How different are Constant Interest Rate Inflation Forecasts from Variable Interest Rate Inflation Forecasts?

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Abstract

The question this paper tries to clarify is whether constant interest rate inflation targeting differs much from variable interest rate inflation targeting. The problem is analysed in the framework of a New Keynesian dynamic stochastic general equilibrium model. Our conclusion is that the monetary authorities that adopt CIR inflation targeting make systematic mistakes in projections of inflation and output gap. They tend to deviate from their assumption of maintaining the constant interest rate within the forecast horizon, which is an observed case in practice. The projections produced under CIR assumptions are only consistent when the economy is in steady state and expected to stay in steady state. The more the economy deviates from steady state, the larger is the mistake they make. When the economy is far from steady state, the degree of relative risk aversion of agents also matters in the extent of the mistake. VIR inflation targeting policy performs better compared to CIR inflation targeting in producing stable and more realistic projections.

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1 Introduction

The key concern of central banks is what strategy to pursue in the conduct of monetary policy. The adoption of the monetary policy framework known as inflation targeting is one of the most interesting developments that have occurred in the past fifteen years. In 1990, the inflation targeting was first introduced in New Zealand and since than has been implemented by more than 20 countries, among them Canada, England, Sweden, and Australia; some newly industrialized and emerging market economies like Brazil, Chile, Israel, Korea, Mexico, South Africa, the Philippines, and Thailand; and some transition economies: the Czech Republic, Hungary, and Poland.

Monetary authorities announce medium-term numerical targets for inflation publicly with a commitment by the monetary authorities to achieve these targets. Svensson (2005) defines the inflation targeting precisely in three points:

1. *An explicit monetary-policy objective in the form of a numerical inflation target.* The main concern is both on the stability of inflation around the target and on the stability of real economy. Therefore, inflation and a real variable like the output gap are among the target variables, which appear in the central bank's explicit or implicit loss function.

2. *An internal decision process, "forecast targeting".* The central bank sets the instrument rate (short term nominal interest rate) such that the forecast of the target variables "looks good" relative to the monetary-policy objective.

3. *A very high degree of transparency and accountability.* In most inflation targeting regimes, the central bank publishes its internal projections and presents the incentives of these forecasts and of its instrument rate decisions with the aim of putting the policy effectively into action and letting external inspection of its performance. An example is the Bank of England's Inflation Report, which is published quarterly.

Bernanke and Mishkin (1997) argue that inflation targeting is a "rule-like" policy strategy, since they restrict the central banks from undertaking policies with undesirable consequences by their forward looking nature, but at the same time tolerate some discretion to deal with unanticipated circumstances. They call it as a framework of "constrained discretion", which results in a number of different implementation methods of the general objective. The best way is the optimization of the policy; however the derived policy responses can be too complicated to be understood by the agents in the economy. One another way is the use of rational expectation forecast of inflation suggested by Svensson (1999b). This way fits with the widely recognised view that monetary policy needs a forward-looking facet. There are many benefits of such a policy, notably they embody the lags in monetary transmission and exploit all information available for predicting future inflation[1].

Besides, as Leitemo (2003) argue, targeting the forecast of inflation could approach to the optimal policy in minimizing the deviations from target and also it is a clear, transparent and intuitive procedure. The inflation target is set for some horizon h, and the policy tries to achieve this target:

$$E_t[\pi_{t+h}] = \pi^*$$

where π_{t+h} is the rational four-quarter inflation rate forecast at horizon h and π^* is the inflation target level. But, in case the horizon is longer than the period in which the policy instruments influence inflation, there are a number of policy instrument paths[2]. Further assumptions are required to attain the fix target. One practically applied specialization is "Constant Interest Rate (hereafter CIR) Inflation Forecast Targeting"[3].

Actually, there are two different approaches of determining CIR projections[4]:

1. The interest rate over the targeting horizon is assumed to be constant under the first approach. Here, forecasts of inflation are calculated for different constant interest rate levels, and the interest rate level that is consistent with the target

[1] Batini and Haldane (1999), Svensson (1999c).
[2] Woodford (2003).
[3] Bank of England and Bank of Sweden used to apply CIR Inflation Forecast Targeting. See Honkapohja and Mitra (2004) for further explanations and differences in their procedures.
[4] Honkapohja and Mitra (2004) call the first approach C_{UK} since it used to be the practice in the UK, and the second one C_S as it seemed to be applied in Sweden.

inflation level is chosen[5].

2. In the second approach, the CIR projection is computed at the interest rate existing before any policy decision and the rate of interest is then adjusted depending on the difference between the CIR projection and the inflation target.

CIR Inflation Forecast Targeting has some advantages which are briefly mentioned above, however, there are good reasons to doubt about its optimality[6]. The interest rate is supposed to be constant only in the derivation of the policy. The interest rate will be adjusted in the forecast horizon, since the inflation target moves forward in time, or within the forecast horizon, new information may appear. The result might be improper forecasts and policy responses.

Alternative instrument rate assumptions are suggested instead of CIR Inflation Forecast Targeting rule after intensive oppositions. Time-varying interest rate rules are among these suggestions, and it can be in the form of a Taylor rule reaction function. Does the CIR rule actually differ from Variable Interest Rate (henceforth VIR) Inflation Forecast Targeting? We try to find an answer to this question in our work.

We set up a New Keynesian Dynamic Stochastic General Equilibrium Model to make our analysis. Our model is basically built on the model proposed by Galí (2002). In our set up, the monetary policy authority employs an interest smoothing Taylor rule, which can also be a representation of CIR inflation targeting of the second type above[7]. In our benchmark model, the economy is exposed to technology shocks and monetary policy shocks. By solving the dynamics of our economy we obtain nice machinery, the laws of motion depending on the endogenous state variables and disturbances, which lets us make projections of interest rate, inflation and output gap under two different monetary policies: CIR inflation targeting and VIR inflation targeting. As we make the projections under CIR inflation targeting, we assume that the central bank commits to its assumption that it keeps the interest rate constant during the forecast horizon. After analysing the results of the benchmark model, we add some other disturbances to our economy: namely a cost-push shock, a preference shock and a government spending shock. We forecast inflation, output gap and deviations in interest rate in the extended model for CIR and VIR inflation targeting

[5]Goodhart (2001), Heikensten (1999).
[6]Svensson (2003a, 2005), Honkapohja and Mitra (2004), Kohn (2000), Woodford (2003, 2004, 2005), Adolfson, Laséen, Lindé, Villani (2005), Leitemo (2003).
[7]See Honkapohja and Mitra (2004).

policies. In addition, we solve our extended model for different values of relative risk aversion coefficient. We try to see if our results depend on how risk averse the agents are.

The general conclusion that we derive in our study is that CIR inflation targeting leads to a systematic mistake in the projections of inflation and output gap, therefore the policy makers tend to deviate from their assumption of constant interest rate during the targeting horizon. Even if the projections are made some periods after a disturbance to the economy, they are still wrong, however, to a smaller degree. The larger the deviation of inflation or output gap from their steady state values, the bigger the extent of their mistake. For instance, in an extremely inflationary economy, they make severely wrong projections. The degree of relative risk aversion also matters if the economy deviates significantly from steady state. If the economy is surrounded with highly risk averse agents, their mistakes in inflation projections are larger under a preference shock or a technology shock. If less risk averse agents exist in the economy, their mistakes in output gap projections are larger when a cost-push shock hits the economy.

The rest of the paper is organized as follows: In the next section we discuss the relevant existing literature which motivates us in our study and which enlightens our way. Section 3 provides the facts so far reached by the scholars working on our topic. We present our benchmark model in Section 4 and in the following section we solve our model. The results obtained are presented in Section 6. Section 7 extends our model by adding further disturbances into the economy, solves this extended model and discusses the corresponding results. We discuss our progress in Section 8, and finally Section 9 concludes.

2 Literature

In the beginning, inflation targeting was declared to be the monetary strategy of several economies with little guidance from the academic literature on monetary policy rules. But since then an academic literature on inflation targeting has arisen. It has been discussed both conceptually (Bernanke and Mishkin (1997), Svensson (1997a, 1997b, 1997d, 1999b, 1999c, 1999d, 2005), Svensson and Woodford (1999, 2005), Woodford (2004, 2005), Giannoni and Woodford (2003b), Bernanke and Woodford (1997)) and empirically (Ammer and Freeman (1995), Bernanke, Laubach, Mishkin and Posen (1998), Mishkin and Schmidt-Hebbel (2001) and papers in Leiderman and Svensson (1995)). They generally assess the appeal of such an approach. This literature finds that an optimal policy regime can be developed bringing together some features of the currently practiced inflation forecast targeting and that of theoretical suggested monetary policies.

Our work is actually related to a branch of the literature that discusses the notion of constant interest rate inflation targeting. In some of the articles listed above, CIR inflation targeting is discussed qualitatively. Woodford (2004, 2005) argues that CIR projections are inappropriate basis for monetary policy, in the way that the inflation target is kept within the target under the assumption of constant interest rates without a commitment to maintaining the interest rates at that constant level over the forecast horizon. Thus, he argues that it cannot improve the extent the private sector anticipates the monetary policy and thus the transparency in communication. Svensson and Woodford (2005) and Woodford (2004, 2005) propose another procedure where the projection that is used to justify current policy will correspond to the bank's own best forecast of how it should act in the future, as the practice of the Reserve Bank of New Zealand.

Svensson (2003, 2005) lists several problems with the CIR assumption, which we will point out in the following section. As another alternative procedure, Svensson (2005) proposes an ad hoc reaction function for the instrument-rate assumption, such as a

Taylor rule.

There are only a few papers in the literature, of which main concern is CIR inflation targeting. Leitemo (2003) is an important study in this area. The first point he emphasizes is the time inconsistency in CIR inflation targeting, that is the central bank will not stick to its constant interest rate assumption during the forecast horizon seeing that new information arrives and the forecast horizon moves forward. In the following forecasting horizon, the inflation may not be on target under the assumption of prevailing interest rate, so central bank needs to alter the interest rate within the previous forecasting horizon. He takes into account that rational forward looking agents realize this discretionary behaviour of the central bank, and develop a method for analyzing CIR forecast targeting. Then, for illustrational purposes, he uses a dynamic New Keynesian model with forward-looking price setting. He investigates the stability of the economy under different degrees of forward looking behaviour of the inflation appended in Phillips curve. He also looks at the dynamic responses of the economy to an output shock and to an inflation shock. His most remarkable finding, which we will later relate to our study, is that the expected development of inflation follows a considerably smoother path than the CIR forecasts indicate. Finally, he tries to find out the optimal forecast horizon, and suggests it to be between five and seven quarters. But if the central bank values interest rate smoothing, it is likely to choose a long forecast horizon which is achieved at the cost of both more real and nominal instability. Honkapohja and Mitra (2004) examine the two different approaches of determining the CIR projections, which are briefly explained in the Introduction part, from the point of view of determinacy and stability under learning. They use a standard New Keynesian model of monopolistic competition and (Calvo (1983)) price stickiness, which can be described by two equations: New Keynesian IS Curve and Phillips Curve. They first introduce the formalization of CIR_{UK} and CIR_S policies[1]. They follow Leitemo (2003) for the CIR_{UK} formulation. The general representation of CIR_{UK} is

$$i_t = \chi_g g_t + \chi_u u_t + \chi_u u_t + \chi_\pi \pi_t \tag{2.1}$$

and for the CIR_S case they obtain

$$i_t = \omega(\varphi_g g_t + \varphi_u u_t + \varphi_u u_t + \varphi_\pi \pi_t) + (1 + \omega\varphi_i) i_{t-1} \tag{2.2}$$

[1] For an explanation of CIR_{UK} and CIR_S see footnote 4 in Introduction part.

where the coefficients depend on the model parameters. Note that CIR_{UK} rule 2.1 is similar to an instrument rule like the classic rule studied by Taylor (1993) and CIR_S rule 2.2 resemble the form of a Taylor type rule with interest rate smoothing which is frequently observed in the data. Woodford (2003)[2] proves that the necessary and sufficient condition for determinacy for a rule of the form 2.2, is that the Taylor principle is satisfied, in case the individual responses to x_t, π_t and i_t are all positive. Depending on the parameter values used, they come to different conclusions. Their general results are, however, as follows: CIR_{UK} policies are particularly susceptible to the problems of indeterminacy and instability under adaptive learning in all versions of the models they examined. CIR_S rule, in contrast, has more tempting features in terms of determinacy and stability in the forward-looking model.

Adolfson, Laseén, Lindé and Villani (2005) were also inspired by the fact that the monetary policy in many countries could be described by a CIR inflation targeting rule of the type presented in equation 2.2. They also stress that a CIR forecast is not compatible with a CIR inflation targeting rule in a forward-looking model for the reason that private agents revise their expectations about policy during the forecast horizon. They remark that some central banks say, nonetheless, that they actually compute forecasts keeping the interest rate constant, which seems inconsistent with the CIR targeting rule they claim they have adopted. Adolfson et al. (2005) evaluate whether CIR projections are perceived as being too far away from those of the estimated policy rule in the model using two variants of the modesty statistic developed by Leeper and Zha (2003). They use an estimated open economy DSGE model as a tool. Their main findings are that CIR forecasts were not modest policy interventions[3] during most of 1999-2002 for the Euro area looking at the forecasts of inflation and output at the eight quarter horizon. Their results are also against the use of CIR forecasts.

We will use a New Keynesian DSGE model as a tool in our study. Our ultimate aim is to compare CIR targeting with VIR targeting. As the evolution of variable interest rate we prefer a Taylor rule with interest rate smoothing. The reason is twofold: first, it is a simple rule in terms of application and communication with the public, so that it is an alternative to CIR targeting with respect to transparency considerations. Moreover, it allows us to compare CIR targeting with VIR targeting

[2] Woodford (2003), Proposition 4.4, p.255.

[3] Adolfson, Laseén, Lindé and Villani (2005) define an intervention to be modest if it does not trigger the agents to revise their expectations about the inflation targeting policy.

on theoretical basis with the finding that CIR policy can be represented by equation 2.2, we will exploit this opportunity here. But we will further assume that, the central bank really sticks to its policy and does not change the interest rate during the forecasting horizon and we will se what is lost or gained by this policy. Note that, we will be comparing the VIR targeting with CIR_S targeting which was suggested to be superior to CIR_{UK}. It is also worth pointing that we assume zero inflation target during our study, which is also the case in the derivation of equation 2.2.

The New Keynesian DSGE model employed in our study is largely built on the model introduced by Galí (2002). The detailed explanation and the solution of the model are presented in section 4 (Model).

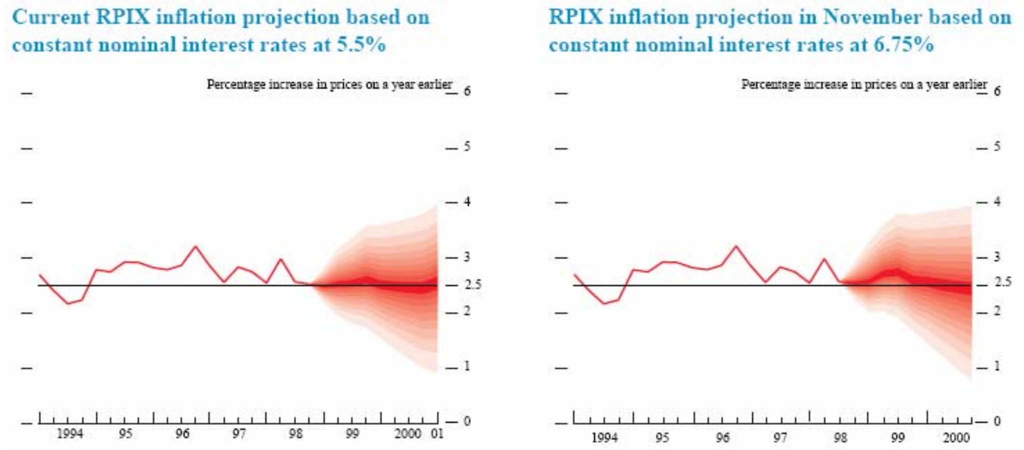
3 Facts

The scholars have reached some common conjectures about CIR inflation targeting. First of all, it is accepted to be an intuitive and clear way of implementing inflation targeting, and it makes the communication with the public transparent. However, there are many arguments against CIR inflation targeting. One of the strongest arguments is that the CIR procedure is time inconsistent. As it is frequently indicated in the previous part, it is highly probable that the interest rate during the forecast horizon is changed although the forecast was done under the assumption of constant interest rates during the horizon.

The examples of this time inconsistency problem exist in the practices of the central banks which apply CIR targeting. Leitemo (2003) demonstrates an example from the February 1999 Inflation Report of the Bank of England. Figure 3.1 illustrates two charts. The one on the left hand side is the projection for the twelve-month RPIX inflation rate assuming that nominal interest rates are constant at 5.5% which prevails at February 1999. The other chart is the November projection, which corresponds to a 1.25% higher interest rate assumption. Both inflation forecasts satisfy the target, but there is a downward trend in the inflation projection in the November forecast in the last year of the forecast horizon. . This trend worried the policy makers so much that they decreased the interest rate from 6.75% to 5.5%, which later increased the inflation forecast as seen on the left-hand-side chart.

Svensson (2003c, 2005) lists the problems with the CIR assumption in inflation targeting, which are generally accepted by academicians. Some of these problems are as follows:

- CIR assumption is unrealistic, which in turn makes the corresponding inflation and output projections unrealistic. These projections are not comparable to the actual outcomes and other projections, so it is hard to evaluate the forecast performance of the central bank.



Source: The Bank of England February 1999 Inflation Report

Figure 3.1: CIR inflation forecasts by the Bank of England

- A CIR often differs from market expectations of future interest rates.
- When market expectations of future interest rates differ from the CIR, central banks typically would not like market expectations to adjust towards the CIR, since it would lead to severe changes in asset prices. So, central banks using CIR projections would not like the private sector to take their assumption seriously, which implies a loss of trust in the central bank.
- CIR paths are not optimal; as a result the corresponding inflation and output-gap forecasts are not the best forecasts of actual outcomes.
- For a CIR, backward looking models are unstable and with increased horizon the inflation and the output-gap projection tends to increase or decrease at an increasing rate, making longer term projections more or less useless. So as to cover the problems with CIR projections, central banks avoid plotting such projections for longer horizons.

