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VCRIX - a volatility index for crypto-currencies

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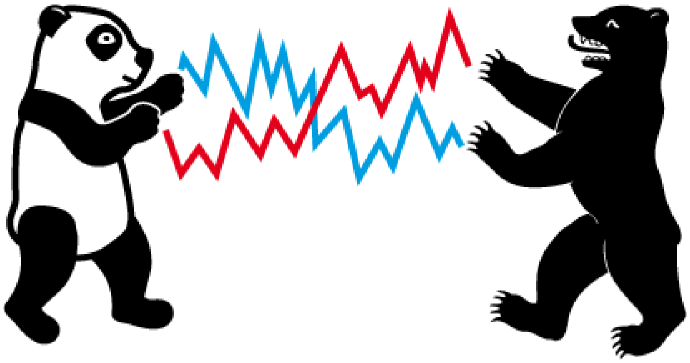

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VCRIX - a volatility index for crypto-currencies*

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Abstract

Public interest, explosive returns, and diversification opportunities gave stimulus to the adoption of traditional financial tools to crypto-currencies. While the CRIX index offered the first scientifically-backed proxy to the crypto-market (analogous to S&P 500), the introduction of Bitcoin futures by Cboe became the milestone in the creation of the derivatives market for crypto-currencies. Following the intuition of the "fear index" VIX for the American stock market, the VCRIX volatility index was created to capture the investor expectations about the crypto-currency ecosystem. VCRIX is built based on CRIX and offers a forecast for the mean annualized volatility of the next 30 days, re-estimated daily. The model was back-tested for its forecasting power, resulting in low MSE performance and further examined by the simulation of VIX (resulting in a correlation of 78% between the actual VIX and VIX

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estimated with the VCRIX model). VCRIX provides forecasting functionality and serves as a proxy for the investors' expectations in the absence of the developed derivatives market. These features provide enhanced decision making capacities for market monitoring, trading strategies, and potentially option pricing.

Keywords: index construction, volatility, crypto-currency, VCRIX

JEL classification: C51, C52, C53, G10

1 Introduction

Since the inception of Bitcoin (BTC) in 2008 the crypto-currency (CC) ecosystem has seen a market capitalization explosion that reached 795 billion USD at its highest point on January 6, 2018 (CoinMarketCap (2018)). Apart from traditional hedge-funds and institutional investors who are interested in diversification, the CC ecosystem saw more than 400 crypto-funds launched during the past three years (next.autonomous.com/cryptofundlist). The rapid growth of BTC price led to persistent talks about "bubble-like" behavior and general skepticism of the market (Hafner (2018), Cheung et al. (2015)), exposing the need for a deeper understanding of the underlying processes driving the valuation of CC. Research in this field was done by Hayes (2017) and White (2015). Traditional market instruments (indices, ratings, investment portfolios) joined the ecosystem, including the early efforts such as CRIX index by Trimborn and Härdle (2018) and exploration of the potential of CC as an investment tool (Petukhina et al. (2018)).

Introduction of BTC futures by the CME and Chicago Board Options Exchange (Cboe) on December 18, 2017 reinforced the positions of CC as a new asset class. The emergence of the derivatives market signaled the need for solid pricing strategies and a reliable (and stable) risk measure. The paper on pricing CC by (Chen et al., 2019) addressed this issue by employing a Stochastic Volatility with a Correlated Jumps model (Duffie et al. (2000)) and using insights on implied volatility dynamics by Fengler et al. (2003) in order to match non-stationarity and local heterogeneity phenomena of CRIX returns.

Industry demand and research revealed the necessity to explore the behavior of the CC volatility further, to provide the final ingredient - a proxy for implied volatility. In traditional markets, implied volatility is measured by volatility indices which can be considered a traditional financial tool. At the end of the 20th century, financial markets of the USA and Europe aimed to capture the global measure of volatility in the respective market, which led to the introduction of VIX or VDAX. The index providers settled on the model most appropriate for the specifics of the behavior of the corresponding derivative. Given the absence of a developed derivatives market, we have to infer the characteristics of the implied volatility from the CC market behavior. The specifics of the latter (high volatility and low liquidity) triggered the development of new investment methods, see Trimborn et al. (2019), further justifying the need for a volatility index, that would capture the unique specifics of CC as an asset class and provide a reliable indicator for the continuously unstable market.

Our research aims to create a VCRIX - a volatility index especially designed for markets akin to the CC ecosystem, see Subsection 3.1. The goal of the proposed VCRIX is the estimation of the risk measurement for the CRIX components and delivery of market status information, analogous to implied volatility indices that capture investors expectations.

Section 2 offers an overview of the used data sets for both traditional and CC markets. Section 3 provides a detailed explanation of the methodology used, including a brief revision of CRIX which was selected as an equivalent for the S&P 500, a note on the existing implied volatility indices and VIX methodology in particular (Subsection 3.2). Subsection 3.3 contains the details on the implied volatility proxy estimation, followed by Subsection 3.4 that clarifies VCRIX model selection and back-testing. Methodological results, details of the VIX simulation conducted to test the selected methodology and final time series are showcased in Section 4. Applications of the proposed volatility index are further explored in Section 5, which contains an example of the trading implementation of VCRIX. Additional observations and a summary of the conducted research are provided in Sections 6 and 7.

2 Data

This research employs CRIX values and traditional financial data, namely S&P 500 index values and VIX, which is the volatility index of Cboe based on the S&P 500. The daily historical closing values of CRIX for the period from Sep 2014 - the emergence of CRIX - to December 2018 (1583 observations, including weekends) were sourced from `thecrix.de` and converted to log-returns.

The daily historical closing prices of the S&P 500 and VIX from 2000 to the end of 2018 (4780 observations) were sourced from `finance.yahoo.com`. It must be pointed out that SPY (ETF on S&P 500 index) has closer relations to VIX by design, as clarified in Subsection 3.3, however, the log-returns of S&P 500 and SPY reveal no difference and thus could be interchangeable for the conducted analysis. The S&P 500 time series were converted to log-returns, VIX values remained as is.

3 Methodology

Implied volatility became a subject of academic research with the development of the derivatives market in the last quarter of the 20th century. The Black and Scholes (1976) model yields implied volatility as a volatility measure because, by definition, the implied volatility is the future volatility expected by the market. However, the market crash of October 1987 that bent the volatility surface of index options into a skewed "volatility smile", motivated an alternative solution that would provide a more accurate fit to market conditions. Bakshi et al. (1997) provide an extensive overview of the further developments in this field, including the stochastic interest rate option models of Merton et al. (1973), the jump - diffusion/pure jump models of Bates (1991), the stochastic volatility models of Heston (1993) and others. While acknowledging the diversity of options pricing models, authors agree on the necessity of matching the selection of one to the goals at hand.

