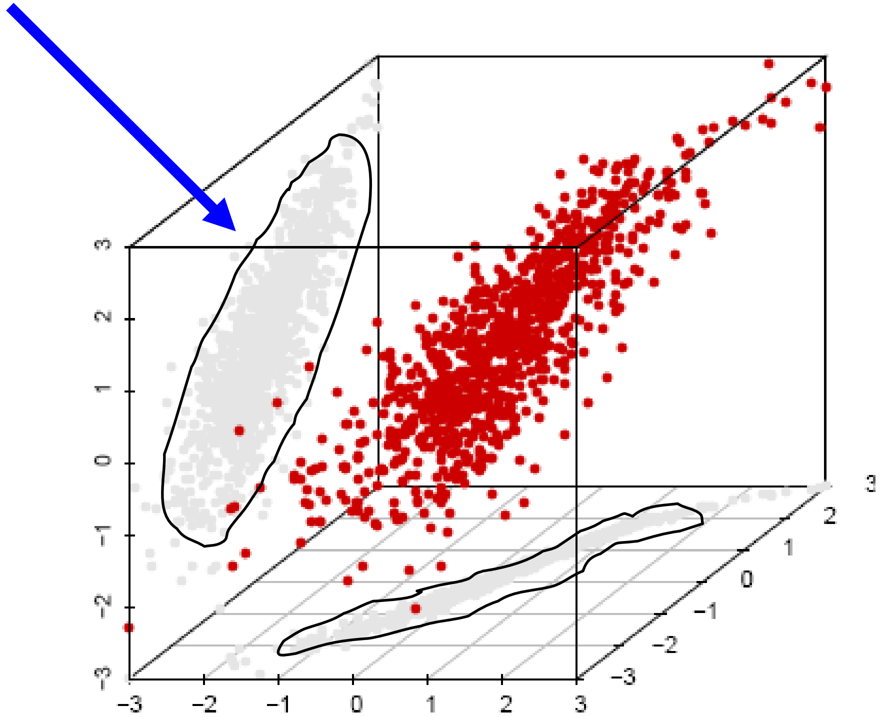






Construction

Covariance



Pricing

Standardized asset log-returns:

$$X_{i,t} = \sqrt{\rho_t} Y_t + \sqrt{1-\rho_t} Z_{i,t},$$

for all, $i = 1, \ldots, d$, where Y_t (systematic risk factor), $\{Z_{i,t}\}_{i=1}^{d}$ (idiosyncratic risk factors) are i.i.d. $N(0,1)$. Hence:

$$(X_{1,t}, \ldots, X_{d,t})^\top \sim N(0, \Sigma_t),$$

with

$$\Sigma_t = \begin{pmatrix} 1 & \rho_t & \cdots & \rho_t \\ \rho_t & 1 & \cdots & \rho_t \\ \vdots & \vdots & \ddots & \vdots \\ \rho_t & \rho_t & \cdots & 1 \end{pmatrix}$$

Gaussian ONE FACTOR model, constant RHO, ITRAXX d = 125 !!

Pricing

This standard ONE FACTOR pricing model assumes that all CDS components move in identical direction.

Wolf pack does not seem to do that when hunting an American Bison, d= 6 !!

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Pricing

- Loss variable of i-th firm until $t \in [0, T]$

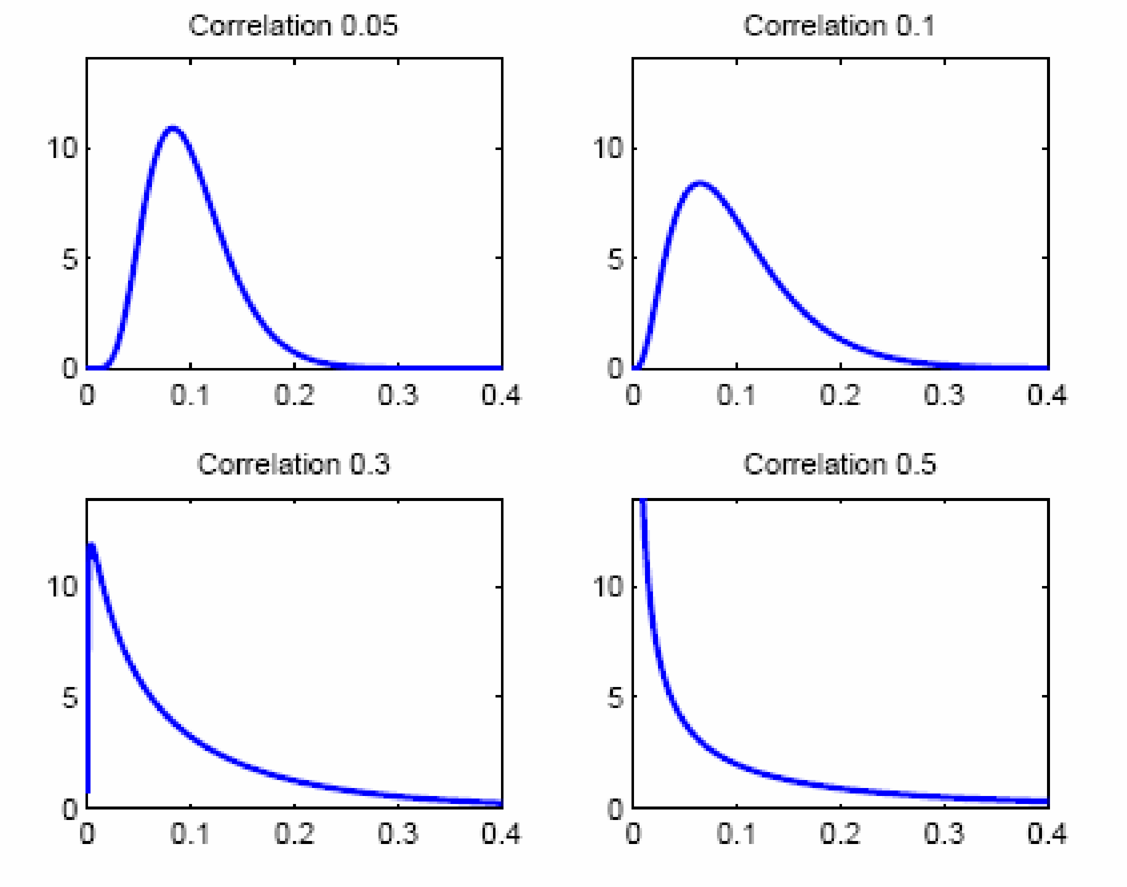
$$\Gamma_{i,t} = I(\sqrt{\rho_t} Y_t + \sqrt{1 - \rho_t} Z_{i,t} < C_t)$$