4 The Model

This section introduces our New Keynesian Dynamic Stochastic General Equilibrium Model with staggered price setting. This model is basically drawn from Galí (2002), who uses a version of the Calvo (1983) model. The model consists of households that supply labor, purchase goods for consumption, and hold money and assets; and firms that hire labor, produce and sell differentiated products in monopolistically competitive goods market. Firms set their own prices but not in each period. Households and firms optimize their preferences. In addition, there is a central bank, which controls the nominal rate of interest rate. The details are all presented below. Our model differs from the model presented in Galí (2002) in some aspects, however. That is, we use a money-in-the-utility function for the preferences of the households and the firms have decreasing returns to scale.

4.1 Households

A continuum of identical households live infinitely long, supply labor, own an equal share of all of the firms that produce the various goods, enjoy leisure, real money balances and consume a single good. The representative household seeks to maximize a discounted sum of utilities:

$$\underbrace{max}_{C_t, N_t, M_t, A_{t+1}} \; E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\chi}{1-\nu} \left(\frac{M_t}{P_t} \right)^{1-\nu} - \frac{N_t^{1+\phi}}{1+\phi} \right] \right\} \tag{4.1}$$

where $0 < \beta < 1$, and β is the intertemporal discount factor.

subject to the period budget constraint

$$\int_0^1 p_t(i) c_t(i) di + M_t + E_t \left[Q_{t,t+1} A_{t+1} \right] \leq M_{t-1} + A_t + \int_0^1 w_t(i) n_t(i) di + \int_0^1 \Pi_t(i) di \tag{4.2}$$

C_t is the composite consumption good which consists of differentiated products produced by monopolistically competetive goods producers and P_t is the corresponding index of the price of the composite good. As Dixit and Stiglitz (1977) suggest, C_t is a constant-elasticity-of-substitution aggregator:

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \tag{4.3}$$

Here there is a continuum of firms, and firm i produces good $i \in [0, 1]$. θ is the price elasticity of demand for good i, and $\theta > 1$.

It is assumed that each of the differentiated goods, denoted by i, uses a specialized labor input in its production. $n_t(i)$ is the quantity of labor of type i supplied. We assume that the representative household simultaneously supplies all of the types of labor:

$$N_t = \int_0^1 n_t(i) di \tag{4.4}$$

In our economy, it is assumed that the financial markets are complete available financial assets completely cover the relevant uncertainty faced by households about future income, prices, taste shocks, etc, so that each household faces a single intertemporal budget constraint.

In the flow budget constraint, M_t denotes the household's nominal holdings of money. A_t is the state contingent value of its nonmonetary portfolio at the beginning of period t. In order not to have arbitrage opportunities, there is a need to have the stochastic discount factor $Q_{t,t+1}$. The relationship between the riskless short-term nominal interest rate i_t and the stochastic discount factor is

$$\frac{1}{1 + i_t} = E_t \left[Q_{t,t+1} \right] \tag{4.5}$$

The gross nominal interest rate is defined as

$$R_t = 1 + i_t \tag{4.6}$$

To complete the setting for the representative household, $w_t(i)$ is the nominal wage of labor of type i in period t. Since the household owns an equal share of all of the firms, $\Pi_t(i)$, the nominal profits from sales of good i, appear in the household's budget constarint.

4.2 Firms

As mentioned before, the firms hire labor and produce and sell differentiated products in monopolistic competitive goods markets. The firms are essentially identical, they all have the same production technology and face demand curves with equal and constant demand elasticities. But they set their prices at different dates. All firms that adjust their price in the same period have the same problem, so they choose the same price. The firms adjust their prices with probability $1-\alpha$, which also means a fraction $1-\alpha$ of all firms adjust their prices. A firm that can adjust its price chooses the price $p_t(i)$ so as to maximize:

$$\underbrace{max}_{p_t(i)} \quad E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j} \left[p_t(i) c_{t+j}(i) - w_{t+j}(i) n_{t+j}(i)\right] \tag{4.7}$$

subject to the production technology

$$y_t(i) = z_t n_t(i)^{1-\gamma} \tag{4.8}$$

The only input of production is labor in our economy, and decreasing returns to scale is assumed with $1-\gamma$ being the labor share. z_t is an exogenous technology factor. For $\log z_t$ an $AR(1)$ process is assumed,

$$\log z_t = \rho_z \log z_{t-1} + \epsilon_{z,t} \tag{4.9}$$

where $0 \leq \rho_z < 1$ and $\epsilon_{z,t}$ is white noise.

Total demand for each good is equal to its production in our model:

$$y_t(i) \quad = \quad c_t(i) \tag{4.10}$$

4.3 Monetary Policy Authority

Suppose the central bank follows a Taylor Rule in order to set the nominal short-term interest rate. Taylor (1993) proposed the original rule and since then variants of it have been shown to be correct empirically. The interest rate reacts to deviations of the previous period's interest rate, inflation and output gap from their steady state values. The nominal interest rate evolves endogenously.

5 Model Analysis

After presenting our model of economy, we turn in this section to the analysis of the model. The model will be implemented in the Toolkit package of Matlab, and the essential steps serving this aim are presented here. First of all the optimization problem of the household and the firms are solved, and then log-linearization of all equations that define the equilibrium are derived. We introduce the concept of inflation and output gap in this section, which play a central role in the New Keynesian sticky price models.

5.1 Households

The decision problem of the household can be handled in two stages. First of all it will always be optimal to purchase the combination of individual goods that minimizes the cost of achieving a certain level of the composite good. First of all, dealing with the problem of expenditure minimization, the household's decision problem is to

$$\underbrace{min}_{c_t(i)} \int_0^1 p_t(i)c_t(i)di \tag{5.1}$$

s.t.

$$C_t \leq \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}} \tag{5.2}$$

The Lagrangian equation for this problem is then,

$$L = \int_0^1 p_t(i)c_t(i)di + \Psi_t \left(C_t - \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}\right)$$

where Ψ_t is the Lagrange multiplier on the constraint. The first order condition for good i is

$$\frac{\partial L}{\partial c_t(i)} = p_t(i) - \Psi_t \frac{\theta}{\theta-1} \underbrace{\left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{1}{\theta-1}}}_{C_t^{\frac{1}{\theta}}} \frac{\theta-1}{\theta} c_t(i)^{-\frac{1}{\theta}} = 0$$

$$c_t(i) = \left(\frac{p_t(i)}{\Psi_t}\right)^{-\theta} C_t$$

The definition of CES aggregator for the composite good then leads to

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$$

$$C_t = \left[\int_0^1 \left(\frac{p_t(i)}{\Psi_t}\right)^{1-\theta} C_t^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}$$

$$C_t = \left(\frac{1}{\Psi_t}\right)^{-\theta} C_t \left[\int_0^1 p_t(i)^{1-\theta} di\right]^{\frac{\theta}{\theta-1}}$$

$$\Psi_t = P_t = \left[\int_0^1 p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} \tag{5.3}$$

The Lagrange multiplier is the aggregated price index for consumption. Then, the demand for good i is:

$$c_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t \tag{5.4}$$

Given the definition of aggregate price index and the demand for good i, it can be showed that the sum of expenditure on individual goods is equal to the expenditure on the composite good:

$$\int_0^1 p_t(i) c_t(i) di = \int_0^1 p_t(i) \left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t di = \frac{C_t}{P_t^{-\theta}} \underbrace{\int_0^1 p_t(i)^{1-\theta} di}_{P_t^{1-\theta}} = P_t C_t$$

In the second stage of the household's decision problem, assuming that they minimize expenditure every period, consumption, labor supply, and nonmonetary portfolio are chosen subject to the budget constraint:

$$P_tC_t + M_t + E_t\left[Q_{t,t+1}A_{t+1}\right] \leq M_{t-1} + A_t + \int_0^1 w_t(i)n_t(i)di + \int_0^1 \Pi_t(i)di$$

In real terms:

$$C_t + \frac{M_t}{P_t} + E_t\left[Q_{t,t+1}\frac{A_{t+1}}{P_t}\right] \leq \frac{M_{t-1}}{P_t} + \frac{A_t}{P_t} + \frac{1}{P_t}\int_0^1 w_t(i)n_t(i)di + \frac{1}{P_t}\int_0^1 \Pi_t(i)di$$

The lagrange equation can be formulated as follows:

$$\begin{aligned} L = \quad & E_t\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\chi}{1-\nu}\left(\frac{M_t}{P_t}\right)^{1-\nu} - \frac{\left[\int_0^1 n_t(i)di\right]^{1+\phi}}{1+\phi} \\ & -\lambda_t\left(C_t + \frac{M_t}{P_t} + E_t\left[Q_{t,t+1}\frac{A_{t+1}}{P_t}\right] - \frac{M_{t-1}}{P_t} - \frac{A_t}{P_t} - \frac{1}{P_t}\int_0^1 w_t(i)n_t(i)di - \frac{1}{P_t}\int_0^1 \Pi_t(i)di\right) \end{aligned}$$

The first order conditions are:

$$\frac{\partial L}{\partial C_t} = C_t^{-\sigma} - \lambda_t = 0$$

$$\lambda_t = C_t^{-\sigma} \tag{5.5}$$

$$\frac{\partial L}{\partial n_t(i)} = -\left[\int_0^1 n_t(i)di\right]^{\phi} + \lambda_t\frac{w_t(i)}{P_t}$$

$$\frac{w_t(i)}{P_t} = \frac{\left[\int_0^1 n_t(i)di\right]^{\phi}}{C_t^{-\sigma}} \tag{5.6}$$

This first order condition corresponds to the intratemporal optimality condition which sets the marginal rate of substitution between leisure and consumption equal to the real wage.

$$\begin{aligned}\frac{\partial L}{\partial A_{t+1}} &= -\lambda_t E_t\left[\frac{Q_{t,t+1}}{P_t}\right] + \beta E_t\left[\frac{\lambda_{t+1}}{P_{t+1}}\right] = 0 \\ \lambda_t &= \beta E_t\left[\frac{\lambda_{t+1}}{Q_{t,t+1}}\frac{P_t}{P_{t+1}}\right]\end{aligned}$$

$$C_t^{-\sigma} = \beta R_t E_t\left[C_{t+1}^{-\sigma}\frac{P_t}{P_{t+1}}\right] \tag{5.7}$$

The equation above is the Euler equation for the optimal intertemporal allocation of consumption.

$$\begin{aligned}\frac{\partial L}{\partial M_t} &= \chi\left(\frac{M_t}{P_t}\right)^{-\nu}\frac{1}{P_t} - \frac{\lambda_t}{P_t} + \beta E_t\left[\frac{\lambda_{t+1}}{P_{t+1}}\right] = 0 \\ 0 &= \chi\left(\frac{M_t}{P_t}\right)^{-\nu}\frac{1}{P_t} - \frac{C_t^{-\sigma}}{P_t} + \beta E_t\left[\frac{C_{t+1}^{-\sigma}}{P_{t+1}}\right] \\ 0 &= \chi\left(\frac{M_t}{P_t}\right)^{-\nu}\frac{1}{P_t} - \frac{C_t^{-\sigma}}{P_t} + \frac{C_t^{-\sigma}}{P_t R_t}\end{aligned}$$

$$\frac{\chi\left(\frac{M_t}{P_t}\right)^{-\nu}}{C_t^{-\sigma}} = \frac{R_t - 1}{R_t} \tag{5.8}$$

The last condition sets the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money.

5.2 Firms

Before analysing the pricing decision of the firm, let us define the total cost for firm i:

$$TC_t(i) = w_t(i)n_t(i) = w_t(i)\left(\frac{y_t(i)}{z_t}\right)^{\frac{1}{1-\gamma}}$$

The nominal marginal cost for firm i is then,

$$MC_t(i) = \frac{\partial TC_t(i)}{\partial y_t(i)} = \frac{1}{1-\gamma} w_t(i) \left(\frac{y_t(i)}{z_t}\right)^{\frac{\gamma}{1-\gamma}} \frac{1}{z_t} = \frac{w_t(i)}{(1-\gamma) z_t n_t(i)^{-\gamma}}$$

$$MC_t(i) = \frac{w_t(i)}{(1-\gamma)\frac{y_t(i)}{n_t(i)}} \tag{5.9}$$

Plugging in the demand for good i (5.4),

$$MC_t(i) = \frac{w_t(i)}{1-\gamma} \left(\frac{\left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t}{z_t} \right)^{\frac{\gamma}{1-\gamma}} \frac{1}{z_t}$$

$$MC_t(i) = \frac{1}{1-\gamma} w_t(i) \left(\frac{p_t(i)}{P_t}\right)^{\frac{-\theta\gamma}{1-\gamma}} C_t^{\frac{\gamma}{1-\gamma}} \frac{1}{z_t^{\frac{1}{1-\gamma}}} \tag{5.10}$$

5.2.1 Collecting the Common Equations

The production technology, the equations derived from the optimizing behavior of households are common in flexible price economy and sticky price economy. Here is a list of these common equations, notice that we added the real marginal cost in the list:

$$\begin{aligned}
\frac{w_t(i)}{P_t} &= \frac{\left[\int_0^1 n_t(i) di\right]^{\phi}}{C_t^{-\sigma}} \\
\frac{\chi \left(\frac{M_t}{P_t}\right)^{-\nu}}{C_t^{-\sigma}} &= \frac{R_t - 1}{R_t} \\
C_t^{-\sigma} &= \beta R_t E_t \left[C_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} \right] \\
mc_t &= \frac{w_t(i)}{(1-\gamma)\frac{y_t(i)}{n_t(i)} P_t} \\
y_t(i) &= c_t(i) \\
y_t(i) &= z_t n_t(i)^{1-\gamma} \\
\log z_t &= \rho_z \log z_{t-1} + \epsilon_{z,t}
\end{aligned}$$

The steady states of the nonstochastic version of the equations above are

$$
\begin{aligned}
\frac{\bar{w}(i)}{\bar{P}} &= \frac{\left[\int_0^1 \bar{n}(i)di\right]^\phi}{\bar{C}^{-\sigma}} = \frac{\bar{n}}{\bar{C}^{-\sigma}} \\
\frac{\chi\left(\frac{\bar{M}}{\bar{P}}\right)^{-\nu}}{\bar{C}^{-\sigma}} &= \frac{\bar{R}-1}{\bar{R}} \\
\bar{C}^{-\sigma} &= \beta\bar{R}E_t\left[\bar{C}^{-\sigma}\frac{\bar{P}}{\bar{P}}\right] \Rightarrow \bar{R} = \frac{1}{\beta} \\
\bar{mc}(i) &= \frac{\bar{w}(i)}{(1-\gamma)\frac{\bar{y}(i)}{\bar{n}(i)}\bar{P}} \\
\bar{y}(i) &= \bar{c}(i) = \bar{c} \\
\bar{y}(i) &= \bar{z}\bar{n}(i)^{1-\gamma} = \bar{z}\bar{n}^{1-\gamma} \\
\log\bar{z} &= \rho_z\log\bar{z} \Rightarrow \bar{z} = 1
\end{aligned}
$$

We will next log-linearize all variables around their steady state values. The aim in this step is to make the equations approximately linear in the log-deviations from the steady state[1]. The entries with a hat are the log-deviations of the corresponding variables from their steady state, in other words the approximate percentage deviation. For the log-deviation of real money balances, we use $\hat{m}_t$. Moreover, the inflation is defined to be $\frac{P_t}{P_{t-1}}$, and it is log-linearized around the steady state value of 0. π_t is the log-deviation of inflation, which is by definition $\hat{p}_t - \hat{p}_{t-1}$.

$$
\begin{aligned}
\hat{w}_t(i) - \hat{p}_t &= \phi\hat{n}_t + \sigma\hat{c}_t &(5.11)\\
\hat{m}_t &= \frac{\sigma}{\nu}\hat{c}_t - \frac{1}{(\bar{R}-1)\nu}\hat{R}_t &(5.12)\\
\hat{c}_t &= E_t[\hat{c}_{t+1}] - \frac{1}{\sigma}\left[\hat{R}_t - E_t[\pi_{t+1}]\right] &(5.13)\\
\hat{mc}_t &= \left(\frac{1+\phi}{1-\gamma} - 1 + \sigma\right)\hat{y}_t - \frac{1+\phi}{1-\gamma}\hat{z}_t &(5.14)\\
\hat{y}_t &= \hat{c}_t &(5.15)\\
\hat{y}_t &= \hat{z}_t + (1-\gamma)\hat{n}_t &(5.16)\\
\hat{z}_t &= \rho_z\hat{z}_{t-1} + \epsilon_{z,t} &(5.17)
\end{aligned}
$$

In the derivation of 5.14, labor supply equation 5.11 and production technology 5.16

[1]For the details of the log-linearization procedure, see Uhlig(1995).

are exploited.