The goal of VCRIX is to capture the expectations of the CC market, much like VIX is offering an uncertainty measurement with regard to the American stock prices. In simplified terms, VIX "predicts" the mean annualized volatility of the S&P 500 for the next 30 days in the future, that is in turn derived from the implied volatility extracted from the S&P 500 ETF swap prices. Absence of a CC analog calls for an alternative solution for VCRIX. In the absence of intrinsic predictive power, VCRIX would also have to be forward-looking, providing a valid estimation of the CC market volatility in the future. The selection of the new methodology thus includes two tasks: estimation of the best implied volatility proxy and further search for the model to exhibit the most consistent predictive performance.

3.1 CRyptocurrency IndeX

S&P 500 and DAX serve as indicators of the current state of American and German markets by aggregating the weighted performance of the most significant listed companies. CRIX, developed by Trimborn and Härdle (2018), plays a similar role for the CC market, providing a statistically-backed market measure, which distinguishes it from other CC indices like Crypto20, CCi30, WorldCoinIndex. At the core of CRIX lies the idea that a fixed number of constituents (as in case of S&P 500) may be a good approach for relatively stable markets, however, with the ever-growing number of CC, practical implementation would demand a filter that keeps out the noise, while preserving the information about the market dynamics. CRIX employs Akaike Information Criterion (AIC, Akaike (1987)) that determine the number of constituents quarterly according to the explanatory power each CC has over the market movements. CRIX was used as a proxy to the CC market before in research papers by Elendner et al. (2018), Klein et al. (2018), Mihoci et al. (2019), and was adopted as a benchmark by commercial projects like Smarter Than Crypto, Crypto20, F5 Crypto Index, and also used by the European Central Bank as a market indicator in the report dedicated to understanding the "crypto-asset phenomenon" (Chimienti et al. (2019)). These use cases confirm the applicability of CRIX as an appropriate basis for VCRIX.

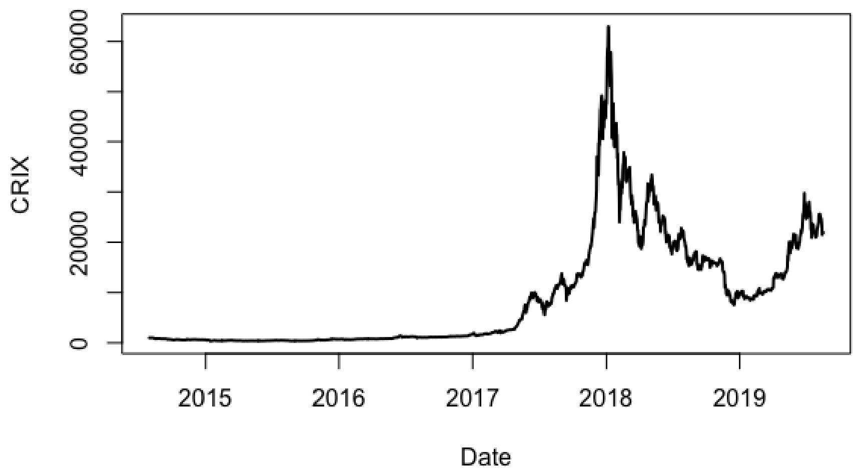

Figure 1: CRIX from Sep 2014 to Aug 2019

CRIXcode

Consequently, the index rules will have a significant impact on the behavior of VCRIX. The initial paper by Härdle and Trimborn (2015) defines CRIX as a Laspeyres index, taking the value of a k asset basket and comparing it against the base period, as indicated in Equation (1):

$$CRIX_t(k) = \frac{\sum_{i=1}^{k} P_{it} Q_{i,t_l^-}}{Divisor(k)_{t_{l^-}}} \tag{1}$$

with P_{it} the price of asset i at time t and Q_{i,t_l^-} the quantity of asset i at time t_l^-(the last time point when Q_{i,t_l^-} was updated). Monthly re-balancing accounts for the changes in the market capitalization of a CC and the number of index components, the *Divisor* ensures that this procedure does not affect the value of CRIX, rather only price changes in its constituents shall be of effect.

3.2 Implied volatility indices

Consideration of the existing volatility indices would constitute a logical step towards the selection of the appropriate solution. As observed by Siriopoulos and Fassas (2009) recent decades saw the rise of the model-free indices (based on model-free implied volatility (MFIV)) that were made possible by highly liquid options markets and readily available model-free implied variances (France, Germany, Japan, Switzerland, the U.K., and the U.S). Major alternatives to the "model-free" approaches are the Black-Scholes (BS) implied volatility and statistical models such as GARCH (Bollerslev (1986)). While MFIV is extracted from the corresponding set of current option prices without the need to assume any specific pricing model, this approach comes along with a range of methodological issues. For example, Biktimirov and Wang (2017) tested both approaches on the subject of forecasting accuracy, and BS implied volatility came out superior both in terms of in-sample "encompassing" models that include several forecasts in the same combined specification and also in out-of-sample forecasting. We consider model-free and model-based methodologies given the available data and above mentioned empirical results.

Introduction of XBT-Cboe BTC Futures by the Cboe in 2017 became the first step in the establishment of the CC derivatives market, thus approaching the possibility of the model-free implied volatility index construction. However BTC futures were not considered for this research due to several reasons: officially listed (Cboe and CME Group) futures do not provide insight into implied volatility of the underlying like option prices do by design, existing data for options is so far only available for BTC from commercial providers like Deribit (2019), not for the broader CC market. Most importantly, the goal of the VCRIX is to grasp the investors' expectations of the whole CC market. As Figure 2 shows, the weight of BTC in CRIX has been remaining below 0.6 most of the time, and thus BTC and its options cannot be considered sufficiently representative.