- Portfolio loss process

$$L_t = \frac{1 - R}{d} \sum_{i=1}^{d} \Gamma_{i,t}$$

where R is the recovery rate equal for all credits in the portfolio.

Pricing



Portfolio loss density for different correlations

Pricing

Credit Default Swap (CDS) is an insurance contract covering the risk that a specified credit defaults.

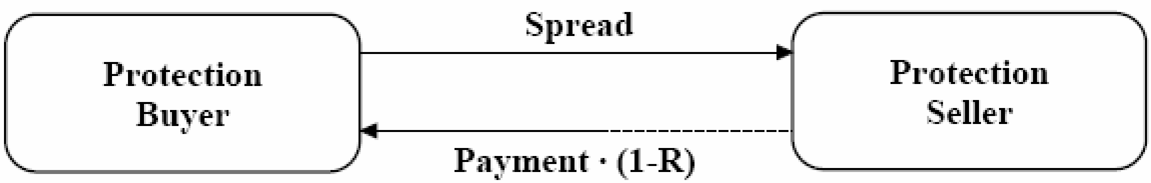


Illustration of a CDS transaction.

Pricing

The probability that the obligor defaults within the time interval [0, t]

$$p(t) = P(\tau \leq t)$$

is called the default probability.

PD = Probability of Default

The obligor's default is modeled as the time until first jump of Poisson process.

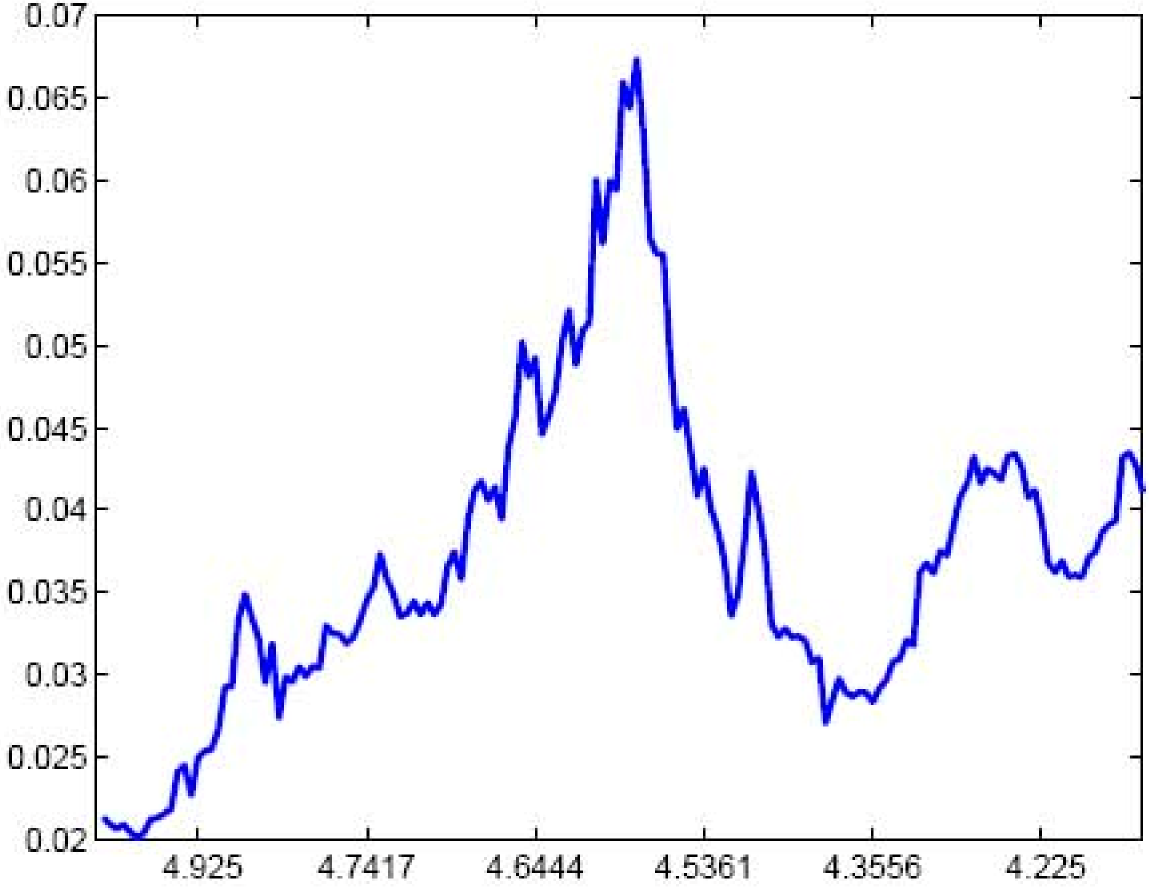
Prussian horse kick data

Ladislaus von Bortkiewicz, 1901 – 1931

Владислав Иосифович Борткевич

Władysław Bortkiewicz

PDs



Probabilities of default of Deutsche Bank, time period 20071022-20080812.