5.2.2 The Case of Flexible Prices

Let's first have a look at the case where all firms are able to choose their prices optimally in every period. They maximize their profits:

$$\underbrace{max}_{p_t(i)} \quad \Pi_t(i) = p_t(i)c_t(i) - w_t(i)n_t(i)$$

The demand equation for good i (5.4) and the production technology (4.8) should be utilized at this point. The profit function can be written as,

$$\Pi_t(i) = p_t(i)y_t(i) - w_t(i)\left(\frac{y_t(i)}{z_t}\right)^{\frac{1}{1-\gamma}}$$

$$\Pi_t(i) = p_t(i)\left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t - w_t(i)\left(\frac{\left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t}{z_t}\right)^{\frac{1}{1-\gamma}}$$

Differentiating with respect to $p_t(i)$ gives:

$$\begin{aligned}
\frac{\partial \Pi_t(i)}{\partial P_t(i)} &= (1-\theta)\left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t + \frac{\theta}{1-\gamma}\frac{p_t(i)^{-\frac{\theta}{1-\gamma}-1}}{P_t^{-\frac{\theta}{1-\gamma}}} w_t(i)\left(\frac{C_t}{z_t}\right)^{\frac{1}{1-\gamma}} = 0 \\
0 &= (1-\theta)\left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t + \frac{1}{1-\gamma} w_t(i)\left(\frac{p_t(i)}{P_t}\right)^{\frac{-\theta\gamma}{1-\gamma}} C_t^{\frac{\gamma}{1-\gamma}} \frac{1}{z_t^{\frac{1}{1-\gamma}}}\theta\left(\frac{p_t(i)}{P_t}\right)^{-\theta}\frac{C_t}{p_t(i)}
\end{aligned}$$

If the equation 5.10 is used, the above equation can be written as

$$0 = (1-\theta)\left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t + MC_t(i)\theta\left(\frac{p_t(i)}{P_t}\right)^{-\theta}\frac{C_t}{p_t(i)}$$

Therefore, the price of good i in the case of fully flexible prices is:

$$p_t(i) = \frac{\theta}{\theta-1} MC_t(i) \tag{5.18}$$

Note that in the flexible price equilibrium, the real marginal costs of all firms are constant and equal. Accordingly, the deviation of real marginal costs from its steady state value is zero when the prices are flexible.

$$\frac{MC_t(i)}{p_t(i)} = \frac{\theta - 1}{\theta} \tag{5.19}$$

$$\hat{mc_t}^f = 0 \tag{5.20}$$

In order to get the deviation of output in the flexible price economy, put 7.1 into 5.14. Other log-linearized equations defining the flexible price equilibrium can be obtained:

$$\hat{y}_t^f = \frac{1+\phi}{1+\phi-(1-\sigma)(1-\gamma)}\hat{z}_t \tag{5.21}$$

$$\hat{n}_t^f = \frac{1-\sigma}{1+\phi-(1-\sigma)(1-\gamma)}\hat{z}_t \tag{5.22}$$

$$\hat{r}_t^f = \hat{R}_t^f - E_t[\pi_{t+1}^f] = \frac{(1+\phi)(\rho_z - 1)}{1+\phi-(1-\sigma)(1-\gamma)}\sigma\hat{z}_t \tag{5.23}$$

5.2.3 The Case of Sticky Prices

All firms adjusting their prices in period t face the same problem:

$$\underbrace{max}_{p_t(i)} \quad E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+j}\left[p_t(i)c_{t+j}(i) - w_{t+j}(i)n_{t+j}(i)\right]$$

subject to

$$c_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t$$

The objective function of the firm is equivalent to the following expression given the definitions 4.8, 4.10 and the constraint defining the demand for good i:

$$\underbrace{max}_{p_t(i)} \quad E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+1}\left[p_t(i)\left(\frac{p_{t+j}(i)}{P_{t+j}}\right)^{-\theta} C_{t+j} - w_{t+j}(i)\left(\frac{p_{t+j}(i)}{P_{t+j}}\right)^{\frac{-\theta}{1-\gamma}} C_{t+j}^{\frac{1}{1-\gamma}} \frac{1}{z_{t+j}^{\frac{1}{1-\gamma}}}\right]$$

First order condition is then,

$$E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+1} \left[(1-\theta) \left(\frac{p_t(i)}{P_{t+j}} \right)^{-\theta} C_{t+j} + \frac{\theta}{1-\gamma} w_{t+j}(i) \left(\frac{p_t(i)^{\frac{-\theta}{1-\gamma}-1}}{P_{t+j}^{\frac{-\theta}{1-\gamma}}} \right) C_{t+j}^{\frac{1}{1-\gamma}} \frac{1}{z_{t+j}^{\frac{1}{1-\gamma}}} \right] = 0$$

$$E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+1} \left[(\theta-1) \left(\frac{p_t(i)}{P_{t+j}} \right)^{-\theta} C_{t+j} \right] = E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+1} \left[MC_{t+j}(i) \theta \left(\frac{p_t(i)}{P_{t+j}} \right)^{-\theta} \frac{C_{t+j}}{p_t(i)} \right]$$

So, a firm trying to maximize its profits will choose the following price:

$$p_t(i) = \left(\frac{\theta}{\theta-1} \right) \frac{E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+1} MC_{t+j}(i) c_{t+j}(i)}{E_t \sum_{j=0}^{\infty} \alpha^j Q_{t,t+1} c_{t+j}(i)} \tag{5.24}$$

5.3 The New Keynesian Phillips Curve

Here we show the derivation of the New Keynesian Phillips Curve.

From the Euler Equation (5.7), we have;

$$Q_{t,t+j} = \beta^j E_t \left[\left(\frac{C_{t+j}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+j}} \right]$$

Plugging this expression into 5.24,

$$(\theta-1) \sum_{j=0}^{\infty} (\alpha\beta)^j E_t \left[\left(\frac{C_{t+j}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+j}} p_t(i) c_{t+j}(i) \right] = \theta \sum_{j=0}^{\infty} (\alpha\beta)^j E_t \left[\left(\frac{C_{t+j}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+j}} MC_{t+j}(i) c_{t+j}(i) \right]$$

Log-linearizing both sides of the equation leads to,

$$\sum_{j=0}^{\infty} (\alpha\beta)^j E_t \left[-\sigma \left(\hat{c}_{t+j} - \hat{c}_t \right) + \hat{p}_t - \hat{p}_{t+j} + \hat{c}_{t+j}(i) + \hat{p}_t(i) \right]$$
$$= \sum_{j=0}^{\infty} (\alpha\beta)^j E_t \left[-\sigma \left(\hat{c}_{t+j} - \hat{c}_t \right) + \hat{p}_t - \hat{p}_{t+j} + \hat{c}_{t+j}(i) + \hat{MC}_{t+j}(i) \right]$$

Therefore, value maximizing firm i will set the price according to the log-linear rule:

$$\hat{p}_t^* = (1-\alpha\beta) \sum_{j=0}^{\infty} E_t \left[(\alpha\beta)^j \hat{MC}_{t+j} \right]$$

$$\hat{p}_t^* = (1-\alpha\beta)\hat{MC}_t + \alpha\beta E_t\left[\hat{p}_{t+1}^*\right] \tag{5.25}$$

From 5.3, the average price in period t satisfies

$$P_t^{1-\theta} = (1-\alpha)(P_t^*)^{1-\theta} + \alpha P_{t-1}^{1-\theta}$$

Log-linearization gives

$$\hat{p}_t = (1-\alpha)\hat{p}_t^* + \alpha\hat{p}_{t-1} \tag{5.26}$$

Multiplying 5.25 with $(1-\alpha)$ and using equation 5.26 we get the realtionship below. Note that we use $\hat{mc}_t$ to for the deviation of real marginal costs in period to from its steady state value.

$$\hat{p}_t - \alpha\hat{p}_{t-1} = (1-\alpha)(1-\alpha\beta)(\hat{mc}_t + \hat{p}_t) + \alpha\beta E_t[\hat{p}_{t+1} - \alpha\hat{p}_t]$$

After some algebra, we arrive at

$$\begin{aligned}\hat{p}_t - \hat{p}_{t-1} &= \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\hat{mc}_t + \beta E_t[\hat{p}_{t+1} - \hat{p}_t] \\ \pi_t &= \beta E_t[\pi_{t+1}] + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\hat{mc}_t\end{aligned}$$

Keep in mind that the labor demand equation (5.14) holds both in the flexible price case and the sticky price case. Making use of this fact and 7.1,

$$\hat{mc}_t = \left(\frac{1+\phi}{1-\gamma} - 1 + \sigma\right)(\hat{y}_t - \hat{y}_t^f)$$

Let output gap be $\hat{x}_t = \hat{y}_t - \hat{y}_t^f$, which is the percent deviation of the real output in case of sticky prices from the level of output which would be relevant in the case of flexible prices. Then

$$\hat{mc}_t = \left(\frac{1+\phi}{1-\gamma} - 1 + \sigma\right)\hat{x}_t$$

We can now obtain the New Keynesian Phillips Curve, which suggests that current inflation depends positively on expected future inflation and negatively on the current output gap:

$$\pi_t = \beta E_t[\pi_{t+1}] + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\frac{1+\phi}{1-\gamma} - 1 + \sigma\right)\hat{x}_t \tag{5.27}$$

5.4 The New Keynesian IS Curve

We know that the Euler equation (5.13) holds both in the flexible price economy and the sticky price economy and $\hat{c}_t = \hat{y}_t$, that is

$$\begin{aligned} \hat{y}_t &= E_t[\hat{y}_{t+1}] - \frac{1}{\sigma}\left[\hat{R}_t - E_t[\pi_{t+1}]\right] = E_t[\hat{y}_{t+1}] - \frac{1}{\sigma}\hat{r}_t \\ \hat{y}_t^f &= E_t[\hat{y}_{t+1}^f] - \frac{1}{\sigma}\left[\hat{R}_t^f - E_t[\pi_{t+1}^f]\right] = E_t[\hat{y}_{t+1}^f] - \frac{1}{\sigma}\hat{r}_t^f \end{aligned}$$

We get the New Keynesian IS Curve by substracting the latter equation from the former:

$$\hat{x}_t = E_t[\hat{x}_{t+1}] - \frac{1}{\sigma}\left(\hat{R}_t - E_t[\pi_{t+1}] - \hat{r}_t^f\right) \tag{5.28}$$

It is implied that the current output gap depends positively on the future expected output gap and negatively on the current real interest rate relative to the flexible price real interest rate.

5.5 Monetary Policy Authority

As it is mentioned when introducing the model, the central bank follows a Taylor rule in the form of

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + \phi_\pi \pi_t + \phi_x \hat{x}_t + \epsilon_{R,t} \tag{5.29}$$

where $\epsilon_{R,t}$ is the monetary policy shock and white noise.

5.6 The New Keynesian Model with Calvo Sticky Prices

We can list now the equations which define the equilibrium of our economy and which will be used in analyzing the stochastic dynamics of our model.

$$
\begin{aligned}
\pi_t &= \beta E_t[\pi_{t+1}] + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\frac{1+\phi}{1-\gamma} - 1 + \sigma\right)\hat{x}_t \\
\hat{x}_t &= E_t[\hat{x}_{t+1}] - \frac{1}{\sigma}\left(\hat{R}_t - E_t[\pi_{t+1}] - \hat{r}_t^f\right) \\
\hat{r}_t^f &= \frac{(1+\phi)(\rho_z - 1)}{1+\phi-(1-\sigma)(1-\gamma)}\sigma\hat{z}_t \\
\hat{m}_t &= \frac{\sigma}{\nu}\hat{x}_t + \left(\frac{\sigma}{\nu}\right)\frac{1+\phi}{1+\phi-(1-\sigma)(1-\gamma)}\hat{z}_t - \frac{1}{(\bar{R}-1)\nu}\hat{R}_t \\
\hat{R}_t &= \phi_R\hat{R}_{t-1} + \phi_\pi\pi_t + \phi_x\hat{x}_t + \epsilon_{R,t} \\
\hat{y}_t^f &= \frac{1+\phi}{1+\phi-(1-\sigma)(1-\gamma)}\hat{z}_t \\
\hat{y}_t &= \hat{y}_t^f + \hat{x}_t \\
\hat{mc}_t &= \left(\frac{1+\phi}{1-\gamma} - 1 + \sigma\right)\hat{y}_t - \frac{1+\phi}{1-\gamma}\hat{z}_t \\
\hat{n}_t &= \frac{1}{(1-\gamma)}(\hat{y}_t - \hat{z}_t) \\
\hat{w}_t &= \phi\hat{n}_t + \sigma\hat{y}_t \\
\hat{z}_t &= \rho_z\hat{z}_{t-1} + \epsilon_{z,t} \\
\epsilon_{R,t} &= \rho_R\epsilon_{R,t-1} + \varsigma_t
\end{aligned}
$$

Notice that the real money demand equation is rewritten using the definition of output gap and flexible price output, and $\hat{w}_t$ in the last equation refers to deviation from real wage rate.

As endogenous state variables, we choose $\hat{x}_t$, π_t, $\hat{R}_t$; as other endogenous variables $\hat{r}_t^f$, $\hat{m}_t$, $\hat{y}_t^f$, $\hat{y}_t$, $\hat{mc}_t$, $\hat{n}_t$, $\hat{w}_t$ and as exogenous states $\hat{z}_t$ and $\epsilon_{R,t}$. The required matrices to implement the model in Toolkit[2] are formed accordingly.

[2]See Uhlig (1995) for the detailed notation.

6 Model Results and Answer

In this section we first calibrate the model, and then we show the impulse responses of the economy to monetary policy and technology shocks. Our next step is illustrating the unconditional estimations of inflation and output gap utilizing the derived properties of our economy under two different scenarios: VIR versus CIR inflation targeting; and pointing at the differences between the two policies.

6.1 Calibration

Our focus is now on the calibration of our model. We follow the lecture notes of Monetary Theory and Policy from Prof. Uhlig (2003, winter semester) to calibrate the model. In fact, some of the parameters in the notes are taken from Galí (2002), Cooley and Prescott (1995), and Mankiw and Reis (2002). Others are just set to match the data, i.e. ν[1]. We set the stochastic discount factor, $\beta = 0.99$. In the baseline calibration, we assume that $\sigma = 2$, while we consider other variations later on. As the inverse of the labor supply elasticity, ϕ, we take the value 1.5. The elasticity of the real money balances, ν, is equated to 2. The fraction of firms that cannot set their prices, α, is decided to be 0.75, which implies an average price duration of one year. This value fits the economic estimates of the parameter α, and the evidence from surveys[2]. The price elasticity of demand is taken as 11, which is compatible with a steady state mark-up of 10% as in Galí, Gertler and López-Salido (2001). They suggest a value of 2/3 for the labor share, hence we set the parameter $\gamma = 1/3$ to satisfy their recommendation.

For the values of coefficients in the Taylor rule with interest rate smoothing, we refer to Söderström, Söderlind and Vredin (2002). They point that the large persistence

[1] I thank Mathias Trabandt for being so kind to tell the exact resources.

[2] The existing survey evidence can be found in Taylor (1999)

in the interest rate cannot be explained by persistence in inflation and output alone, which a simple Taylor rule suggests. So they estimate a Taylor rule of the type used in our model. They use quarterly data in their estimation procedure, which is consistent with the fact that our periods refer to one quarter. Accordingly, we directly take the values proposed by Söderström et al. (2002), that is for the coefficient of inflation rate, $\phi_\pi = 0.466735$, for the coefficient of output gap, $\phi_x = 0.277151$ and for the coefficient of lagged interest rate, $\phi_R = 0.711$. As it is seen, the short term interest rate is quite persistent. These values are very close to those estimated by Rudebusch (2002b).

Finally, the values of the shock parameters are set. The autocorrelation coefficient of technology shock policy, ρ_z is chosen to be 0.95, and the standard deviation of the technology shock (σ_z) in percentage units is 0.712. Since the monetary shock is assumed to be a white noise process, we set $\rho_R = 0$. The standard deviation of the monetary policy shock, σ_R is assumed to be equal to the value estimated by Söderström et al. (2002). We summarize our calibration for our baseline model in Table 6.1

Parameter	Value	Economic Interpretation
β	0.99	Stochastic discount factor
σ	2	Coefficient of relative risk aversion
ϕ	1.5	Inverse of the labor supply elasticity
ν	2	Elasticity of the real money balances
α	0.75	The fraction of firms that cannot set their prices
θ	11	Price elasticity of demand
$1-\gamma$	2/3	Labor share
ϕ_π	0.466735	Coefficient of inflation rate in the Taylor rule
ϕ_x	0.277151	Coefficient of output gap in the Taylor rule
ϕ_R	0.711	Coefficient of lagged interest rate in the Taylor rule
ρ_z	0.95	Autocorrelation coefficient of the technology shock
σ_z	0.712	Standard deviation of the technology shock
ρ_R	0	Autocorrelation coefficient of the monetary policy shock
σ_R	0.35	Standard deviation of the monetary policy shock

Table 6.1: Calibration of the baseline model

6.2 Impulse Responses

6.2.1 Technology Shock

Figure 6.1 shows the impulse responses to a favorable technology shock. The positive technology shock increases the marginal productivity of employers, so that the output increases. But the increase in output in case of sticky prices is not as much as its increase in the flexible price case, since the firms cannot adjust their prices immediately under sticky prices. Hence, a negative output gap occurs. The favorable technology shock leads marginal costs of the firms to decline. The firms which can set their prices prefer to lower the price level in the short run, which causes a negative deviation in inflation. Following the fall in aggregate price level, the demand will go up. But the extent of increase in productivity is larger than the increase in demand; as a result the aggregate labor declines. The negative deviation in employment is consistent with the observations of Galí (1999) and Basu, Fernald and Kimball (1998), however, the response of employment depends on the parameter values considered[3] (Galí (2002)). The real wages move upwards. Employees face a trade off between income and substitution effects, and in this case income effect dominates, they work less consume more. In order to mitigate the negative impulse responses of inflation and output gap, the monetary policy authority drops the interest rate. The economy reaches its steady state after a while, i.e. the impulse responses are dampened.

6.2.2 Monetary Policy Shock

The impulse responses to a monetary policy shock is illustrated in Figure 6.2. The unexpected monetary tightening increases the short term nominal interest rate. The rise in the interest rate means that, the opportunity cost of holding money decreases. So people consume less, work less and increase their money holdings. The raised interest rate acts contractionary, and a decrease in output and output gap is observed. The firms which can adjust their prices decrease the price level following the fall in output, so the aggregate price level falls. The marginal cost and the real wages fall as well. The effects of a monetary policy shock are not persistent, since it is assumed to be a white noise process in our setting. After one quarter, the impulse responses

[3] Galí (2002) shows that the negative response of employment holds for a large subset of parameter values considered, however it is not a necessary implication.