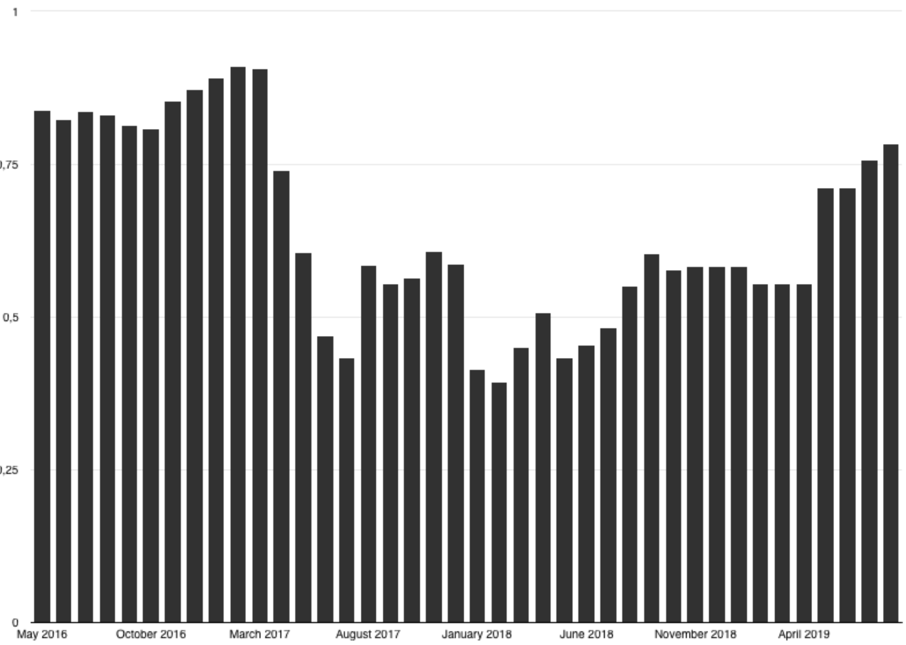

Figure 2: Weight of BTC as a constituent of the CRIX over time

Given the outlined limitations of the CC derivatives market, we settle for a model-based index, that is capable of capturing the predictive power of a traditional volatility index. The VIX by Cboe for the US market was selected as a guidance and benchmark. VIX is acknowledged by the established CC players as a standard for the implied volatility modeling: in 2019 one of the biggest CC derivative trading platforms Ledger X - a US company regulated by CFTC (United States Commodity Futures Trading Commission) - introduced an implied volatility index for BTC called LXVX (Cointelegraph (2019)), announcing its inheritance to VIX (LXVX (2019)).

The current VIX methodology was developed based on the pioneering research of Whaley (1993), Neuberger (1994), Madan et al. (1998), Demeterfi et al. (1999) and Britten-Jones and Neuberger (2000) among others. It estimates the implied volatility of option prices on the S&P 500 by taking strikes and option prices as inputs. With exchange-traded S&P 500 variance swap rate as its underlying, VIX became a proxy for market volatility (Cboe (2009)):

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T} \left[\frac{F}{K_0} - 1 \right]^2 \tag{2}$$

$$VIX = \sigma * 100, \tag{3}$$

where T is time to expiration, F is a forward index level from index option prices, K_0 is a first strike price below F, K_i is a strike price of the ith OTM option (on average the range of i is between 1 and 500, reflecting the composition of the S&P 500) , $Q(K_i)$ is the midpoint of the bid-ask spread for each option with strike K_i, ΔK_i is an interval between strike prices (half the difference between the strike on either side of K_i) and R the risk-free interest rate to expiration.

3.3 Implied volatility proxy

VCRIX is designed to measure and proxy the lacking implied volatility in the CC market, hence it has to be based on a model, capable of capturing the predictive power of a traditional implied volatility index like VIX. In order to select an appropriate proxy for VIX, one has to check the dynamics of the underlying, in particular the annualized historical rolling volatility of SPY log-returns over 30 days (VIX measures how much the market thinks the S&P 500 Index will fluctuate in the 30 days from the time of each tick, according to Cboe (2009)). Equation (4) displays the rolling volatility method (r_t being a daily return of an asset on day t and $\hat{\mu}$ an estimated mean daily return over the 30 day period). In case of historical volatility, the σ would define the volatility of the last day of the month, while for forward volatility the same calculation will account for the volatility of the first day of the month. It should be pointed out that we are not using the notion of forward volatility as in Taleb (1997), namely, how implied volatility differs for related financial instruments with different maturities. In this case, the "forward" part only bears the idea of adjusting the time span of the traditional rolling volatility measure to be forward-looking (results are displayed in Figure 4).

$$\sigma_t = \sqrt{\frac{1}{30} \sum_{i=t-30}^{t-1} (r_i - \hat{\mu})^2} * \sqrt{252} * 100 \tag{4}$$

3.4 Model selection and back-testing

The dataset of CRIX log-returns was transformed into annualized daily volatility based on 30-day rolling windows (CC are traded everyday, unlike traditional securities). We considered both univariate and multivariate models, however, the latter did not prove superior in approximating the selected time series and for the sake of brevity this case will not be described in this paper. Thus the choice was made in favor of univariate models. 273 values of the dataset were set aside for back-testing, which corresponds to 20% of the dataset. We considered the following models that describe the volatility dynamics:

1. GARCH family (tested by Hansen and Lunde (2005), French et al. (1987), Antoniou and Holmes (1995)
   - GJR
   - EGARCH
   - EWMA
2. Heterogeneous Auto-Regressive (HAR) model (introduced by Corsi (2009) and tested by Chiriac and Voev (2011), Busch et al. (2011), Patton and Sheppard (2015))
3. neural network-based Long short-term memory cell (LSTM) models (Hochreiter and Schmidhuber (1997)

The latter represents a comparatively new approach to volatility modeling. The LSTM architecture belongs to the Recurrent Neural Networks family and has been extensively used (together with Gated Recurrent Units) for the modeling of sequential data like text or time series. Its complex architecture provides interesting forecasting opportunities that have been explored and proven useful by Kong et al. (2017), Pichl and Kaizoji (2017), Kim and Won (2018), Luo et al. (2018). Figure 3 provides a visual comparison of the 3 best-performing models: HAR (specified in Equations ((9)-(11)), EWMA model (specified in Equation (5), where $\sigma^2_{i,t+1}$ is the variance of CRIX log-returns ($r_{i,t}$) in the next period and the decay factor λ=0.96) and LSTM model (15 epochs, 3 layers of 365 neurons, specified in Equation (6) in its simplified form, where $\hat{\theta}$ signifies the complex set of parameters that are optimized during the training of the neural network).

$$\sigma^2_{i,t+1} = \lambda\sigma^2_{i,t} + (1-\lambda)r^2_{i,t} \tag{5}$$

$$\sigma^2_{i,t+1} = f_{\hat{\theta}}(\sigma^2_{i,t}) \tag{6}$$

As can be observed from Figure 3, all three models learn to anticipate the behaviour of the 30-day rolling volatility of CRIX quite well, however, the similar peaks from August to October expose their limited ability to timely reflect a sudden splash in the CC market. LSTM proves to be particularly vulnerable in its predictive capacity. This could be further remedied by the more complex architecture and increased training time, making the modeling more computationally costly. Given the non-substantial role of LSTM in the further implementation of VCRIX and the fact that the detailed explanation of the LSTM methodology with regards to financial forecasting has been provided previously in papers by Chen et al. (2015), Heaton et al. (2017), Fischer and Krauss (2018), we omit the detailed explanation of the LSTM application.