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Loss of a tranche

$$= \begin{cases} 0, & L_t < l_j, \\ L_t - l_j, & l_j \le L_t \le u_j, \\ u_j - l_j, & L_t > u_j. \end{cases}$$

Example Let j be the mezzanine tranche with the lower attachment point 6% and the upper attachment point 9%. Then

Loss of the portfolio	2	7	10
Loss of the tranche	0	1	3

CDO Premium

The premium s_j of tranche j is chosen in such a way that

1. fixed (premium) leg PL_j – the payments that tranche holders receive,
2. floating (protection) leg DL_j – the payments that tranche holders pay

are equal:

$$PL_j = DL_j.$$

The premiums are constantly observed in the market!

Implied Correlation

Implied correlation is found by inverting a pricing model for CDOs and searching for a correlation parameter that match the quoted spread of a tranche.

If Gaussian one factor model was correct, then the implied correlation ρ_j from s_j would be approximately constant across tranches and equal ρ.

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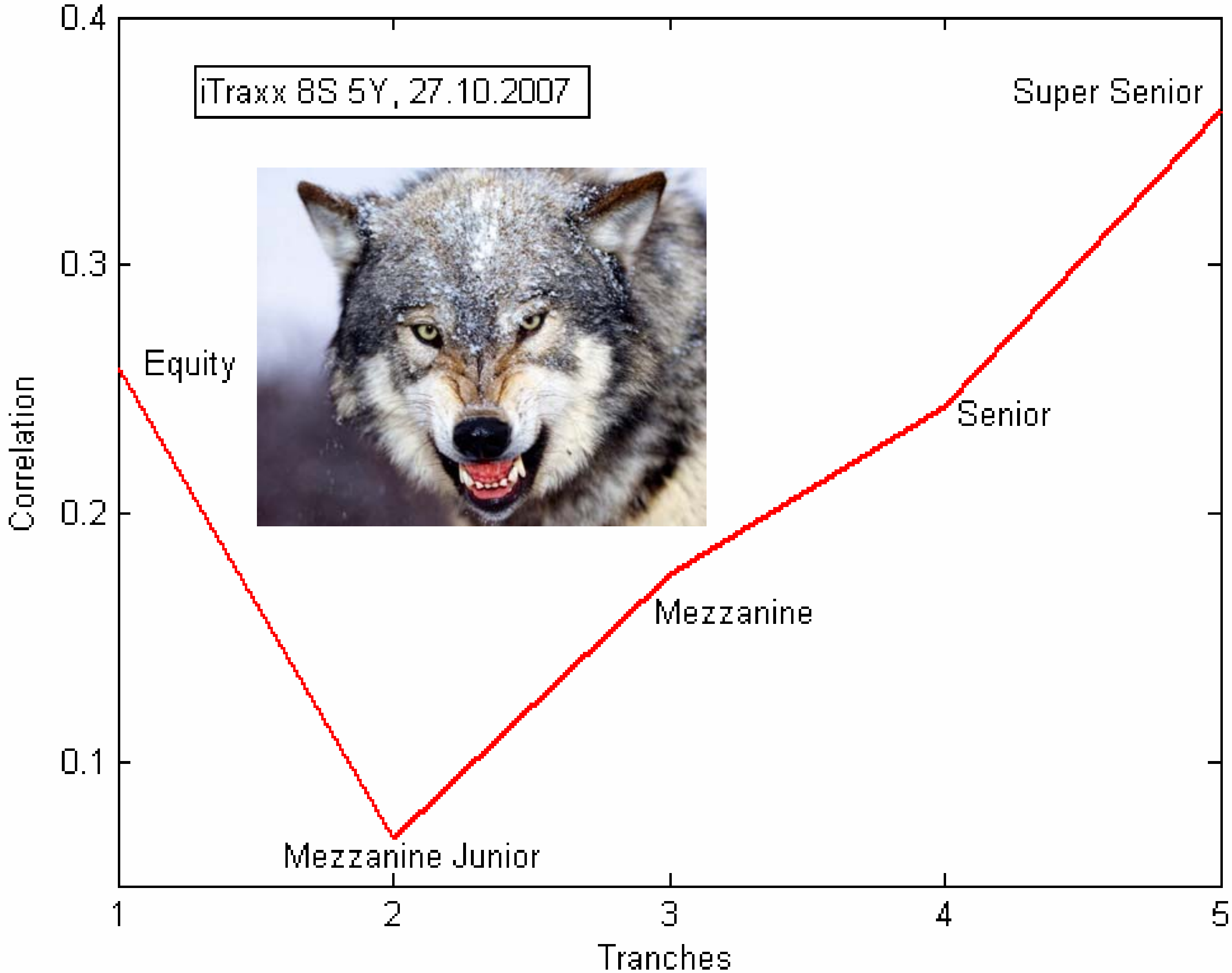
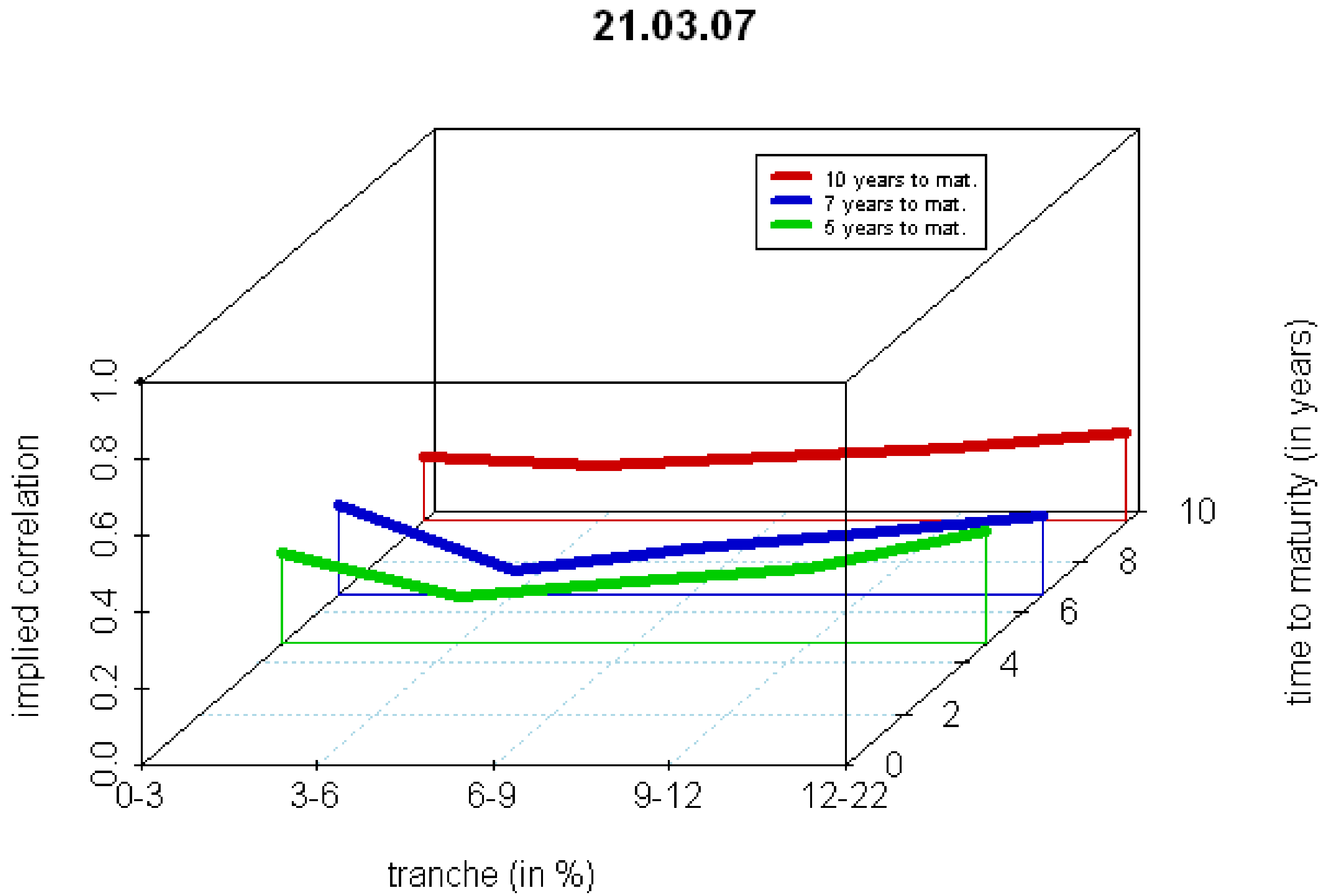
Pricing