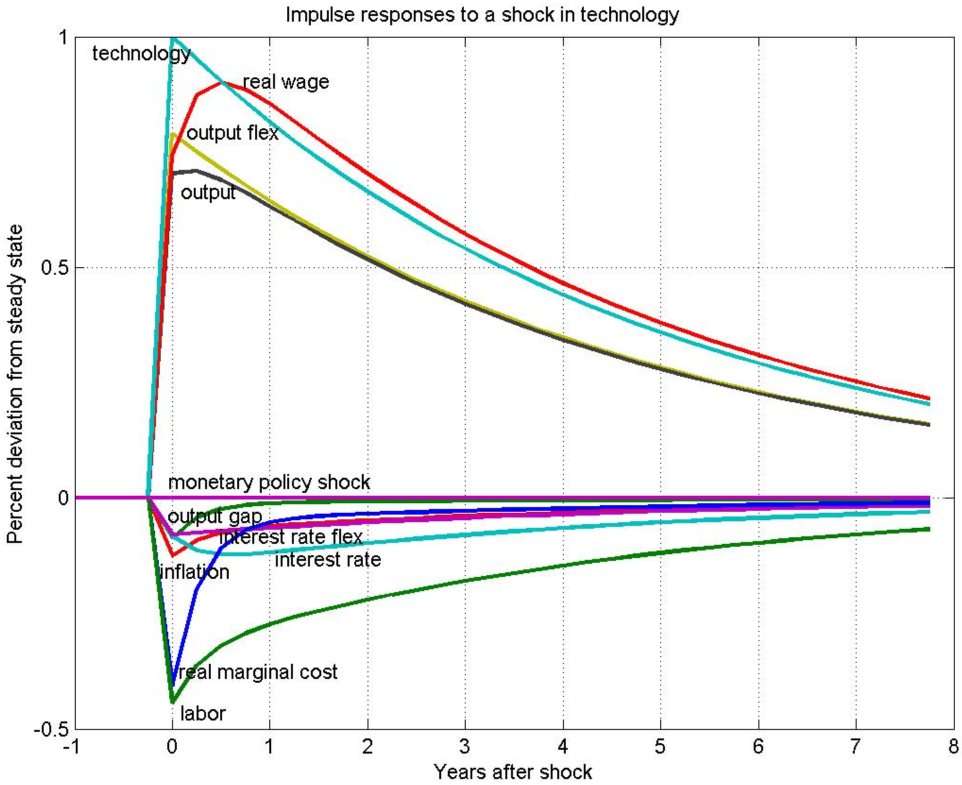


Figure 6.1: Impulse responses to a technology shock

approach zero, and the economy reaches its steady state.

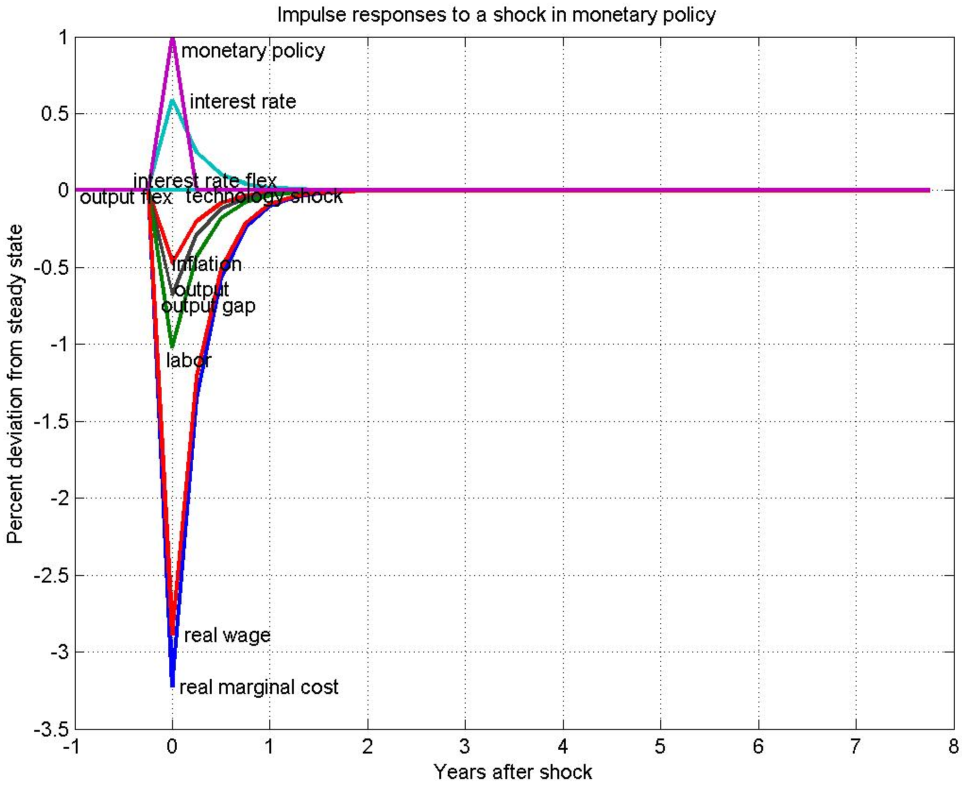


Figure 6.2: Impulse responses to a monetary policy shock

6.3 Inflation and Output Gap Forecasts

At this point, we know how the economy would evolve in case of a positive technology shock and a monetary policy shock. The Toolkit provides us the recursive equilibrium law of motion for the economy:

$$\begin{aligned} X_t &= PX_{t-1} + QZ_t \\ Y_t &= RX_{t-1} + SZ_t \\ Z_t &= NZ_{t-1} + \epsilon_t \end{aligned}$$

where X_t is a vector of endogenous state variables, Y_t is a vector of other endogenous variables, Z_t is a vector of exogenous stochastic processes and P, Q, R and S are the solution matrices. We have a nice machinery to make a forecast of inflation and output gap. We base our projections on unconditional forecasts, so $E_t[\epsilon_t] = 0$.

We forecast inflation and output gap under two scenarios: first we let the interest rate evolve according to verified law of motion; second we assume that the central bank applies constant interest rate inflation targeting and sticks to its assumption that interest rate remains constant during the targeting horizon ($t = 10$ in our case).

Note that, to be able make forecasts, we need to make an assumption about the starting points at date $t - 1$, i.e. the state of the economy at date $t - 1$. Then, the determined law of motion lets us to estimate the projections by solving the system forward. We generate our projections by a Matlab code, which uses the law of motion of our economy and requires the starting points of inflation, interest rate and output gap at date $t-1$. The code calculating the forecasts can be found in the CD supplied. First, we assume that our economy is in steady state at date $t - 1$, and at date 0 a technology shock hits. The central bank makes its projections at date 0, when the shock hits. Figure 6.3 shows, how the central bank estimates the path of the deviation of the interest rate from its steady state under the two scenarios. Notice that, the central bank, which applies CIR inflation targeting, supposes that the deviation of the interest rate from its steady state value persists during the forecast horizon, keeping the interest rate constant at its $t = 0$ value.

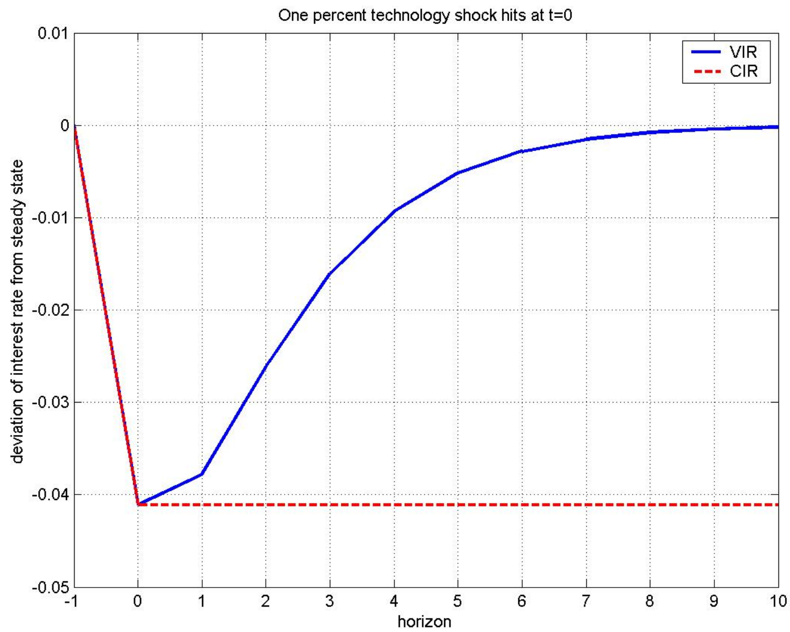


Figure 6.3: Deviation of interest rate from its steady state

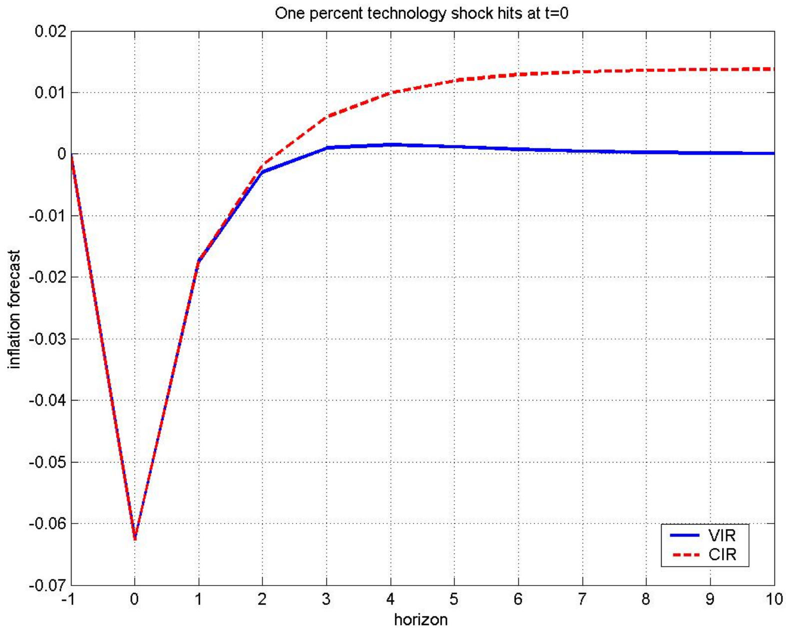


Figure 6.4: Inflation Forecast

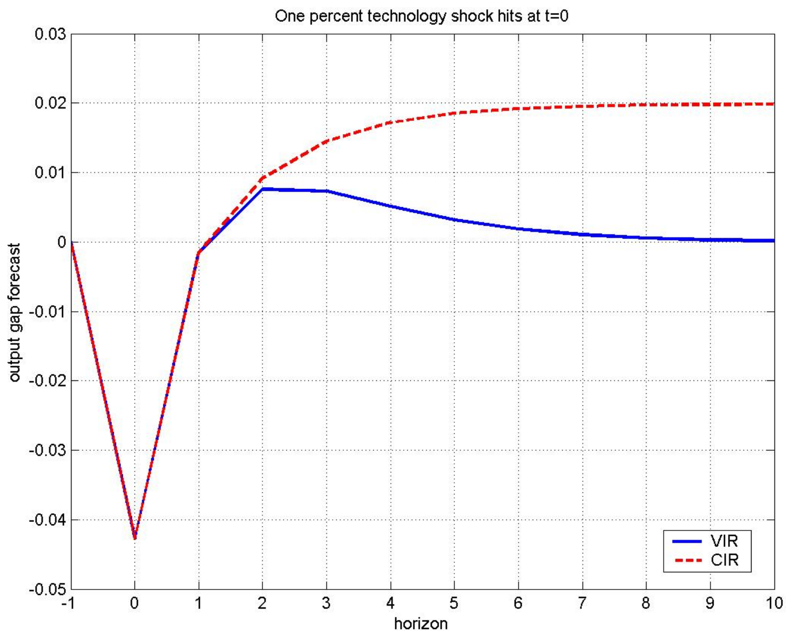


Figure 6.5: Output Gap Forecast

We illustrate the inflation and output forecasts in Figure 6.4 and Figure 6.5 respectively. It is observed that when the interest rate is let to vary during the targeting horizon, the inflation and output gap arrive at their steady state values. However, when the interest rate is assumed to be constant, the central bank makes a systematic mistake in the projections of inflation and output gap. The policy makers will see the falling output gap and diminishing aggregate prices, and decrease the interest rate at date 0. But when they keep the interest rate constant at date 0 value, the persisting negative deviation of interest rate during the forecast horizon will result in an inflationary economy, with positive output gap and an inflation above the target[4]. The positive output gap triggers positive inflation all through the forecast horizon and vice versa. The central bankers then tend to deviate from their assumption about constant interest rates, and increase the interest rate in order to lower output gap and inflation. They are likely to deviate from their commitment, which will later on indulge the credibility and reliability of the central bank.

Now we will look at another situation, we assume that the technology shock hit two periods ago. The central bank makes the forecasts of inflation and output gap at date 0. The path of interest rate, inflation and output gap are the same when they have a time-varying interest rate assumption. The CIR targeting central bank takes the value of the interest rate at date 0 to be constant (see Figure 6.6), compared to the previous case the effect of the technology shock is more moderate and the deviation of interest rate from steady state is less. As a result, the systematic mistakes in the forecasts of inflation and output gap are now milder, however, the mistakes still exist (see Figure 6.7 and Figure 6.8). The inflation is still above the target. It could be the case that the central bank made the projections at a date, when the economy almost reached its steady state, then the inflation would be on target and the central bank would not realize its mistake (see Figure 6.9), in other words the central bankers would not be inclined to change the interest rate they set.

[4]The target inflation interval of the Bank of England is $2.5\% \pm 0.5\%$. Following the practice of the Bank of England, we let inflation to deviate 0.5% up or down from the steady state value of zero inflation.

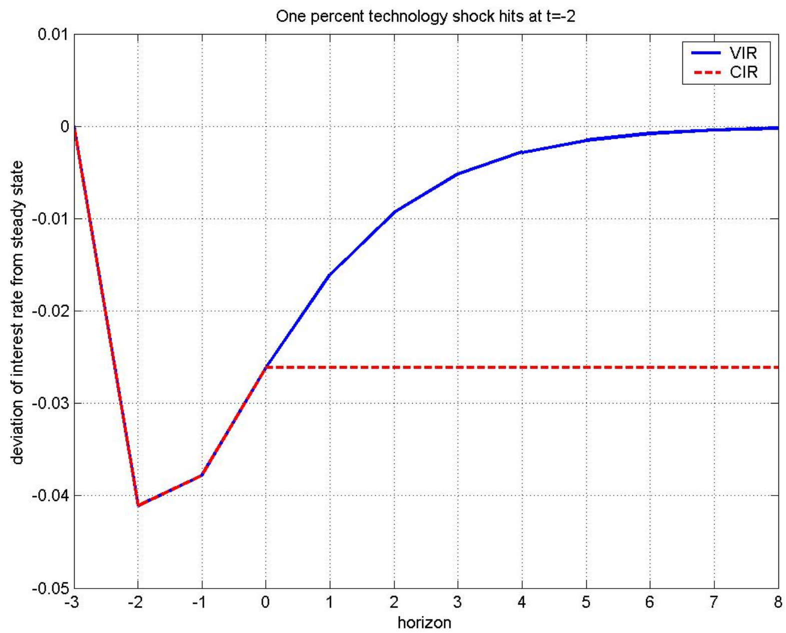


Figure 6.6: Deviation of interest rate from its steady state (technology shock at t=-2)

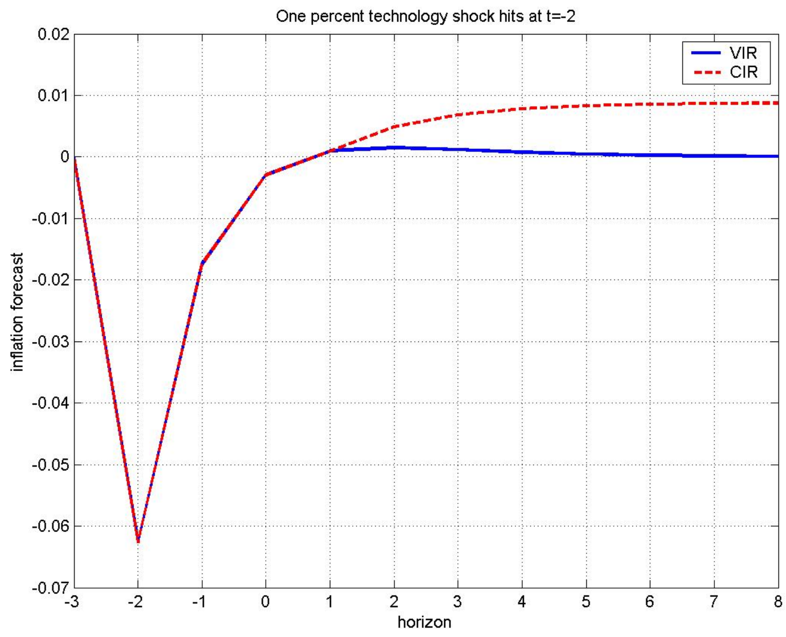


Figure 6.7: Inflation Forecast (technology shock at t=-2)

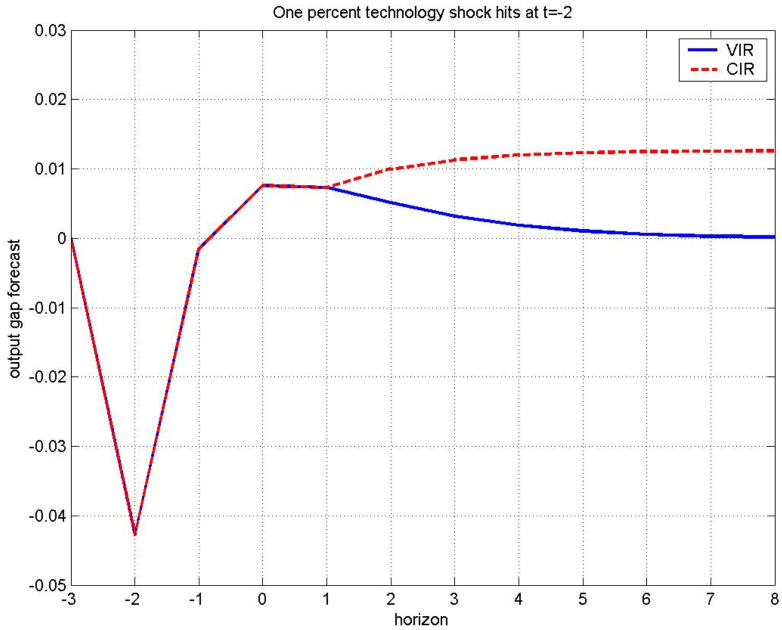


Figure 6.8: Output Gap Forecast (technology shock at t=-2)

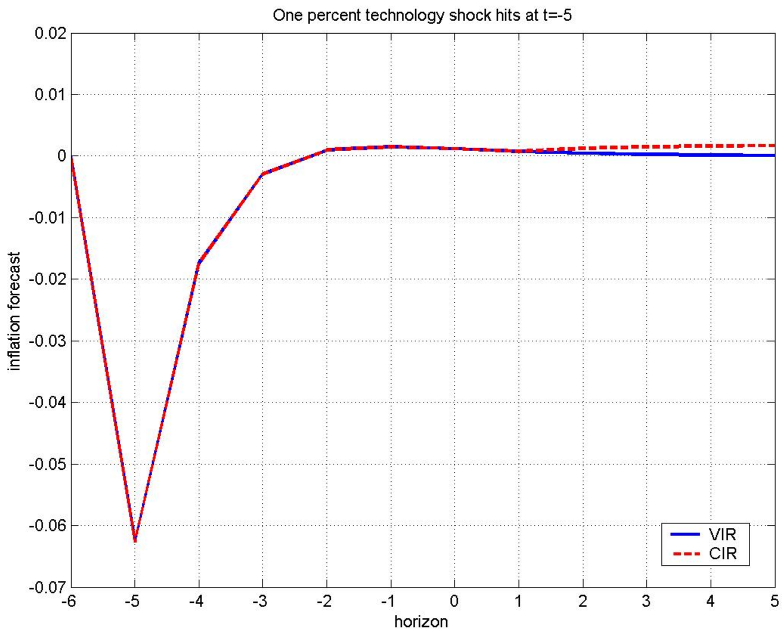


Figure 6.9: Inflation Forecast (technology shock at t=-5)

7 Variations

The basic model we analysed is nice to see the properties of the economy and understand essentially how CIR inflation targeting differs from VIR inflation targeting. However, there are no other disturbances that might cause movements in output gap and inflation than the monetary policy shock and the productivity shock. It might be worth to bring the model closer to the real world economies by introducing a few more stochastic disturbances, such as a preference shock, a cost-push shock and a government spending shock. The shocks will not be added exogenously[1] to our model; they will all evolve endogenously to have better micro-foundations. In the extended model, we will once more check how CIR inflation targeting differs from VIR inflation targeting and see if the difference is more severe or not.