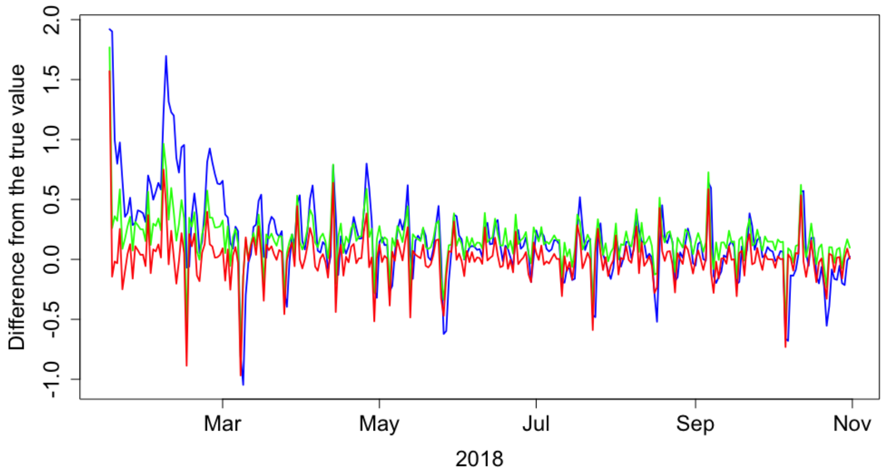

Figure 3: Difference between the true (30-day rolling volatility of CRIX) and the HAR, EWMA and LSTM models

Metric	**HAR**	**EWMA**	**LSTM**
Correlation	0.99	0.99	0.97
MSE	0.03	0.06	0.16
MAE	0.11	0.19	0.30
Mincer Zarnowitz R-adj	0.98	0.98	0.94

Table 1: Evaluation of the predicted values of 30-day annualized rolling volatility of log-returns on CRIX (daily re-estimation)

4 Simulation and assessment

During the model back-testing, the HAR and the EWMA models performed very closely. EWMA consistently underestimated the volatility but registered the up and down shifts faster. The LSTM frequently overestimated the volatility, which is coherent with the higher values that are picked up by VIX in comparison to the rolling volatility as showcased in Figure 3.

According to the results in Table 1, the HAR model was selected as the best predictive performer with correlation 0.99, MSE 0.03, and MAE 0.11. It should be specified that the original HAR model, Corsi (2009), is built on the premise that traders conduct their activities according to the strategies based on different frequencies (high-frequency trading, daily traders, weekly, monthly), which in turn affects the overall market volatility at certain points in time. As the CC market is young and presumably still dominated by sporadic non-expert traders (due to the pseudo-anonymity of most CC, justification of this assumptions remains challenging), presenting an informed judgment at this stage is rendered impossible by the implicit anonymity of most CC and its users. The recent analysis for potential herding behavior by Bouri et al. (2018) and Gama Silva et al. (2019) touches on this topic, without providing actual analysis of the traders' practices.

In the absence of data on CC traders' behavior, we have made the assumption that the traditional practices could potentially be applied for the CC case. This led us to make two adjustments to the original HAR model. 30-day historical rolling volatility (annualized, as shown in Equation (7) was used instead of realized volatility (it was selected as a most representative to proxy VIX).

$$RV_t^d = \sigma_t = \sqrt{\frac{1}{30}\sum_{i=t-30}^{t-1}(r_i - \hat{\mu})^2} * \sqrt{365} * 100 \tag{7}$$

Similarly to Equation (4), r_t is a daily return of CRIX on day t and $\hat{\mu}$ an estimated mean daily return over the past 30 days (we keep the span to 30 days as CC are traded without the weekends), meanwhile, the number of days was changed to 365 for the same reason. Further on we will refer to σ_t^2 as daily realized volatility RV_t^d to maintain the usual HAR notation.

The change of 5 (weekly) and 21 (monthly) trading frequencies to 7 and 30 days respectively is reflected in the calculation of weekly and monthly volatilities

(Equations (8) and (9)).

$$RV_t^w = \frac{1}{7}(RV_t^d + RV_{t-1}^d + ... + RV_{t-6}^d) \tag{8}$$

$$RV_t^m = \frac{1}{30}(RV_t^d + RV_{t-1}^d + ... + RV_{t-29}^d) \tag{9}$$

The final version of VCRIX is forward-looking and offers a forecast of the mean annualized daily volatility for the next 30 days. The index is re-estimated daily based on the realized daily volatility. The Equations (10) and (11) offer the actual methodology where the forecast - RV_{t+1}^d - is estimated with a regression given the daily RV_t^d (initially estimated with 30-day rolling window), weekly RV_t^w and monthly RV_t^m volatilities that are recalculated daily.

$$RV_{t+1}^d = \alpha + \beta^d RV_t^d + \beta^w RV_t^w + \beta^m RV_t^m + \omega_{t+1} \tag{10}$$

$$VCRIX_t = \frac{RV_{t+1}^d}{Divisor} \tag{11}$$

The initial value of VCRIX is set to 1000, following the convention set by CRIX. A *Divisor* is introduced in order to account for the jumps that might occur due to the change in the number of constituents every month. The *Divisor* is set to a certain value on the first day to transform the estimated volatility to 1000 points of VCRIX. *Divisor* remains the same over the month. Every month the constituents can change. In this case, the value of VCRIX from the last day of the month will be transferred to the first day of the next month, after that the *Divisor* will be reevaluated in order to reflect the value for transformation.

In order to provide an additional justification for the selected methodology, a VIX simulation was performed. It comprised the application of the selected HAR model to log-returns of the S&P 500 instead of CRIX.

After establishing the CRIX as the underlying for VCRIX and selecting VIX as a benchmark for the evaluation of the CC volatility index, we proceeded with selection of the appropriate implied volatility proxy in the absence of CC derivatives market. The time series (Figure 4) analysis showed the correlation of 0.89 between VIX and historical volatility, while the correlation between VIX and forward-looking volatility was 0.78. Given the scale of the differences, it is obvious that both historical and forward-looking volatilities fail to grasp the exact variation of VIX. This gap grows in crisis periods (as it can be seen for 2009) but shrinks back during market cool-down.