Gaussian one factor model with constant correlation

Compound Correlation



Film of compound correlations over time

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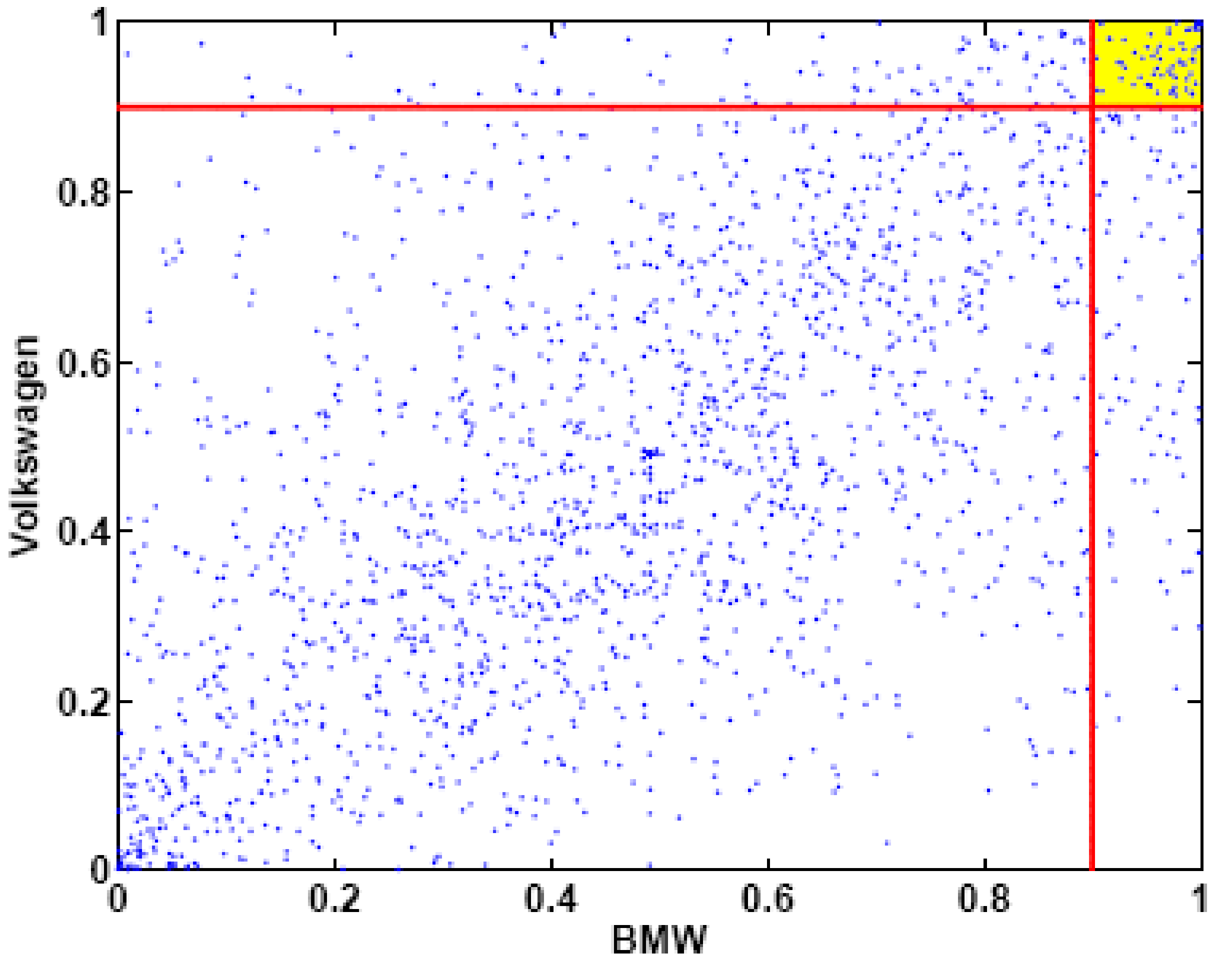
Gaussian Multi-Factor Model