In this section, we will look at a sensitivity analysis simultaneously. We aim to see how our results are dependent on the risk aversion degree of the households. The coefficient of relative risk aversion, σ, will be assigned different values making the households more (σ=5) and less (σ=1) risk averse.

7.1 Introducing Economic Disturbances

The first shock introduced is a preference shock which has an effect on the representative household's utility from consumption, and the second one is a stochastic wage mark-up[2] that influences the amount of labor supplied by the household. Via the inclusion of the latter disturbance, it turns out that we have a cost-push shock in our economy. The utility function of the household (4.1) is adjusted by incorporating a

[1] Clarida, Galí, Gertler (1999) show that an exogenous cost-push shock allows the new Keynesian model to generate a meaningful policy problem.

[2] Clarida, Galí and Gertler (2001) put forward adding a stochastic wage mark-up so as to have a stochastic shock in the New Keynesian Phillips Curve.

random preference shock and a random wage mark-up:

$$\underbrace{max}_{C_t,N_t,M_t,A_{t+1}} \quad E_t\left\{\sum_{t=0}^{\infty}\beta^t\left[\frac{(\varphi_t C_t)^{1-\sigma}}{1-\sigma}+\frac{\chi}{1-\nu}\left(\frac{M_t}{P_t}\right)^{1-\nu}-\frac{e^{u_t}N_t^{1+\phi}}{1+\phi}\right]\right\} \tag{7.1}$$

where

$$\begin{aligned}\log\varphi_t &= \rho_\varphi \log\varphi_{t-1}+\epsilon_{\varphi,t}\\ u_t &= \rho_u u_{t-1}+\epsilon_{u,t}\end{aligned}$$

and $\epsilon_{\varphi,t}$ and $\epsilon_{u,t}$ are white noise processes.

This modification changes the first order conditions:

$$\begin{aligned}\lambda_t &= C_t^{-\sigma}\varphi_t^{1-\sigma}\\ \frac{w_t(i)}{P_t} &= \frac{\left[\int_0^1 n_t(i)di\right]^{\phi}e^{u_t}}{C_t^{-\sigma}\varphi_t^{1-\sigma}}\\ C_t^{-\sigma} &= \beta R_t E_t\left[C_{t+1}^{-\sigma}\frac{\varphi_{t+1}^{1-\sigma}}{\varphi_t^{1-\sigma}}\frac{P_t}{P_{t+1}}\right]\\ \frac{\chi\left(\frac{M_t}{P_t}\right)^{-\nu}}{C_t^{-\sigma}\varphi_t^{1-\sigma}} &= \frac{R_t-1}{R_t}\end{aligned}$$

One further alteration in the model is that the government is supposed to finance government expenditure on the individual goods. The aggregate goods market equilibrium condition turns into:

$$y_t = c_t + g_t \tag{7.2}$$

The government expenditure is assumed to follow an exogenous stochastic process:

$$\log g_t = \rho_g \log g_{t-1} + \epsilon_{g,t}$$

The log-linearization of the new list of equations yields:

$$\begin{aligned}\hat{w}_t(i)-\hat{p}_t &= \phi\hat{n}_t+\sigma\hat{c}_t+u_t-(1-\sigma)\hat{\varphi}_t\\ \hat{m}_t &= \frac{\sigma}{\nu}\hat{c}_t-\frac{1}{(\bar{R}-1)\nu}\hat{R}_t-\frac{(1-\sigma)}{\nu}\hat{\varphi}_t\\ \hat{c}_t &= E_t[\hat{c}_{t+1}]-\frac{1}{\sigma}\left[\hat{R}_t-E_t[\pi_{t+1}]\right]+\frac{\sigma-1}{\sigma}\left[E_t[\varphi_{t+1}]-\varphi_t\right]\end{aligned}$$

$$
\begin{aligned}
\hat{mc}_t &= \left(\frac{1+\phi}{1-\gamma} + \frac{\sigma}{\frac{\bar{c}}{\bar{y}}} - 1\right)\hat{y}_t - \frac{1+\phi}{1-\gamma}\hat{z}_t - \frac{\sigma\frac{\bar{g}}{\bar{y}}}{\frac{\bar{c}}{\bar{y}}}\hat{g}_t - (1-\sigma)\hat{\varphi}_t + u_t \\
\hat{y}_t &= \frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{g}}{\bar{y}}\hat{g} \\
\hat{\varphi}_t &= \rho_\varphi\hat{\varphi}_{t-1} + \epsilon_{\varphi,t} \\
\hat{g}_t &= \rho_g\hat{g}_{t-1} + \epsilon_{g,t} \\
u_t &= \rho_u u_{t-1} + \epsilon_{u,t}
\end{aligned}
$$

where $\bar{g}/\bar{y}$ is the share of government spending in the aggregate output, and $\bar{c}/\bar{y}$ is the share of consumption of households.

Note that in the extended model the real marginal costs of firms in the flexible price equilibrium are not constant, but equal to the stochastic wage mark-up which affect the real wages:

$$\hat{mc}_t^f = u_t$$

Using the last statement, the marginal cost can be represented in terms of output gap and the stochastic wage mark-up:

$$\hat{mc}_t = \left(\frac{1+\phi}{1-\gamma} + \frac{\sigma}{\frac{\bar{c}}{\bar{y}}} - 1\right)\hat{x}_t + u_t$$

This implies that the New Keynesian Phillips Curve becomes

$$\pi_t = \beta E_t[\pi_{t+1}] + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\frac{1+\phi}{1-\gamma} + \frac{\sigma}{\frac{\bar{c}}{\bar{y}}} - 1\right)\hat{x}_t + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}u_t$$

In this formulation u_t is the source of disturbance in the New Keynesian Phillips Curve, which is often called *cost-push shock, inflation shock* or *price shock* in the literature.

The New Keynesian IS Curve should also be adjusted. The goods market equilibrium equation is used to eliminate $\hat{c}_t$, and then the definition of output gap is utilized to replace $\hat{y}_t$. We get

$$\hat{x}_t = \quad E_t[\hat{x}_{t+1}] - \frac{\bar{c}}{\bar{y}}(\frac{1}{\sigma})\left(\hat{R}_t - E_t[\pi_{t+1}]\right) + \frac{\bar{c}}{\bar{y}}(\frac{\sigma-1}{\sigma})\left[E_t[\hat{\varphi}_{t+1}] - \hat{\varphi}_t\right]$$

$$-\frac{\bar{g}}{\bar{y}}\left[E_t[\hat{g}_{t+1}] - \hat{g}_t\right] + \left[E_t[\hat{y}^f_{t+1}] - \hat{y}^f_t\right]$$

Making use of the output gap definition and the goods market equilibrium once more, we can write the money demand equation:

$$\hat{m}_t = \frac{\sigma}{\nu\frac{\bar{c}}{\bar{y}}}\hat{x}_t + \frac{\sigma}{\nu\frac{\bar{c}}{\bar{y}}}\hat{y}^f_t - \frac{1}{(\bar{R}-1)\nu}\hat{R}_t - \frac{(1-\sigma)}{\nu}\hat{\varphi}_t - \frac{\sigma\frac{\bar{g}}{\bar{y}}}{\nu\frac{\bar{c}}{\bar{y}}}\hat{g}_t$$

Let us now bring the equations which define the equilibrium of our extended economy together:

$$\begin{aligned}
\pi_t &= \beta E_t[\pi_{t+1}] + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}\left(\frac{1+\phi}{1-\gamma} + \frac{\sigma}{\frac{\bar{c}}{\bar{y}}} - 1\right)\hat{x}_t + \frac{(1-\alpha)(1-\alpha\beta)}{\alpha}u_t \\
\hat{x}_t &= E_t[\hat{x}_{t+1}] - \frac{\bar{c}}{\bar{y}}(\frac{1}{\sigma})\left(\hat{R}_t - E_t[\pi_{t+1}]\right) + \frac{\bar{c}}{\bar{y}}(\frac{\sigma-1}{\sigma})\left[E_t[\hat{\varphi}_{t+1}] - \hat{\varphi}_t\right] \\
&\quad -\frac{\bar{g}}{\bar{y}}\left[E_t[\hat{g}_{t+1}] - \hat{g}_t\right] + \left[E_t[\hat{y}^f_{t+1}] - \hat{y}^f_t\right] \\
\hat{m}_t &= \frac{\sigma}{\nu\frac{\bar{c}}{\bar{y}}}\hat{x}_t + \frac{\sigma}{\nu\frac{\bar{c}}{\bar{y}}}\hat{y}^f_t - \frac{1}{(\bar{R}-1)\nu}\hat{R}_t - \frac{(1-\sigma)}{\nu}\hat{\varphi}_t - \frac{\sigma\frac{\bar{g}}{\bar{y}}}{\nu\frac{\bar{c}}{\bar{y}}}\hat{g}_t \\
\hat{R}_t &= \phi_R\hat{R}_{t-1} + \phi_\pi\pi_t + \phi_x\hat{x}_t + \epsilon_{R,t} \\
\hat{y}_t &= \frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{g}}{\bar{y}}\hat{g} \\
\hat{y}^f_t &= \left(\frac{1+\phi}{1-\gamma}\hat{z}_t + \frac{\sigma\frac{\bar{g}}{\bar{y}}}{\frac{\bar{c}}{\bar{y}}}\hat{g}_t + (1-\sigma)\hat{\varphi}_t\right) / \left(\frac{1+\phi}{1-\gamma} + \frac{\sigma}{\frac{\bar{c}}{\bar{y}}} - 1\right) \\
\hat{y}_t &= \hat{y}^f_t + \hat{x}_t \\
\hat{mc}_t &= \left(\frac{1+\phi}{1-\gamma} + \frac{\sigma}{\frac{\bar{c}}{\bar{y}}} - 1\right)\hat{x}_t + u_t \\
\hat{n}_t &= \frac{1}{(1-\gamma)}(\hat{y}_t - \hat{z}_t) \\
\hat{w}_t &= \phi\hat{n}_t + \sigma\hat{c}_t + u_t - (1-\sigma)\hat{\varphi}_t \\
\hat{z}_t &= \rho_z\hat{z}_{t-1} + \epsilon_{z,t} \\
\hat{\varphi}_t &= \rho_\varphi\hat{\varphi}_{t-1} + \epsilon_{\varphi,t} \\
\hat{g}_t &= \rho_g\hat{g}_{t-1} + \epsilon_{g,t} \\
u_t &= \rho_u u_{t-1} + \epsilon_{u,t} \\
\epsilon_{R,t} &= \rho_R\epsilon_{R,t-1} + \varsigma_t
\end{aligned}$$

As endogenous state variables, we choose again $\hat{x}_t$, π_t, $\hat{R}_t$; as other endogenous variables $\hat{m}_t$, $\hat{y}_t^f$, $\hat{y}_t$, $\hat{mc}_t$, $\hat{n}_t$, $\hat{w}_t$, $\hat{c}_t$ and as exogenous states $\hat{z}_t$, $\hat{\varphi}_t$, $\hat{g}_t$, u_t and $\epsilon_{R,t}$.

7.2 Calibration

We added three new shocks to our system and we need to calibrate the new parameter values to solve the extended model. Table 7.1 shows the values of new parameters. All the other parameter values remain the same. But in order to see how our results depend on the risk aversion degree of the households, we will set once $\sigma = 1$, and once $\sigma = 5$. The values of preference shock parameters are based on Ireland (2004). We follow Collard and Dellas (2005) for the share of government spending; and autocorrelation coefficient and standard deviation of the government spending shock.

Parameter	Value	Economic Interpretation
$\bar{c}/\bar{y}$	0.8	Share of consumption
$\bar{g}/\bar{y}$	0.2	Share of government spending
ρ_φ	0.947	Autocorrelation coefficient of the preference shock
σ_φ	4.05	Standard deviation of the preference shock
ρ_g	0.97	Autocorrelation coefficient of the government spending shock
σ_g	2	Standard deviation of the government spending shock
ρ_u	0.5	Autocorrelation coefficient of the wage mark-up
σ_u	0.66	Standard deviation of the wage mark-up

Table 7.1: Calibration of the extended model

7.3 Impulse Responses

7.3.1 Cost-Push Shock

When a cost-push shock hits the economy, the immediate effect is an increase in the real marginal costs of the firms and increase in the real wages (remember that the introduction of the cost-push shock is in fact through a stochastic wage mark-up). The firms will prefer to set their prices higher, which results in an inflationary economy. The firms which cannot adjust their prices lower their output, so the aggregate output as well as the output gap falls. The consumption also decreases, and less labor is needed to produce a less amount of output. The impulse responses are shown in Figure 7.1.

One remarkable observation in the impulse response is that, there is a short run trade off between inflation and output stabilization. In the baseline model, a central bank which stabilized the inflation could also stabilize output gap. However, when a cost-push shock is introduced, they face a trade off[3]. They set a higher interest rate with the aim of reducing the inflation, but this triggers the fall in output. Output gap is only closed, after the inflation reaches its steady state value.

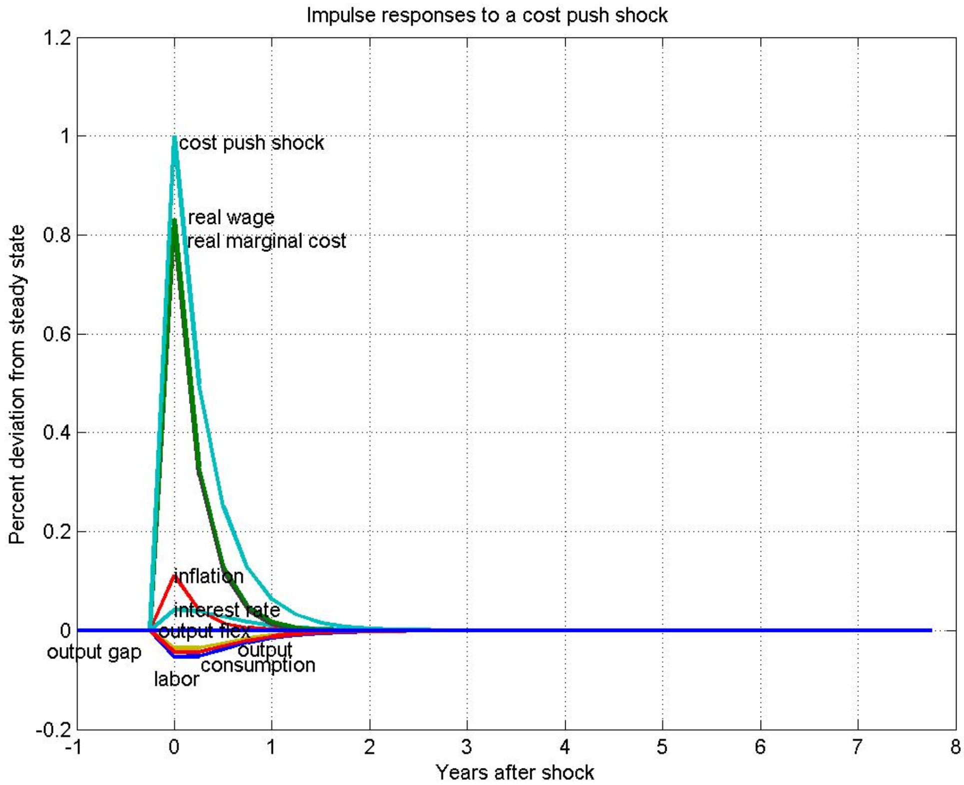


Figure 7.1: Impulse responses to a cost-push shock

[3]See Clarida, Galí, Gertler (1999) and Galí (2002).

7.3.2 Preference Shock

The preference shock affects the marginal utility from consumption that households enjoy. In our set up with $\sigma = 2$, a positive preference shock decreases the marginal utility from consumption. Therefore, households consume less. Since the demand declines, the firms produce less output, and the ones which can set their prices lower the price level. The firms need less labor now. The firms which cannot update their prices diminish their output more than the others, so a negative output gap occurs. The monetary policy authority lowers the interest rate in order to stabilize inflation and output gap. See Figure 7.2 for the evolution of impulse responses in case of a preference shock.