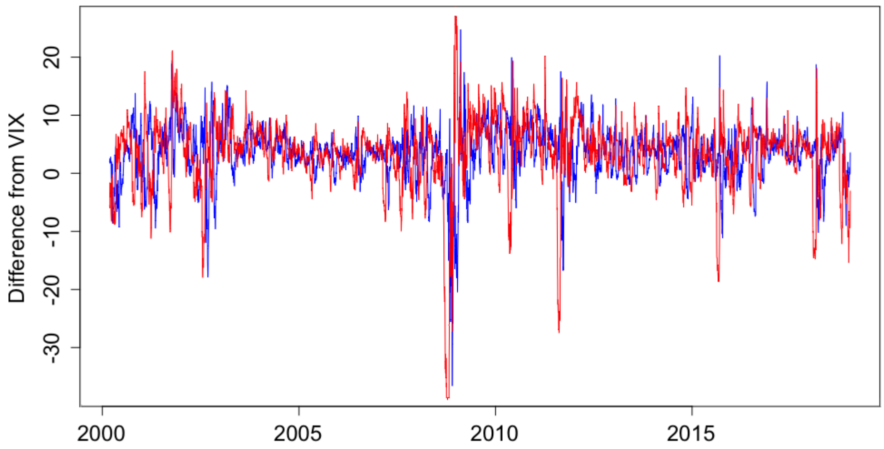

Figure 4: Difference between VIX and historical and forward-looking volatilities (30 calendar days)

Further analysis with linear regression showed that historical volatility could explain 80% of the VIX variance. Thus the historical 30-day rolling volatility of S&P 500 log-returns was selected as the best proxy for VIX. Following this decision and the goal of granting VCRIX predictive capabilities, the time series of 30-day historical rolling volatility of CRIX log-returns was constructed and used as a true value in back-testing of several predictive models that were estimating the annualized volatility one day ahead. According to the evaluation metrics, as shown in Table 1, the HAR model was selected as a basis for VCRIX. Further on this model was tested in the simulation of actual VIX using the S&P 500 log-returns instead of CRIX log-returns. The resulting pair of time series showcased the correlation of 89%, thus justifying the model selection.

Days of lag	**Correlation**	**MDA**
Day-on-day	0.89	51%
21 days	0.89	64%
42 days	0.87	73%

Table 2: Evaluation of the simulation of VIX using VCRIX methodology, comparison of true and simulated values

The simulation of VIX exhibited correlation of 89% and a Mean Directional

Accuracy (MDA) of 51% rising to 64% in case lag of 21 days is considered, as indicated in Table 2. Figure 5 and Figure 6 showcase the difference between the estimated values and actual VIX. These results led us to believe that the chosen methodology does indeed provide a solid estimation of the implied volatility in the absence of the derivatives market.

Figure 7 displays the time series of VCRIX from Jan 2015 to Aug 2019 and the smoothed conditional means (LOESS) red line with a span of 0.5, it is added to offer a long term review on volatility.

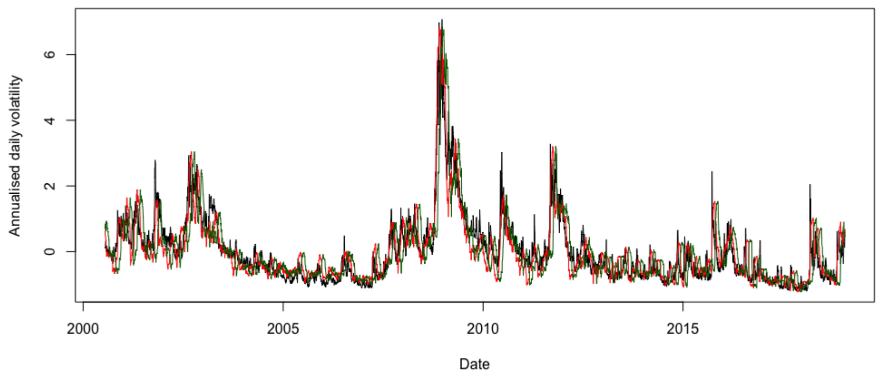

Figure 5: VIX estimated with HAR model on scaled daily volatility of SPY log-returns, VIX estimated with HAR with 21 days lag and true **VIX** values from 2000 to 2019

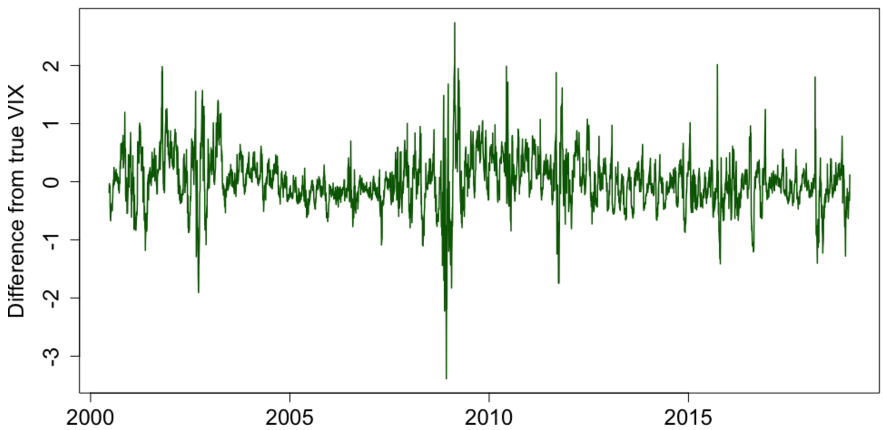

Figure 6: Difference between true and estimated VIX, values from 2000 to 2019. One can observe that the proposed model lags in catching the big spikes but performs well when market volatility is lower.