Three factor model

$$\Sigma = \begin{pmatrix}
\boxed{\begin{matrix} 1 & \cdots & \rho_2 \\ & \ddots & \\ \rho_2 & \cdots & 1 \end{matrix}} & \begin{matrix} \rho_1 & \cdots & \cdots \\ \vdots & & \\ \rho_1 & & \end{matrix} & & \begin{matrix} \cdots & \cdots & \rho_1 \\ & & \vdots \\ & & \vdots \end{matrix} \\
\begin{matrix} \rho_1 & \cdots & \rho_1 \\ \vdots & & \\ \vdots & & \end{matrix} & \boxed{\begin{matrix} 1 & \cdots & \rho_2 \\ & \ddots & \\ \rho_2 & \cdots & 1 \end{matrix}} & & \\
& & \ddots & \\
& & \boxed{\begin{matrix} 1 & \cdots & \rho_3 \\ & \ddots & \\ \rho_3 & \cdots & 1 \end{matrix}} & \begin{matrix} & & \vdots \\ & & \vdots \\ \rho_1 & \cdots & \rho_1 \end{matrix} \\
\begin{matrix} \vdots & & \\ \vdots & & \\ \rho_1 & \cdots & \cdots \end{matrix} & & \begin{matrix} & & \rho_1 \\ & & \vdots \\ \cdots & \cdots & \rho_1 \end{matrix} & \boxed{\begin{matrix} 1 & \cdots & \rho_3 \\ & \ddots & \\ \rho_3 & \cdots & 1 \end{matrix}}
\end{pmatrix}$$

Upper Tail Dependence



Copula

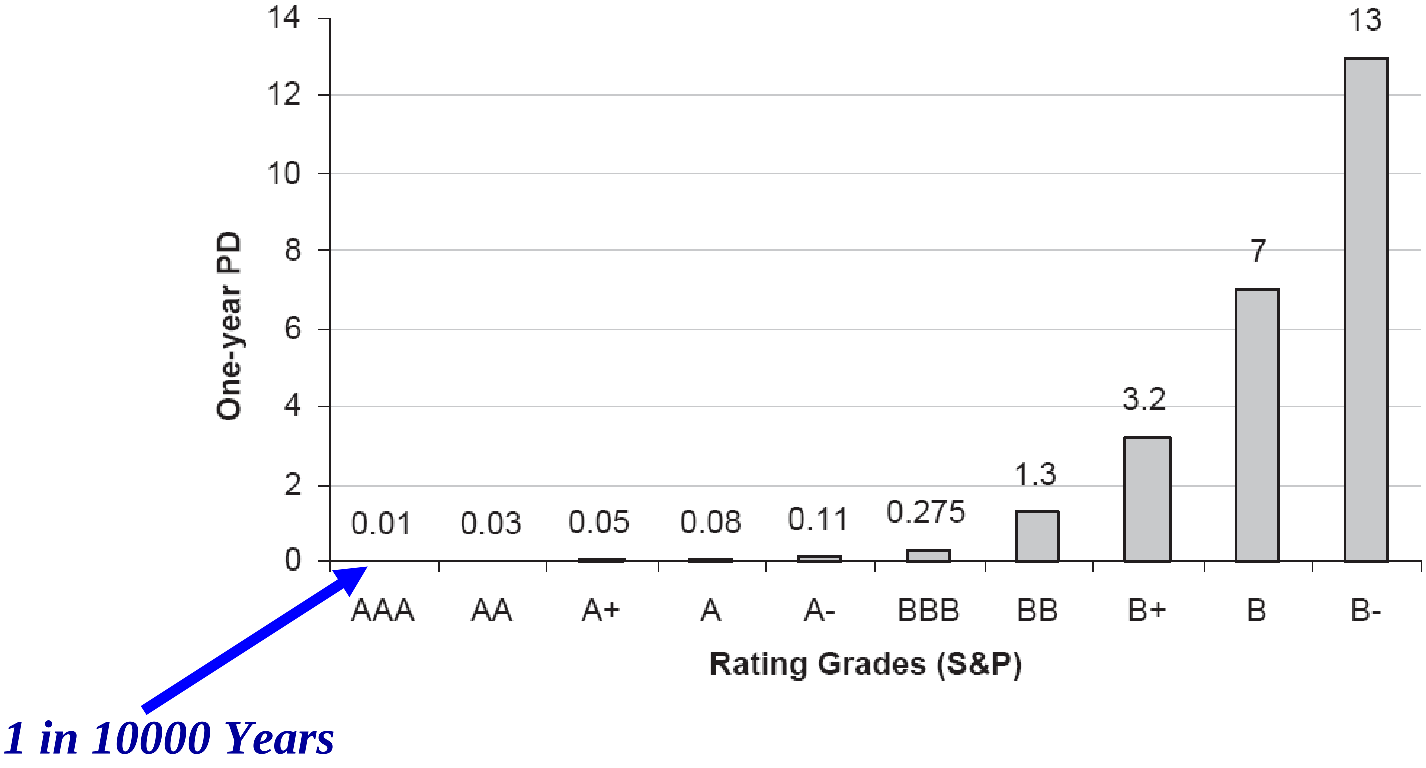
For a distribution function F with marginals $F_{X_1}, \dots, F_{X_d}$. There exists a copula $C : [0,1]^d \to [0,1]$, such that