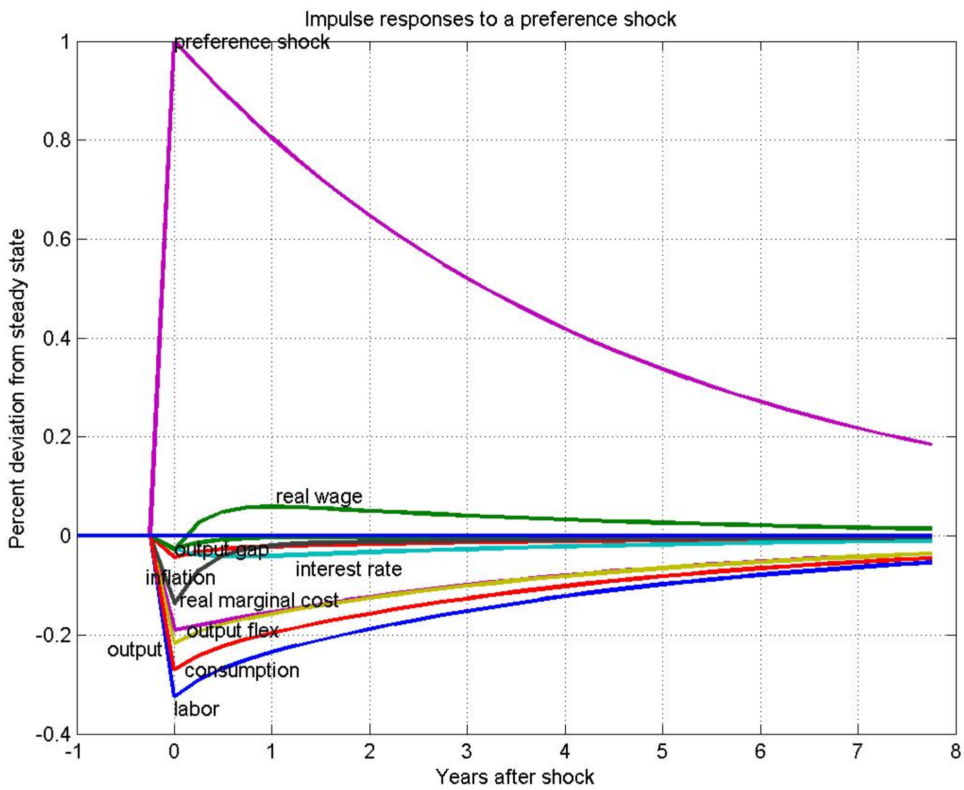


Figure 7.2: Impulse responses to a preference shock

7.3.3 Government Spending Shock

When the government increases its expenditures, there is less for households to consume. The consumption deviates negatively. The marginal utility from consumption increases, and people would like to consume more. Hence, they have to work more in order to produce more. They are even willing to work when their wages are lower. The output and the marginal costs increase. The government spending shock acts slightly inflationary, and also a minor output gap arises. Under such a situation, the central bank increases the interest rate to some extent. Figure 7.3 illustrates the effects of a government spending shock.

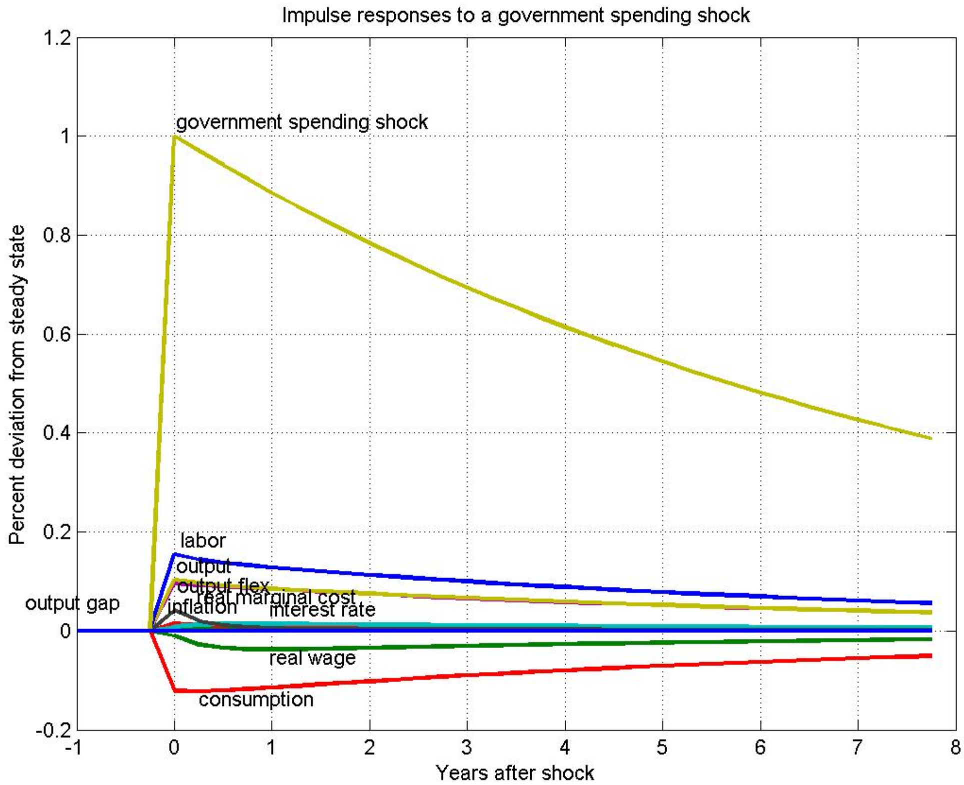


Figure 7.3: Impulse responses to a government spending shock

7.3.4 Technology and Monetary Policy Shocks

The impulse responses of the extended model to a technology shock and to a monetary policy shock are essentially the same with a slight difference in their values. The economic intution behing the shocks are the same with the baseline model, but to remind the evolution of the economy we provide in this section only the figures.

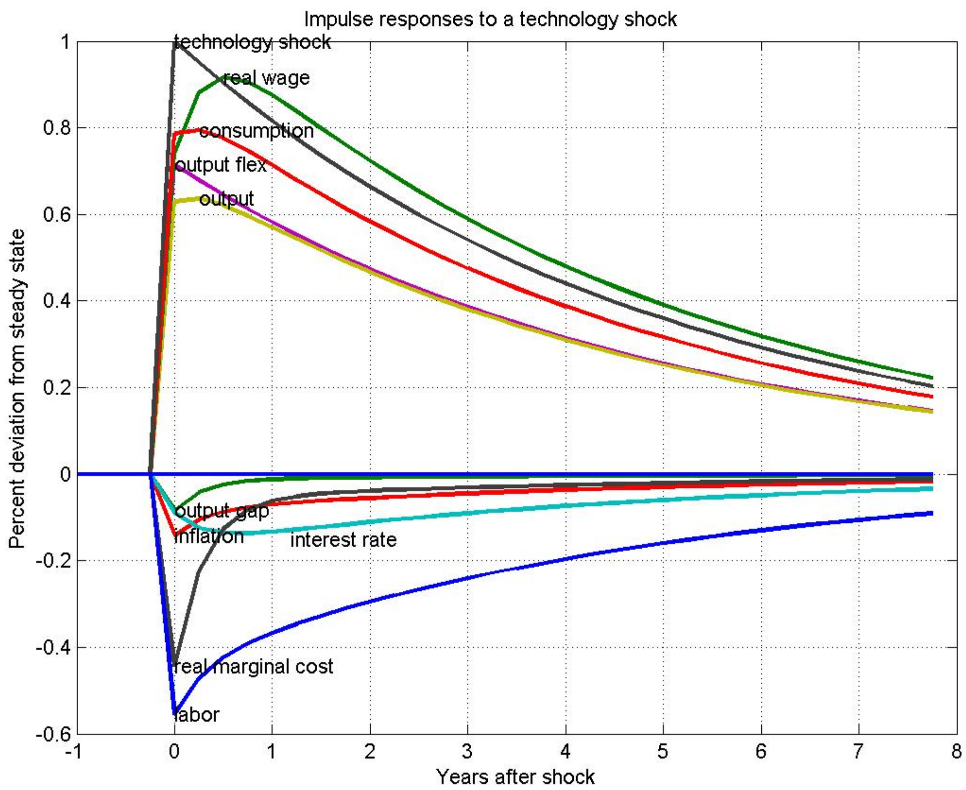


Figure 7.4: Impulse responses to a technology shock (extended model)

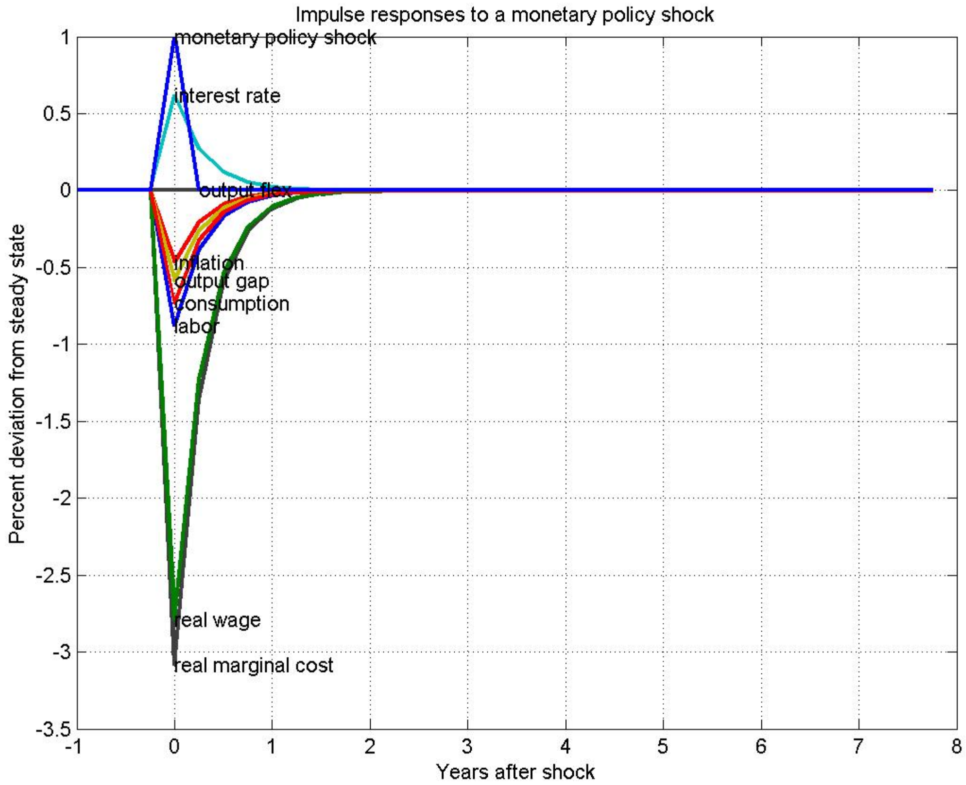


Figure 7.5: Impulse responses to a monetary policy shock (extended model)

7.4 Inflation and Output Gap Forecasts

In this section, we do the same exercise that we did for the benchmark model. We forecast inflation and output gap under CIR targeting and VIR targeting policies. The way how we make our projections are explained for the benchmark case. The economy is in steady state before a shock hits, and at date 0 a stochastic disturbance (or many) appears. Different from the benchmark case; we perform a sensitivity analysis simultaneously. We check how the projections change if households are more risk averse ($\sigma = 5$) or less risk averse ($\sigma = 1$). The impulse responses of the economy with households having different degrees of risk aversion are given in Appendix. The figures below show the estimated deviation of interest rate, inflation forecast and output gap forecast for each stochastic shock case. Our general result about CIR targeting is still valid. VIR inflation, and output gap reaches steady state and always on the target. However, the central bank makes wrong projections at all times under CIR targeting, and the inflation is not on target most of the times.

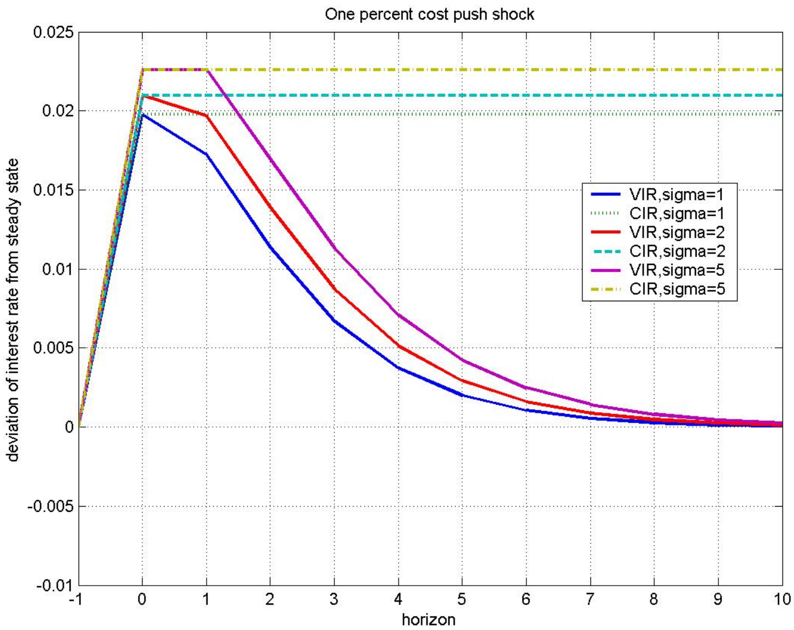


Figure 7.6: Deviation of interest rate from its steady state (cost-push shock)

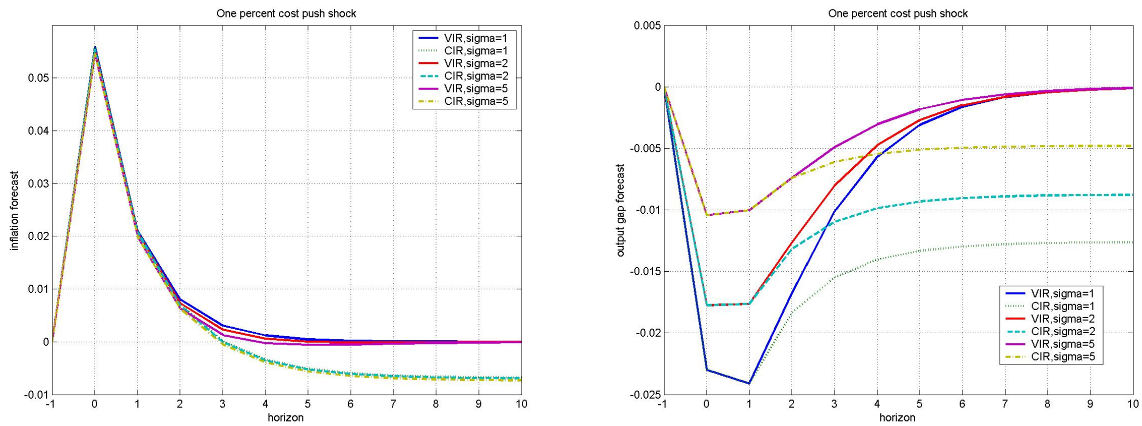


Figure 7.7: Panel A: Inflation Forecast Panel B: Output Gap Forecast
. (cost-push shock)

Before we start discussing our findings, we want to remind that the impulse responses for the cases of $\sigma = 1$ and $\sigma = 5$ can be found in Appendix.

When a cost-push shock comes into view, inflation deviates positively and does not depend on the risk tolerance of households. What is dependent on risk tolerance is the output gap and therefore interest rate. The households, who are less risk averse, decrease their consumption more than the households, who are more risk averse. Because more risk averse people would like to smooth their consumption, so they do not like harsh deviations of their consumptions. Thus, the demand declines more when people are less risk averse, resulting in a larger fall of output and output gap. Hence, the interest rate should be larger when individuals are less risk averse. Note that the inflation and output gap stabilization trade off exists in all cases. Finally, as mentioned before, whatever the risk tolerance level of individuals is, policy makers are wrong in their forecasts of output gap if they apply CIR inflation targeting. But the extent of their mistake increases, if the individuals are less risk averse, since the deviation of output is larger in this case. (See Figure 7.6 and 7.7).

First of all, notice that in our setting, the preference shock does not affect the economy if individuals have log utility in consumption, i.e. $\sigma = 1$. As we discussed before, a preference shock lowers the marginal utility from consumption, and in fact if individuals are more risk averse the reduction in marginal utility from consumption is larger, which means risk averse individuals decrease their consumption more compared to less risk averse individuals. Subsequently, the firms decrease their prices more, when the economy is surrounded with risk averse households. The firms also reduce their production more in this case, and we observe more negative output gap. As a result, the interest rates are smaller. The central bank, which has a CIR targeting policy makes larger mistakes of inflation projections when a preference shock hits an economy, where more risk averse individuals exist. (See Figure 7.8 and 7.9).

The Figures 7.10 and 7.11 show that, the difference of projections are not so much different from each other for different risk aversion parameters. The reason is that, the government spending shock causes such a minor inflation and a minor output gap that a significant difference does not take place.