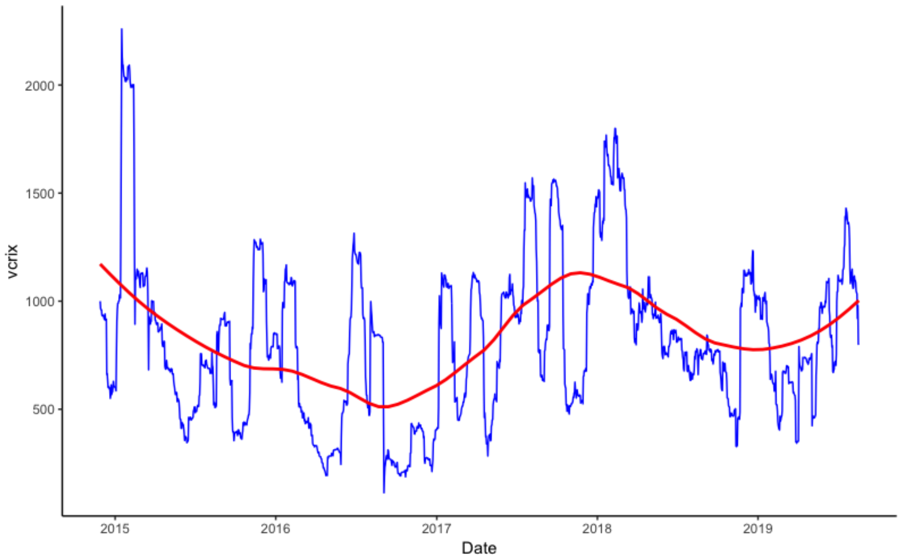

Figure 7: VCRIX and LOESS-smoothed mean (span=0.5)

VCRIX

5 Trading implementation

As the CC market develops and new financial instruments based on CC appear, VCRIX can become increasingly employed in trading strategies. As one of the examples, an inverse volatility ETF is a financial product that allows investors to gain exposure to volatility, and thus hedge against portfolio risk, without having to buy options.

Regardless of the absence of the above mentioned derivative instruments, volatility-based trading strategies may still be employed and tested. Conventional short-term reversal strategies have been explored and perfected by scholars and industry practitioners (Lehmann (1990), Jegadeesh (1990), Blitz et al. (2013)) over the years. We have employed a number of modified reverse volatility trading strategies with an example provided below. As an input, we employ VCRIX for daily volatility estimation and LOESS of VCRIX (as a variation of MA, different spans represented in Figure 8) as a benchmark.

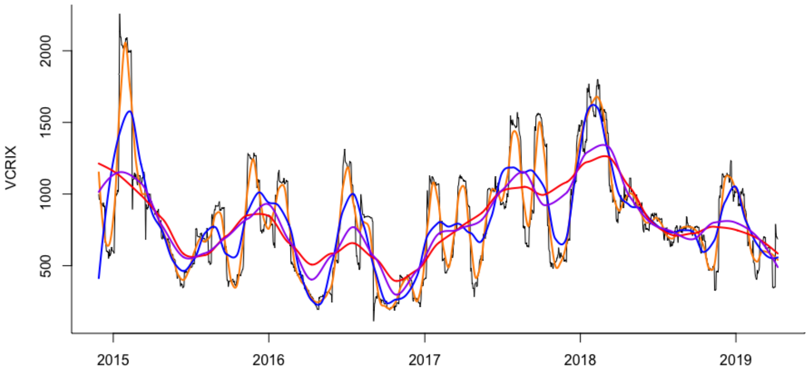

Figure 8: VCRIX and the LOESS-smoothed mean of VCRIX, with span=0.05, span=0.1, span=0.2, span=0.25

VCRIXloess

LOESS is a non-parametric operator that yields a smooth function by locally minimizing the variance of the residuals or prediction error (Cleveland (1979)). For each value of x, the value of $f(x)$ is estimated by using its neighboring sampled (known) values (quite similarly to a knn algorithm). In the case of LOESS, the tunable parameter is the span that will determine the smoothness of the resulting estimate, with a broader span resulting in higher bias and narrower span offering higher variance.

Figure 9 provides an illustration of a trading strategy that is based on long-cash signals generated by the relationships between the daily VCRIX value and its two LOESS curves (span=0.25 and span=0.20). In further notation we indicate the span with the subscripts, as in Figure 9, constructed with the use of $LOESS_{0.25}$ and $LOESS_{0.20}$.

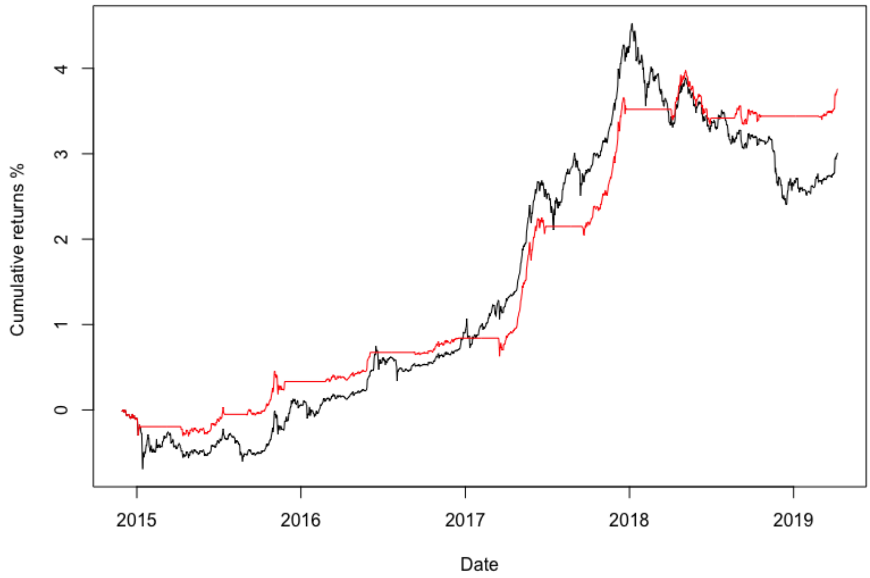

Figure 9: Cumulative returns of the trading strategy with $LOESS_{0.25}$ and $LOESS_{0.20}$ versus the cumulative **returns on CRIX**

VCRIXtrading

The strategy gets its signals from the LOESS-smoothed mean of VCRIX. The trading strategy, Algorithm 1, dictates to go long in cash when the volatility measured by VCRIX is high and go long in an ETF on CRIX when the volatility measured by VCRIX is low. We compare if the volatility is high or low by the LOESS-smoothed mean of VCRIX. A LOESS with a broad span gives a long term smoothed average for VCRIX, whereas a LOESS with smaller span gives the short term average. In particular we go Long in a CRIX ETF when the short term volatility is low compared to the long term one, $LOESS_i \geq LOESS_j$, and vice versa go Long in cash when the short term volatility is comparably high, $LOESS_i < LOESS_j$, see Algorithm 1.

Algorithm 1 : Trading strategy

Set: $i, j \in \{0.05, 0.1, 0.15, 0.2, 0.25\}$, $i > j$

Input: LOESS_i, LOESS_j, CRIX ETF

Output: Investment product y

```
if LOESS_i ≥ LOESS_j then
    y = CRIX ETF
else LOESS_i < LOESS_j
    y = Cash
end if
```

By construction the choice of the span of LOESS is critical for the performance of the trading strategy. We construct the LOESS for the spans 0.05, 0.1, 0.15, 0.2, 0.25, and compare the results with the following measures:

1. **cumul.returns**: the aggregate gain over the observed time period up to the final day of trading.