$$F(x_1, \dots, x_d) = C\{F_{X_1}(x_1), \dots, F_{X_d}(x_d)\} \qquad (11)$$

Copula
Copula
Kopuła
Kopula
关联结构
連辞
الارتباط الصلة
코플러

Rating

- **MCRA Modular Credit Risk Analysis**



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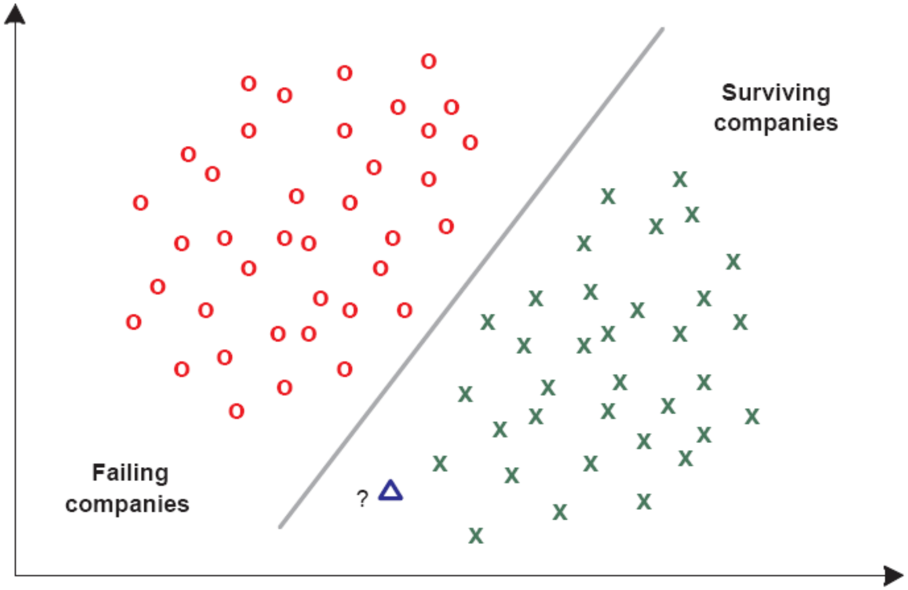
Rating

- **Linear discriminant analysis**
- **CDS are based on company ratings**
- **Rating technolog is applied statistics**

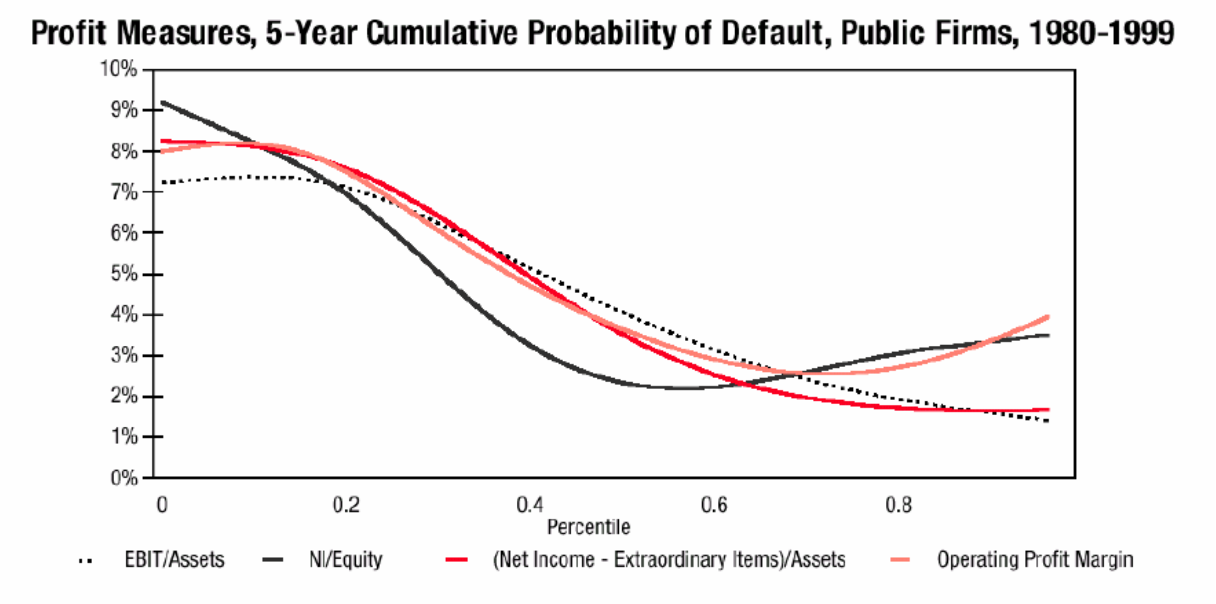


Nettokapitalgewinn

Zinsdeckungsquote

Rating

- 5Y cumulative PD 1980-1999
- Effect on PD is non linear
- Effect on PD is non monotone