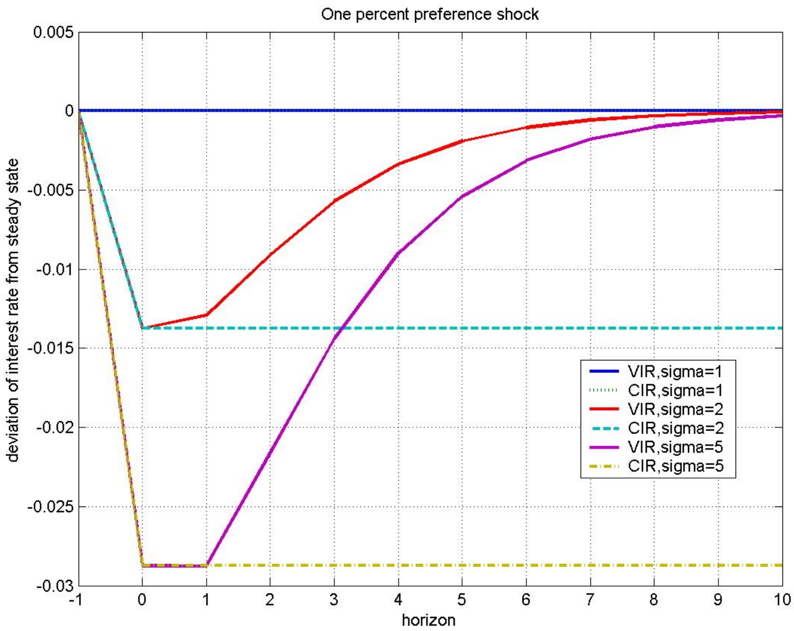


Figure 7.8: Deviation of interest rate from its steady state (preference shock)

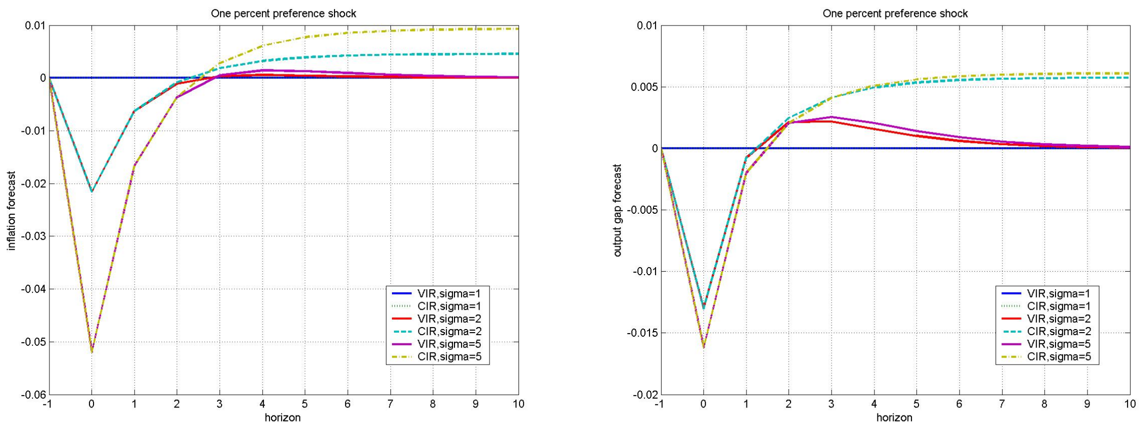


Figure 7.9: Panel A: Inflation Forecast Panel B: Output Gap Forecast
. (preference shock)

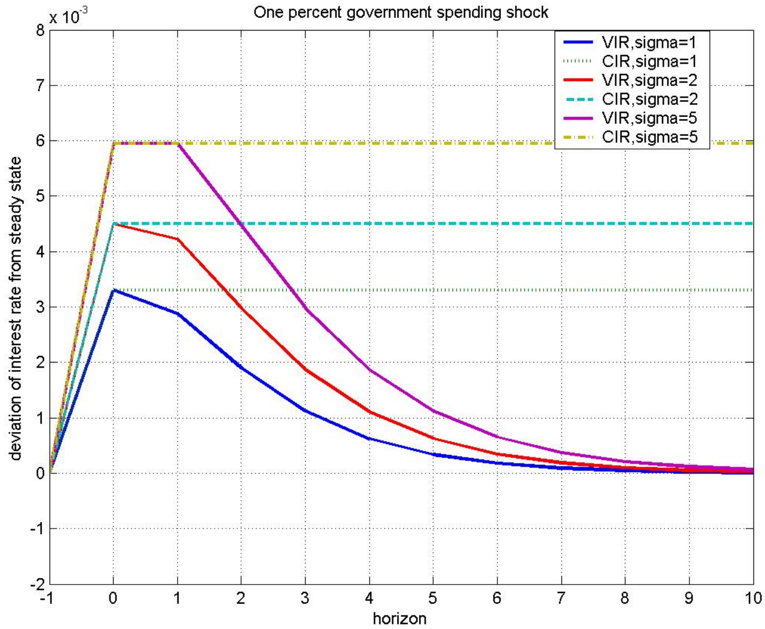


Figure 7.10: Deviation of interest rate from its steady state (government spending shock)

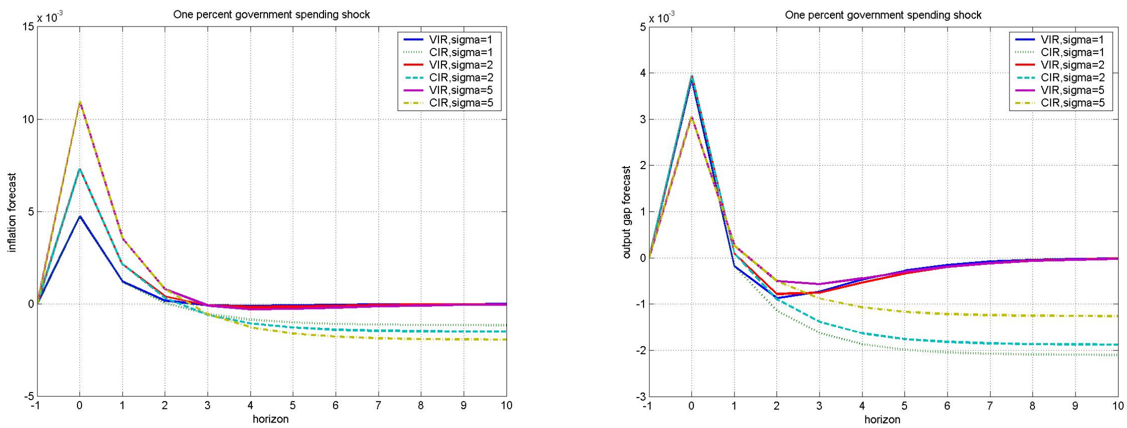


Figure 7.11: Panel A: Inflation Forecast Panel B: Output Gap Forecast . (government spending shock)

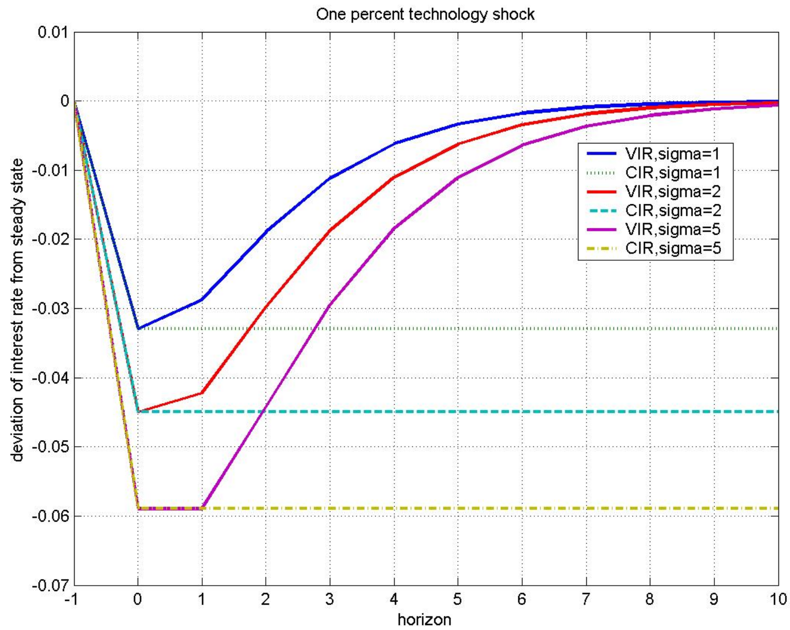


Figure 7.12: Deviation of interest rate from its steady state (technology shock)

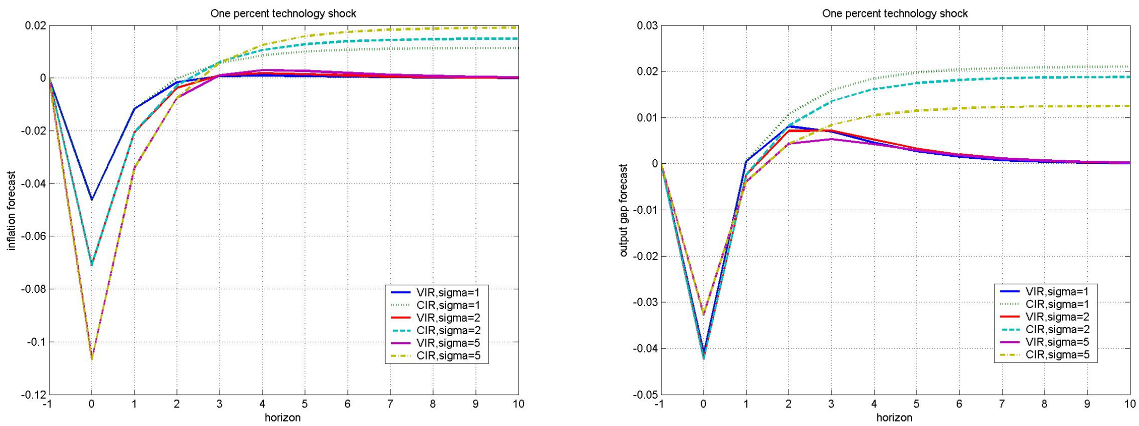


Figure 7.13: Panel A: Inflation Forecast Panel B: Output Gap Forecast
. (technology shock)

Figure 7.12 and Figure 7.13 show the projections of the central bank, in case a technology shock hits the economy. The more risk averse the individuals are, the more negatively deviates the inflation. We can derive this by an economical intuition. The technology shock increases productivity in the economy, and less risk averse households would like to enjoy more consumption today. Whereas the more risk averse individuals aim to smooth their consumption and they do not increase their consumption today as much as the less risk averse individuals. The firms want to encourage more risk averse people to consume more, since they produce more thanks to improved productivity; hence they decrease their prices more if the households are risk averse. Subsequently, the monetary policy authority sets a lower interest rate if there exists risk averse individuals in the economy; moreover, if they have a CIR targeting policy, they make more serious mistakes in inflation projections compared to the central banks which work in an economy where less risk averse agents exist.

With our projection machinery on hand, we can make projections for several scenarios, that is we can assume that different combinations of shocks come into sight together and affect the economy in different ways. We present in our paper, one of the worst possible cases: a positive unit of cost-push shock, government spending shock and preference shock hits our economy together with a negative technology shock. The projections made for such a situation can be seen in Figure 7.14 and Figure 7.15. This combination of shocks act extremely inflationary (an inflation of 12% is expected!). As Panel A in Figure 7.15 illustrates, in such an inflationary economy, the CIR targeting central banks are susceptible to severe mistakes of inflation projection, and the degree of risk aversion of households does not matter that much. The second remarkable observation in this economy is that, the output gap follows an interesting path. At date 0, the firms which cannot adapt the fall in productivity, causes a positive output gap, but in the succeeding periods, possibly the effect of the cost-push shock dominates; and we observe negative output gap and a trade off between inflation stabilization and output gap stabilization. The mistakes in output gap projection of CIR targeting central banks are quite large, and depend on the risk tolerance of individuals. The less risk averse the individuals are, the larger the mistake. The last inferences for the periods following date 0 were already derived above, when we analyzed the projections in case of a cost-push shock. This also confirms that, after the first period, cost-push shock dominates. Finally, as expected in such an inflationary economy, the interest rates are quite large.

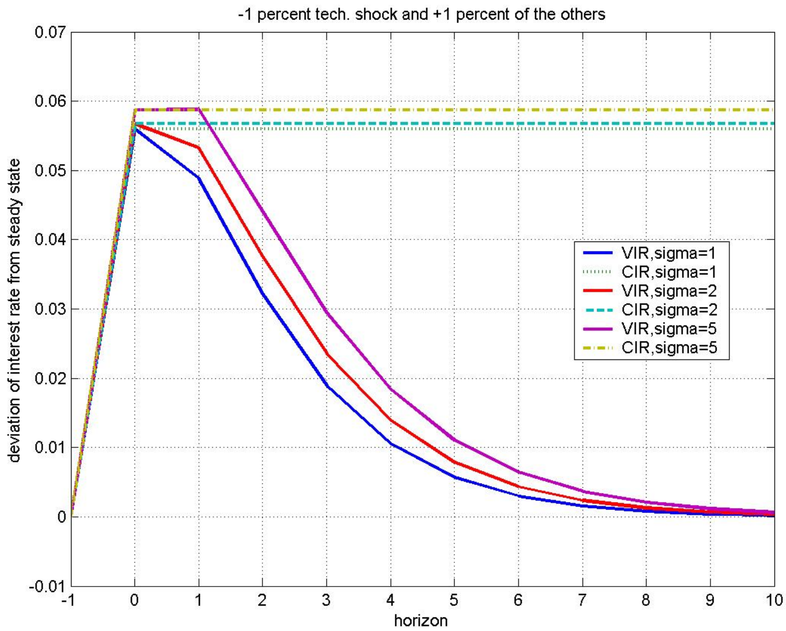


Figure 7.14: Deviation of interest rate from its steady state (all shocks, (-) technology shock)

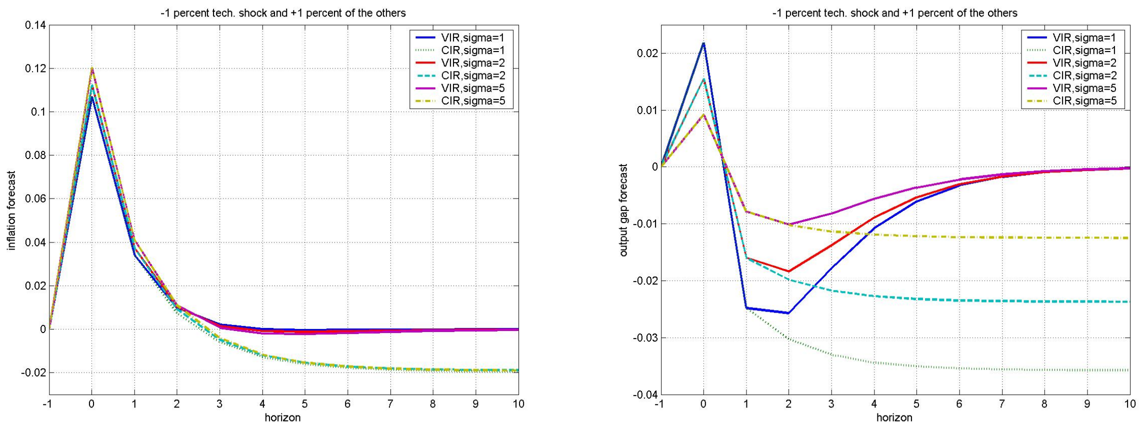


Figure 7.15: Panel A: Inflation Forecast Panel B: Output Gap Forecast
. ((all shocks, (-) technology shock))

8 Discussion

My analysis shows that CIR inflation targeting is a problematic assumption which leads policy makers to make systematic mistakes in inflation and output gap projections. The central banks which adopt CIR assumption face a dilemma of deviating from their policy by tending to change the interest rate within the forecast horizon. The forecasts made under the assumption of CIR are consistent if and only if the economy is in steady state and expected to stay in steady state. The more the economy deviates from steady state, the more severe is the systematic mistake made by the central bank adopting CIR policy. The degree of relative risk aversion is also a determinant of the extent of the mistake, when there are disturbances to the economy which cause large deviations from steady state.

The central banks adopting the CIR inflation targeting monetary policy should sooner or later give up this policy and instead use a time varying interest rate inflation targeting. A Taylor rule type monetary policy could do better, at least closer to the optimal policy.

In fact, I would like to make a welfare analysis to compare CIR inflation targeting and VIR inflation targeting. Then, I would prove quantitatively that VIR policy would be closer to optimal and would work better. Moreover, I would like to extend my model by introducing the central bank. I would assign a loss function to minimize, or a welfare function to maximize. A combination of a loss/welfare function and a Taylor type rule could possibly give the best projections of inflation and output gap. It would also be easy to communicate thanks to the Taylor type rule employed.

Another question which still stays in my mind is that, how constant interest rate inflation targeting could be so effective in lowering the inflation in practice. This proves that the expectations of private sector play an important role in the evolution of inflation. It is possible that, the results change if we employ a model where expectation formation of agents plays a role.

Last but not least, there is always a good reason to question the monetary policy and to improve it. CIR inflation targeting is not the best monetary policy, and there are better alternatives to it like VIR inflation targeting.

9 Summary and Concluding Remarks

Inflation forecast targeting is a nice improvement in monetary policy. There is an explicit, quantitative target that can be shared with the public. It has worked well lowering inflation and stabilizing inflation and output gap. Especially, constant interest rate inflation targeting is advocated to be a simple, clear policy which helps the private sector to anticipate future policy. But what if it is in fact misleading? The question we analyse in this paper is in brief how constant interest rate inflation targeting differs from variable interest rate inflation targeting. During our analysis, we ask further questions like how the risk aversion degree of individuals affects the difference between the two policies and how different disturbances influence the outcomes.

We analyse our questions in the framework of a New Keynesian Dynamic Stochastic General Equilibrium Model. An interest rate smoothing type of Taylor rule is used by the central bank. It can well be an illustration of CIR inflation targeting (Honkapohja and Mitra (2004)). We solve the model, and get the dynamics of the model. We use the laws of motion we obtain as a tool to make projections of future inflation and output gap. We make forecasts under two monetary policies: CIR inflation targeting and VIR inflation targeting. It is assumed that CIR targeting authorities stick to their assumption about maintaining the prevailing interest rate during the forecast horizon. As extensions to the model, we introduce additional disturbances, i.e. a cost-push shock, a preference shock and a government spending shock. We broaden our analysis and make projections of different states of the economy, again for CIR inflation targeting and VIR inflation targeting. Moreover, we look at the situations where highly risk averse individuals exist in the economy and where less risk averse agents surround the economy.

We see in the end that CIR inflation targeting monetary authorities make systematic mistakes in their projections of inflation and output gap. They have a propensity to disregard the assumption they made during the formulation of the future policy, and change the interest rate within the targeting horizon, which is an observed case in practice. Although they might be making forecasts some periods after a shock hit the economy, they are still wrong. Their mistakes are then smaller, but still exist. The main determinant of the severity of their errors is the amount of deviation of inflation or output gap from their steady state values. If they deviate too much from steady state, CIR inflation targeting policy makers make larger mistakes. The most extreme scenario could be an inflationary economy, which is also illustrated in our study. When the economy is far from steady state, the degree of relative risk aversion also matters. A preference shock and a technology shock cause bigger mistakes in inflation projections, if there are highly risk averse agents in the economy. A cost-push shock leads to larger mistakes in output gap projections if less risk averse agents exist in the economy.