2. **mean.returns**: the mean of the daily trading strategy returns.

3. **takeover.days**: the percentage of days when the cumul.returns are higher for the trading strategy than for CRIX.

4. **Sharpe.ratio**: compares the mean of the returns of the trading strategy over the standard deviation of the returns of the trading strategy, reflecting extra return per unit of increase in risk.

The results are presented in Table 3. The rows are named by the two LOESS-smoothed means involved in the trading strategy. CRIX returns are offered for reference. The left LOESS measures the long term VCRIX volatility and the right one the shorter-term one, in Algorithm 1 indicated as i and j respectively.

We observe the Sharpe ratio is best when we measure the long term volatility over a longer window, meaning for higher values of LOESS-smoothed means, e.g., $i = 0.20$ and $i = 0.25$. We found the best results, in terms of the Sharpe ratio, for the pair of LOESS spans 0.25 and 0.15, as well as 0.25 and 0.20. The second pair performs best, followed by spans 0.25 and 0.15 in terms of cumulative returns as well as takeover days (these trading strategies are more often above the one for a CRIX

	cumul.returns	mean.returns	takeover.days	Sharpe.ratio
CRIX	3.00%	0.19%	NA	0.0484
$\text{LOESS}_{0.10} \sim LOESS_{0.05}$	0.83%	0.05%	27.24%	0.0202
$\text{LOESS}_{0.15} \sim LOESS_{0.05}$	2.18%	0.14%	45.64%	0.0583
$\text{LOESS}_{0.20} \sim LOESS_{0.05}$	2.36%	0.15%	45.89%	0.0661
$\text{LOESS}_{0.25} \sim LOESS_{0.05}$	2.96%	0.19%	55.05%	0.0810
$\text{LOESS}_{0.15} \sim LOESS_{0.10}$	3.43%	0.22%	75.52%	0.0948
$\text{LOESS}_{0.20} \sim LOESS_{0.10}$	2.95%	0.19%	63.97%	0.0867
$\text{LOESS}_{0.25} \sim LOESS_{0.10}$	3.41%	0.21%	66.92%	0.1009
$\text{LOESS}_{0.20} \sim LOESS_{0.15}$	3.51%	0.22%	57.88%	0.1013
$\text{LOESS}_{0.25} \sim LOESS_{0.15}$	3.58%	0.23%	49.15%	**0.1039**
$\text{LOESS}_{0.25} \sim LOESS_{0.20}$	**3.76%**	**0.24%**	**68.05%**	0.1029

Table 3: Comparison of trading strategies with several LOESS-smoothed means of VCRIX.

ETF). The trading strategy, see Algorithm 1, works in this case in the following way: We go long in a CRIX ETF when $LOESS_{0.25} > LOESS_{0.10}$ and long in cash when $LOESS_{0.25} < LOESS_{0.10}$. Similarly, for 0.25 and 0.20, the trading strategy receives signals if: We go long in a CRIX ETF when $LOESS_{0.25} > LOESS_{0.20}$ and long in cash when $LOESS_{0.25} < LOESS_{0.20}$.

As it can be observed from the graph, Figure 9, and Table 3, for 67% of the days the strategy outperforms the benchmark. As an additional benefit for the portfolio balancing, the variation of the trading strategy is lower than one of CRIX returns. Regardless of the downturn that takes place during the 2017 boom, the results after the cool-down remain superior to the plain CRIX returns, which suggests the viability of VCRIX as a trading tool.

6 Discussion

From the beginning, one of the biggest complexities in crypto-trading came from the absence of clear pricing strategies: what is BTC worth? How do we estimate the value of new coins? Are coins under- or over-appreciated? (Yermack (2015)).

While mechanics and potential implications of CC in financial economics are being explored Härdle et al. (2019), there is still no established consensus over the evaluation methods. Nowadays agents are often left with nothing but the information on the overall market "feeling" about the CC, which is communicated by the rise and fall of the price, in other words, It is volatility.

VCRIX captures the volatility jumps that correspond to the development of the CC-ecosystem and can tell a story of the CC adoption (Figure 11). We observe spikes of interest in BTC in 2015, winter and summer of 2016 when BTC was slowly making its way to the attention of the general public. The large scale swings in price would not constitute a significant shock in absolute values, but when something that was still considered a digital maverick rose in value from roughly 400 USD to 1000 USD within a year (Business Insider, 2016 (*"Bitcoin is still storming higher"*)), investors noticed. VCRIX further captures the beginning of the first massive growth wave (also captured well by the CRIX in Figure 10) and development of altcoins (ETH, LTH, and others).

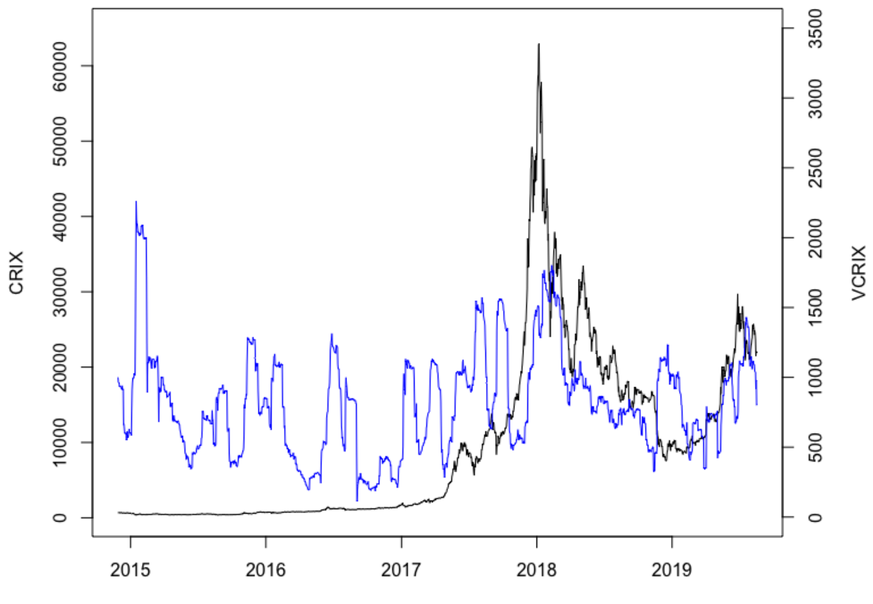

Figure 10: CRIX and VCRIX

2017 became the year of massive volatility (VCRIX showcases the values that can be interpreted as daily volatility of 140%). These levels of uncertainty were largely caused by the major legislative shifts that were happening in countries-juggernauts of CC movement: China, Korea, Japan, and the USA. Additionally, BTC was going through the heated debates on the SegWit (Segregated Witness) fork that was supposed to improve the speed and cost of BTC transactions. The fork was implemented in August, 2017 and led to the emergence of BTC Cash due to a certain number of big miners disagreeing with the implementation. These volatility spikes yet proved to be minor in comparison with the major market meltdown that happened at the beginning of 2018, when prices of most currencies on average suffered an 80% drop (CoinMarketCap (2018)). 2018 was considered to be a stabilization period when governments and financial corporations were getting on-board, however, the end of 2018 saw another volatility spike, majorly driven by the "holiday race" and uncertainty driven by "Constantinople fork" that is expected from Ethereum at the beginning of 2019.