EBIT/Assets

Nettokapitalgewinn

Operating Profit margin

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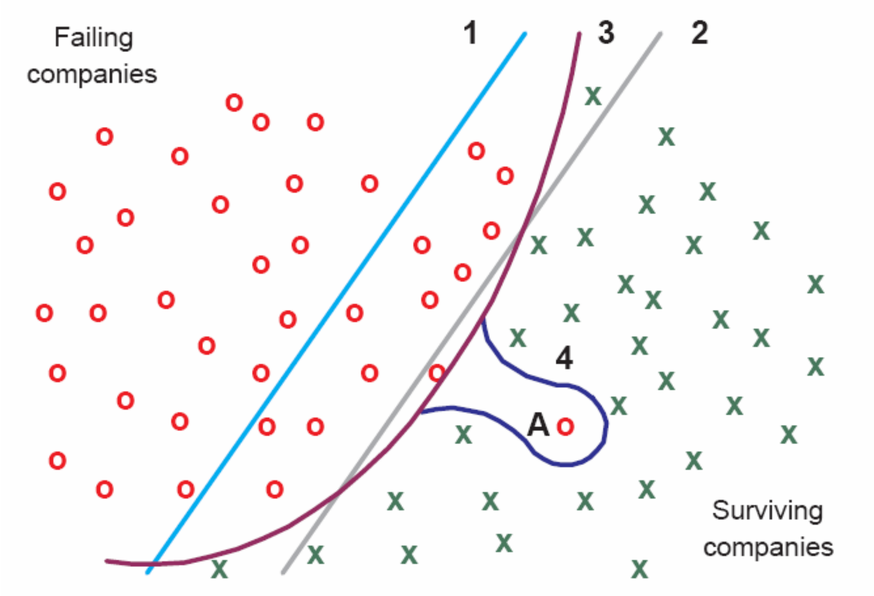
Rating

- **Separation in feature space not linear**
- **How to find the best separating curve**
- **Complexity vs. Precision**