Actually, we share the common arguments in the literature. The CIR inflation targeting is a problematic framework and the central banks should not employ it as a monetary targeting policy. CIR inflation targeting policy leads to time inconsistency trouble which threatens the reliability of the central bank. CIR is an unrealistic assumption, so the resulting projections of inflation and output gap are also unrealistic. The projections produced under CIR assumptions are only consistent when the economy is in steady state and expected to stay in steady state. It would be preferable to base policy on projections conditioned on time varying instrument rate paths. The central banks would better adopt a Taylor rule type policy, which is also a practical communication device and produces better and more realistic projections of inflation and output gap.

10 Appendix

10.1 Impulse Responses of the Extended Model for different σ values

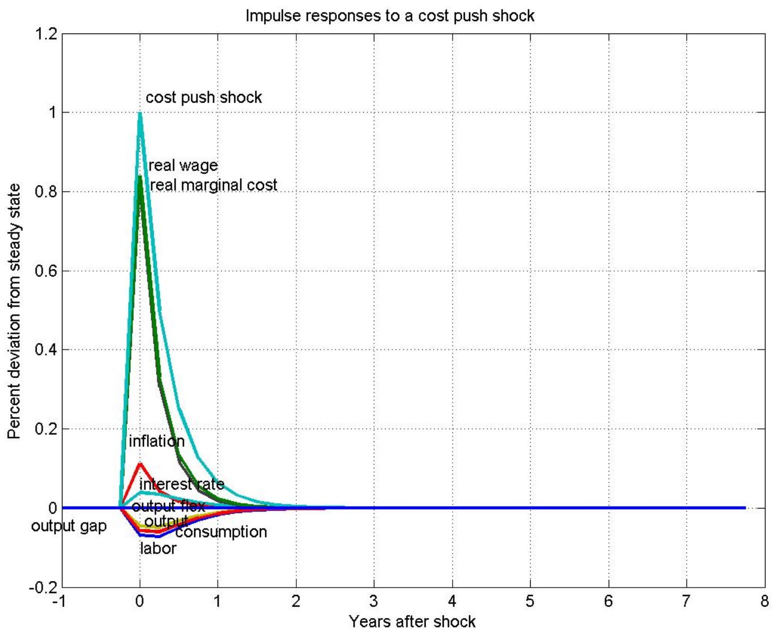


Figure 10.1: Impulse responses to a cost-push shock, $\sigma = 1$

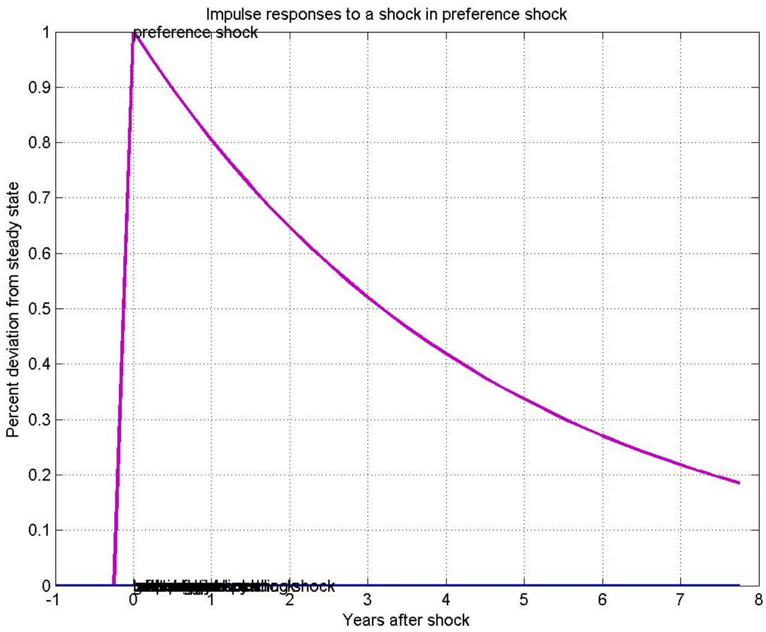


Figure 10.2: Impulse responses to a preference shock, $\sigma = 1$

Note that, with a log utility of consumption ($\sigma = 1$), a preference shock does not effect our economy. This is a result of how we incorporate the preference shock to the model[1].

[1] See "Variations" part.

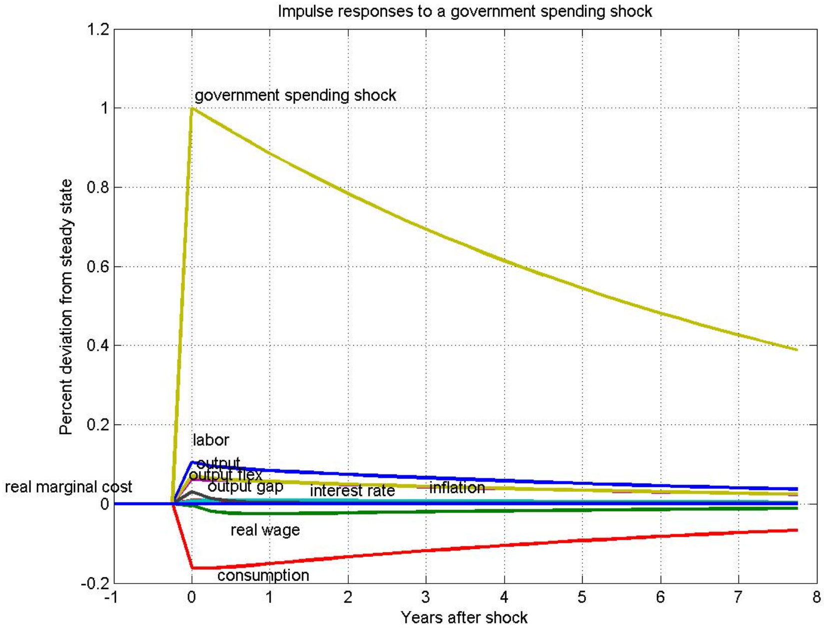


Figure 10.3: Impulse responses to a government spending shock, $\sigma = 1$

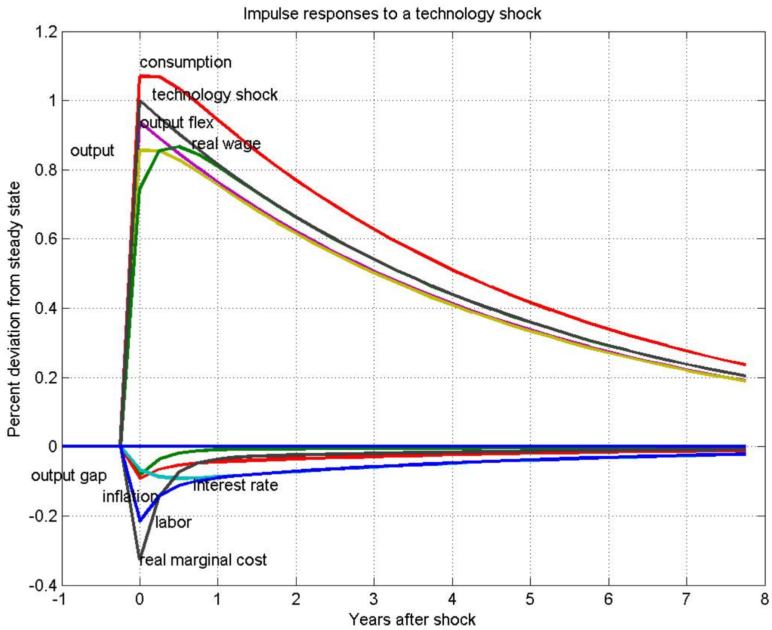


Figure 10.4: Impulse responses to a technology shock, $\sigma = 1$

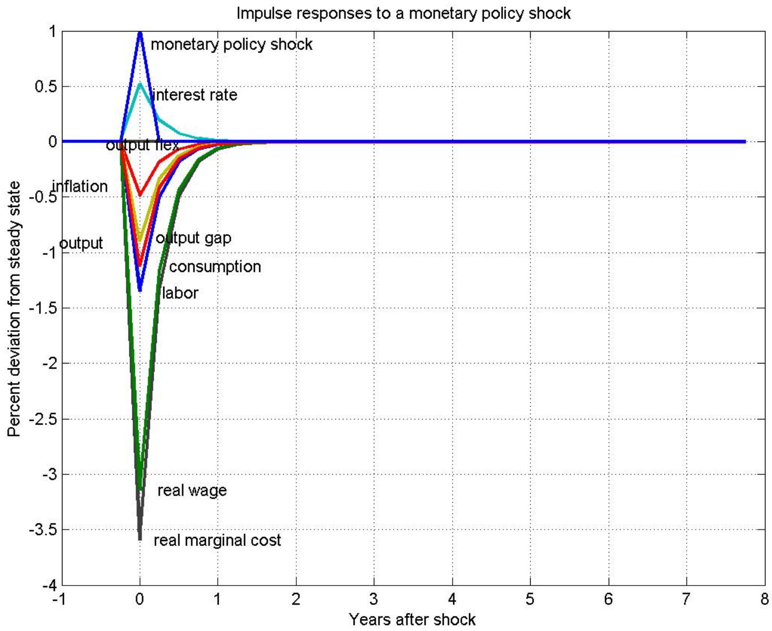


Figure 10.5: Impulse responses to a monetary policy shock, $\sigma = 1$

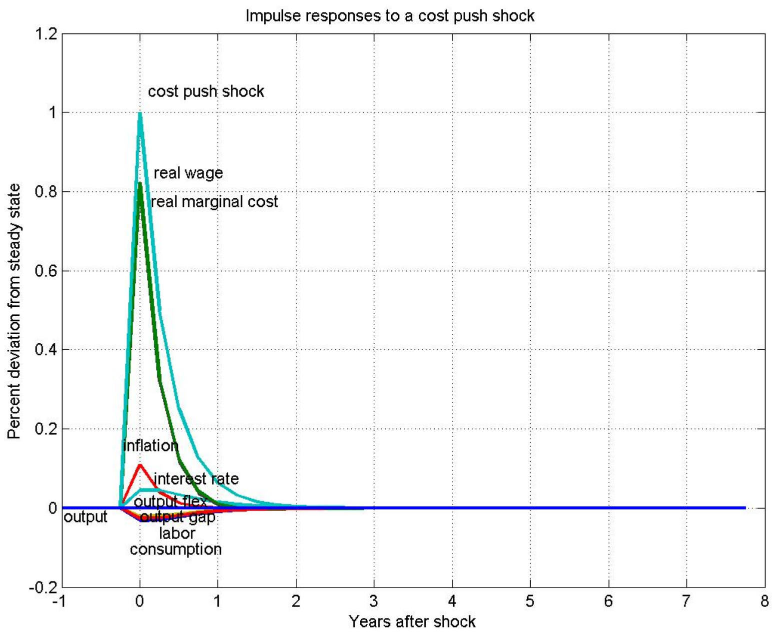


Figure 10.6: Impulse responses to a cost-push shock, $\sigma = 5$

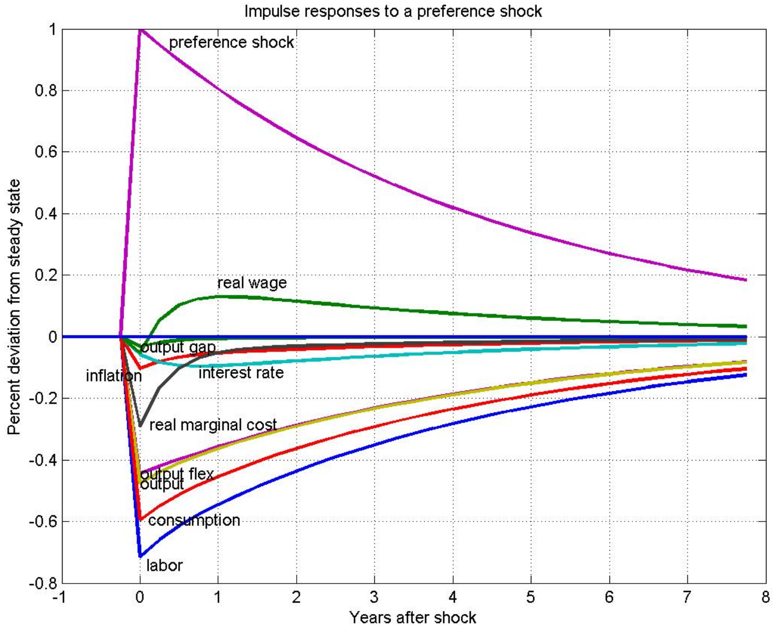


Figure 10.7: Impulse responses to a preference shock, $\sigma = 5$

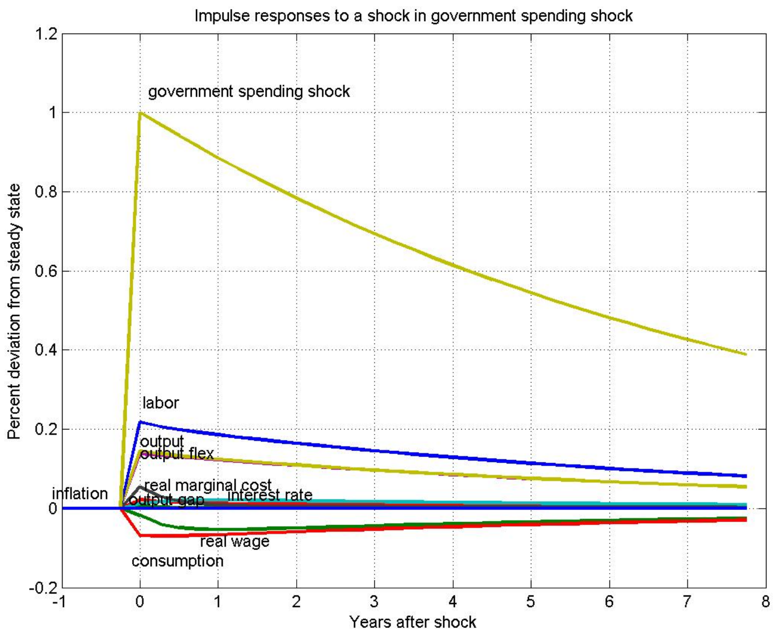


Figure 10.8: Impulse responses to a government spending shock, $\sigma = 5$

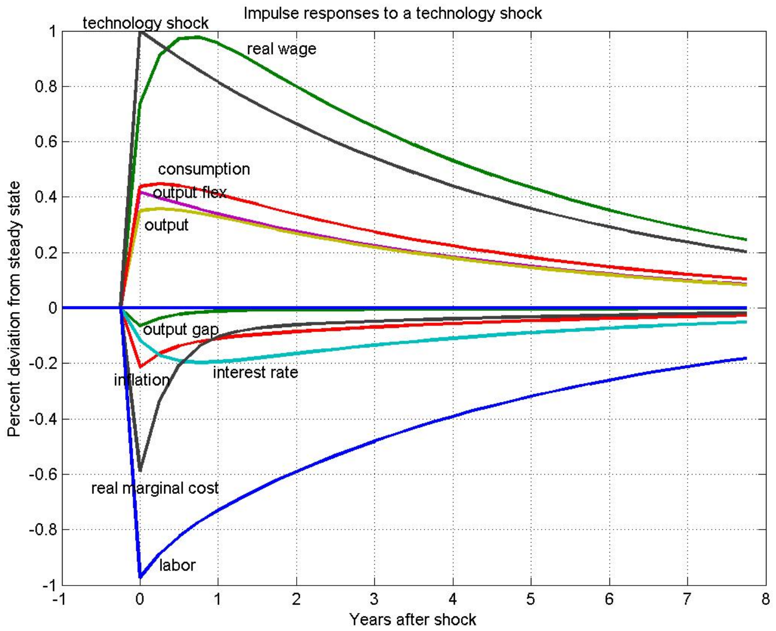


Figure 10.9: Impulse responses to a technology shock, $\sigma = 5$

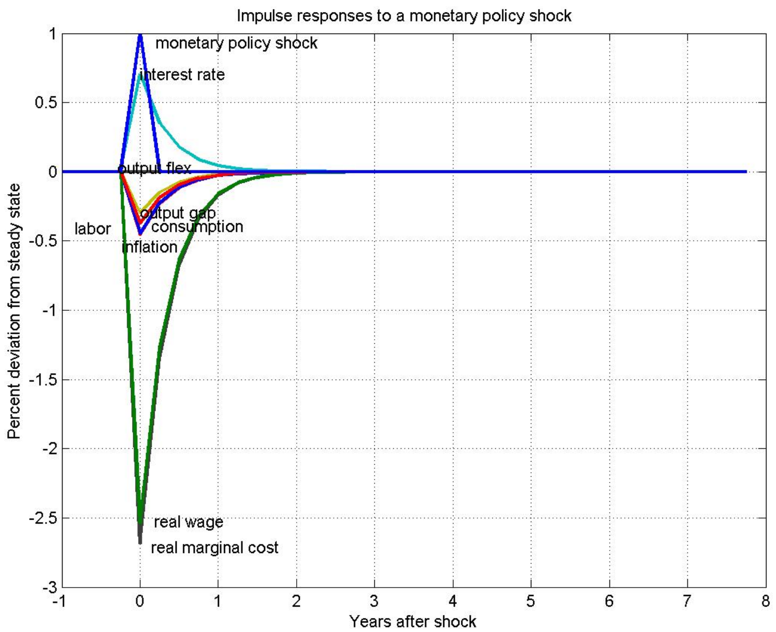


Figure 10.10: Impulse responses to a monetary policy shock, $\sigma = 5$

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Declaration of Authorship

I hereby confirm that I have authored this master thesis independently and without use of others than the indicated resources. All passages, which are literally or in general matter taken out of publications or other resources, are marked as such.

Burcu Erdogan

Berlin, 27th December 2005