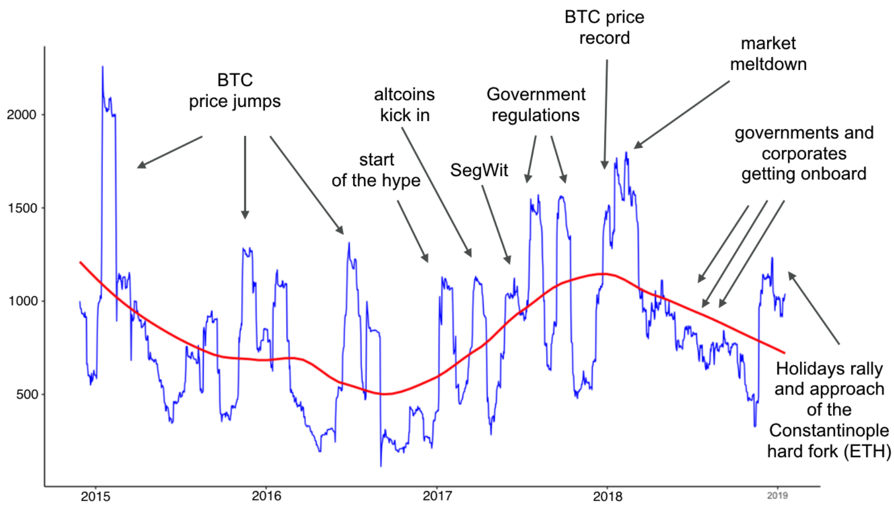

Figure 11: VCRIX interpretation

Pattern analysis of the VCRIX graph allows to distinguish a pattern that could be allegedly interpreted as a signal to large volatility spikes. Volatility clusters take the "triple spike" shape with the first spike indicating the upcoming large wave - this structure can be observed throughout 2016 and 2017, with the biggest wave at the end of 2018, taking a form of a tall "triple spike". This structure fades throughout 2018 during the settle-down, however, one may expect that the spike at the beginning of 2019 may be interpreted as the signal to a large wave of volatility coming during summer and autumn of 2019 (this prognosis was made during the writing of the paper in Spring of 2019). As of August 2019, this forecast proved correct (Figure 12), although the interpretation requires further economic investigation and cannot be used as a forecasting tool without additional scrutiny.

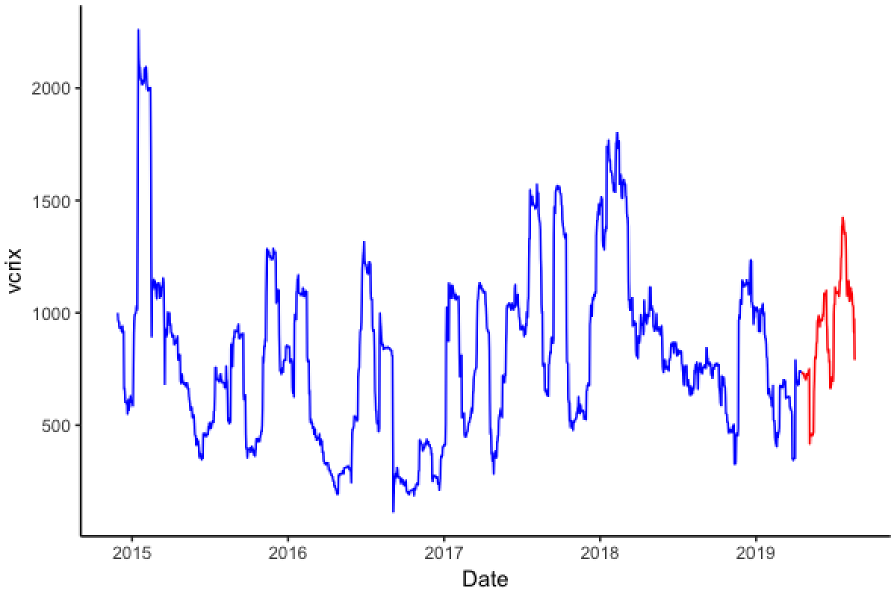

Figure 12: VCRIX and realization of the forecasted volatility spike

The search for an implied volatility proxy performed in Section 3.3 showed that VIX tends to overestimate the realized volatility. As it would seem, there is some information about market expectations that is not explained by the historical volatility. The excessive uncertainty would be expected to have strong relationships with returns that happen at the point of the highest delta. Given the design of VIX, one may expect it to contain additional signal about the emotional status of the market that tends to overreact in times of uncertainty. Interestingly enough, the LSTM predictive model also tends to overestimate the volatility. The neural network-based models are known for the capability to pick up underlying trends that are omitted in traditional financial models, however, the "black box" nature of models render clear interpretation complicated.

7 Conclusion

We have set the goal of capturing the expectations on the CC market (represented by CRIX) through the construction of an implied volatility proxy in the absence of the derivatives for the majority of CC. The "fear index" of the American stock market - VIX - was selected as guidance and benchmark. Analysis of the relationships between VIX and volatility of the underlying assets provided an insight for the selection of a mentioned proxy - the historical rolling volatility of SPY. Following this finding, the rolling volatility of log-returns of CRIX was calculated. The HAR model proved to be best for the estimation of the daily volatility of CRIX log-returns, offering the MSE of 0.03 and a 99% correlation with the 30 day-rolling volatility of CRIX log-returns. This model was further tested in a simulation, where it was used to estimate VIX. An impressive 89% correlation was achieved, thus proving the fitness of the selected methodology to the announced goal. The established VCRIX provides a daily forecast for the mean annualized volatility of the next 30 days. Authors intend to conduct further research to capture the observed excessive volatility that is captured by derivative-based indices like VIX and presumably stems from the behavioral component of option pricing.

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