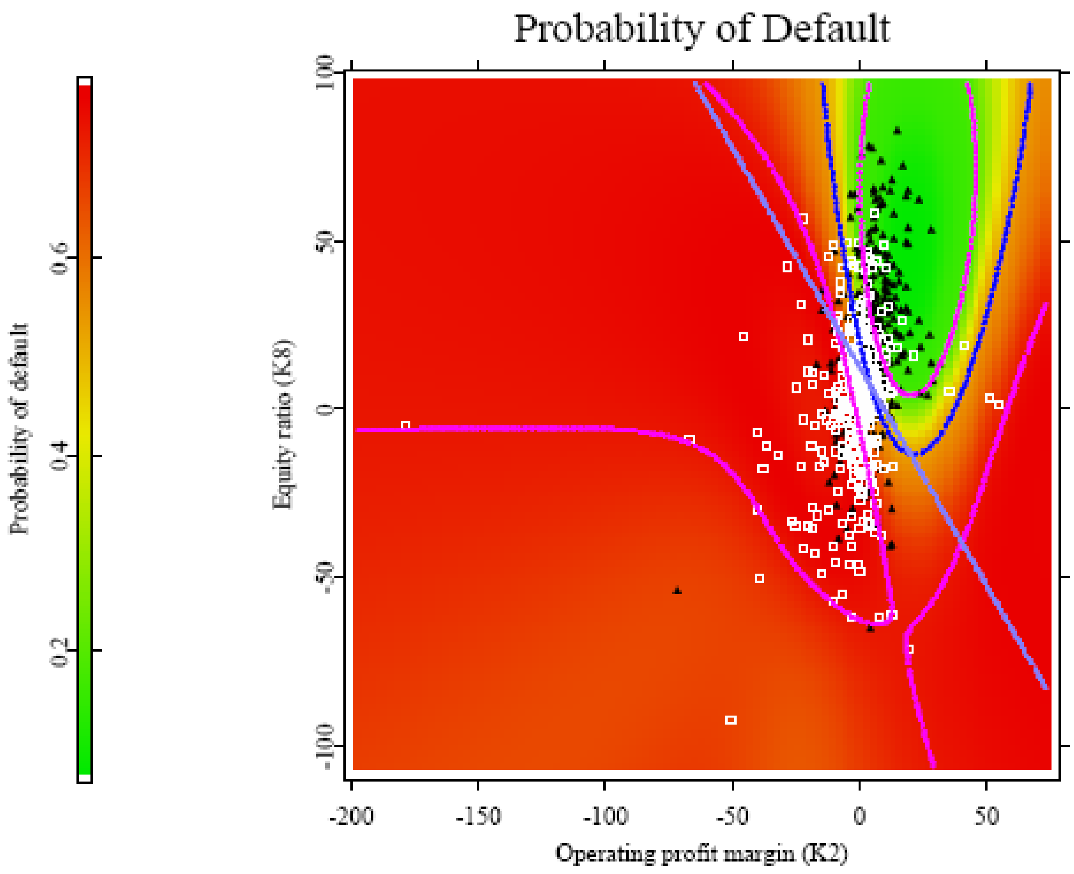
Nettokapitalgewinn

Zinsdeckungsquote

Rating

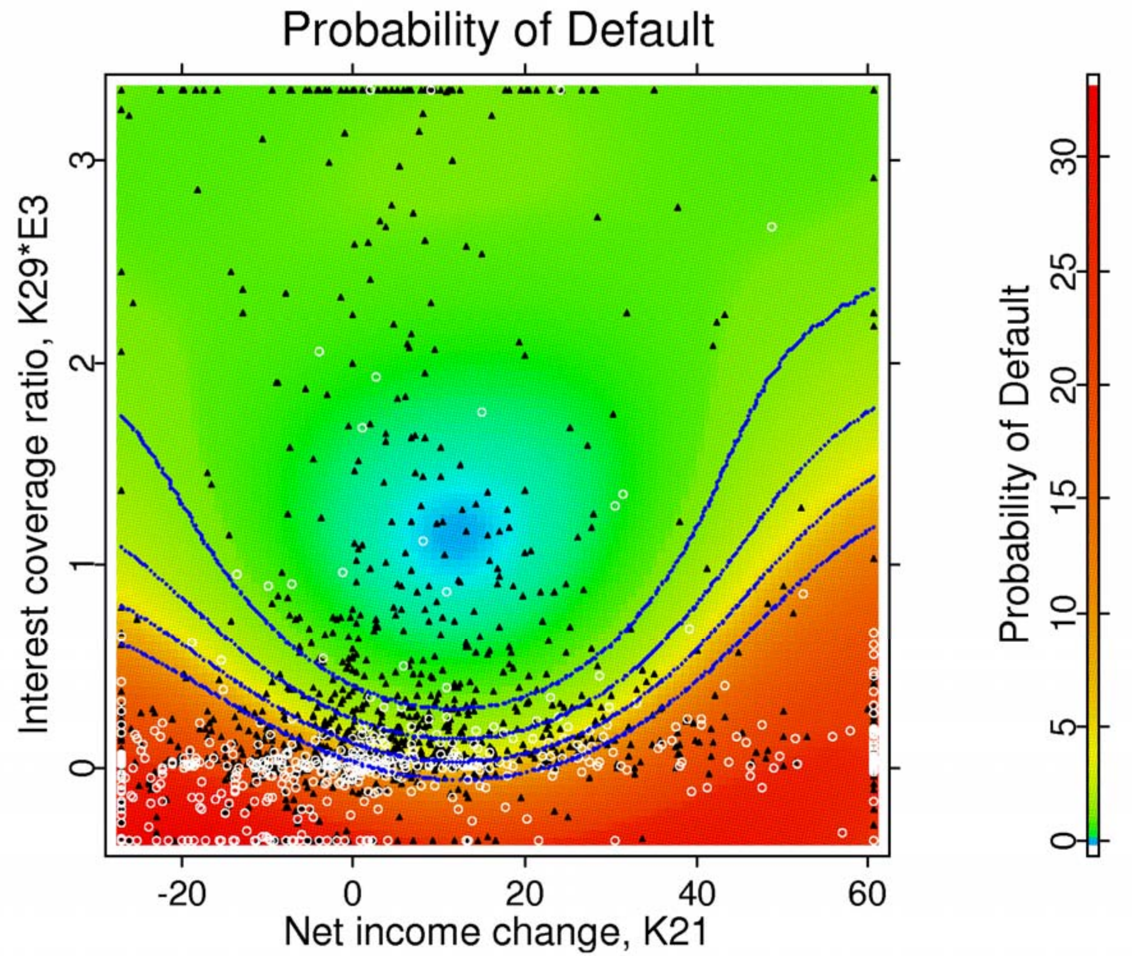
Wolf pack moves non linear!!

Support Vector Machines (SVM) produce better separation between companies.



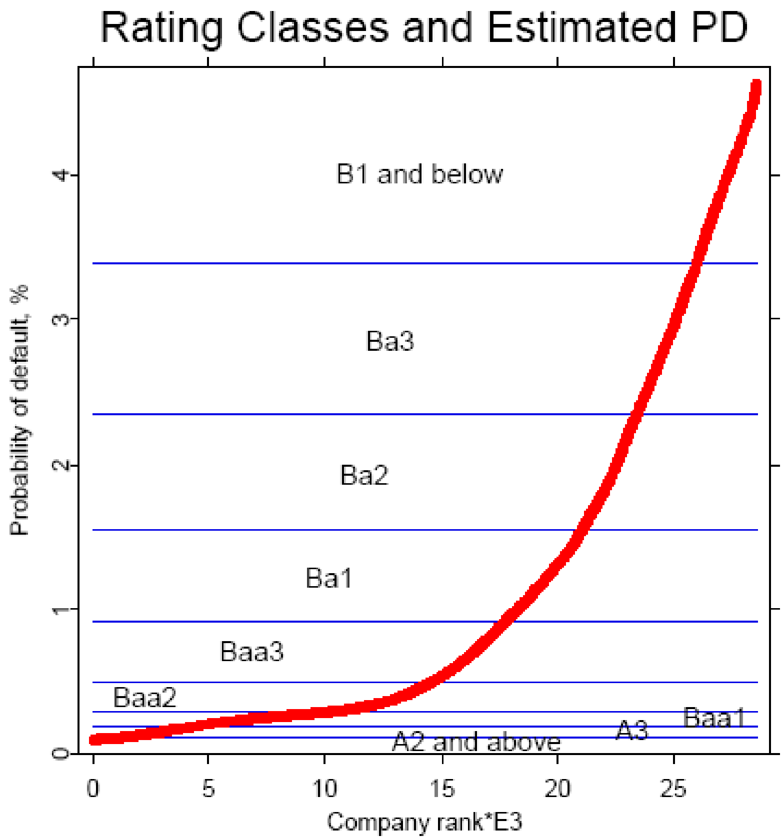
Rating

- **Support Vector Machines**



Rating

▣ MCRA Modular Credit Risk Analysis



Taming

- **A rating method (flaw #4) must be applicable: SVM was extensively tested with Bundesbank data**
- **A rating method must be stable: SVM delivers a stable and unique solution**
- **A rating method must be stationary: SVM produces PD estimates with different data**
- **A rating method must be a forecaster: SVM exceeds in accuracy both DA and Logit**

Taming

- Copulae model (flaw #1, 2) dependency more general than Gaussian elliptical
- Tail dependence cannot be produced with Gaussian Model
- Dependencies change over time
- Need to simulate non stationary processes

Taming

- **PDs (flaw #3) are different for different CDS**
- **More general cdfs for the default case**
- **Multifactor Gaussian models as a proxy**
- **Hierarchical Archimedian Copulae**

Taming

- **Enlarge the statistical view on dependency**
- **Think of tail dependence**
- **Move to different PDs for each risk factor**
- **Extend rating technology to nonlinearity**

- **Simulate wolf pack moves in the prairie**

How to tame CDOs?

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