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Understanding the Role of Housing in Inequality and Social Mobility

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Abstract

Housing typically takes up a major proportion of households' expenditure, and thus it certainly plays a critical role in shaping the pattern of income inequality and social mobility. Whether high housing price-to-rent ratio will amplify inequality and inhibit social class upgrading is still a controversial issue in the existing literature. In this paper, we develop a partial equilibrium lifecycle framework to address these issues. Agents in our economy are divided into two social classes according to the initial human capital level inherited from their parents. Those who belong to upper class will draw their innate abilities from a distribution that first order stochastically dominates those from lower class. Throughout the entire lifecycle, agents make endogenous human capital investment and housing tenure decisions. We calibrate the model to mimic some stylized facts in the the real world counter part. Our simulation results indicate an inverse-U pattern between housing price-to-rent ratio and measures of income inequality, and as well as a U-shape pattern between price-to-rent ratio and social mobility measured by Shorrocks Index. The implication is that housing tends to amplify the inequality and slow down the social mobility when houses can only be purchased by a small group of agents in the economy. Moreover, our results also suggest that better quality of education as a result of a higher return to human capital investment tends to dampen the role of housing.

Keywords: Income Inequality; Social Mobility; Price-to-rent ratio.

JEL Classification:

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1 Introduction

The seminal work by Piketty in 2014 presented a "U-Shaped" pattern of inequality for the past century. In particular, the last forty years have seen a rapid rise in top income inequality in the United States (Keister and Moller, 2000; Piketty and Saez, 2006; Saez, 2009). Right before the Subprime Mortgage Crisis in 2007, the highest concentration of income since 1979 was observed when the top 1% of households accounted for almost 20 percent of total income (Sherman and Stone, 2010). Many studies have been done about the possible causes of the rising inequality: Morris and Western (1999) believed that the supply side demographic shifts can be the contributors to the changes of the income distributions. Harrison and Bluestone (1990) regarded the economic restructuring could be one of the reasons. Skill-biased technological change (Bound and Johnson, 1989) and institutional shifts (Levy and Temin, 2007) have also been considered.

During the same time span, housing price and housing value to rent in the U.S climbed to the top until the financial crisis of 2007-2008 (Campbell et al., 2009). Housing typically takes up a major proportion of households' expenditure, and thus it certainly plays a critical role in shaping the pattern of income inequality. However, the role of housing in the rise of social inequality is basically ignored in the previously mentioned studies. There are three aspects of housing in the U.S. that may impact the distributions of income and have been discussed the most in the literature: housing affordability (Chi and Laquatra, 1998; Mimura, 2008; Quigley and Raphael, 2004), homeownership (Chi and Laquatra, 1998), and subprime lending (Belsky et al., 2007; Stegman et al., 2007). Based on empirical analysis, Rognlie (2014) argued that indeed, there existed an upward trend of net capital share after 1984 as mentioned in Piketty (2014); however, when he disaggregated the capital share into the housing sector and the non-housing sector, only the housing sector was found to have contributed to the inequality increases.

It seems that we need a bridge to connect housing and inequality. Cantó et al. (2000)

and Andrews and Leigh (2009) showed that there existed a strong trade-off between income inequality and social mobility. Aaberge et al. (2002) exhibited that the U.S., which suffered from a higher degree of inequality, had lower mobility of earnings than the Scandinavian countries. Whether high housing Price-to-Rent ratio will amplify inequality and inhibit social class upgrading is still a controversial issue in the existing literature. We infer that social mobility plays an important role. Methods of measuring social mobility have been developed with time: Shorrocks (1978) , Maasoumi and Zandvakili (1986) , Jarvis and Jenkins (1998), Schluter and Trede (2003), Nichols and Favreault (2009). However, in this paper, we still adopt the most commonly used Shorrocks Index as our measure method. Kennickell and Starr-Mccluer (1997), Charles and Hurst (2003), Aaberge et al. (2002), Klevmarken et al. (2003) reports measurement of social mobility. In our study, we try to match our simulation results with the study of Kennickell and Starr-Mccluer (1997).

In this chapter, we develop a partial equilibrium life-cycle framework to address the relationships among inequality, social mobility and housing. For the basic setting of the model we follow Benabou (1994). Agents in our economy are divided into two social classes, the upper class and the lower class according to the initial human capital level inherited from their parents. Those who belong to the upper class will draw their innate abilities from a distribution that first order stochastically dominates those from the lower class. Throughout the entire lifecycle, agents make endogenous human capital investment and housing tenure decisions. We calibrate the model to mimic some stylized facts in the the real world counter part. Our simulation results indicate an inverted-U pattern between housing price-to-rent ratio and measures of income inequality, and as well as a U-shape pattern between price-to-rent ratio and social mobility measured by Shorrocks Index. The implication is that housing tends to amplify the inequality and slow down the social mobility when houses can only be purchased by a small group of agents in the economy. Moreover, our results also suggest that better quality of education as a result of a higher return to human capital investment tends to dampen the role of housing.

The rest of the paper is structured as follows. Section 2 lays out the theoretical framework. Section 3 illustrates both the benchmark's simulation results and the counterfactual results. Section 4 presents our conclusions.

2 Model

Time is discrete and infinite indexed by $t = 0, 1....$ The economy is populated by overlapping generations of individuals. The size of each cohort is normalized to be 1. Each individual lives two periods: the Youth and the Old-age. Each individual gives birth to one child at the end of her middle-age. The new born child is endowed with the same human capital as her parent through at-home learning (Benabou, 1994). We will describe the timeline in detail later. Individuals are also heterogeneous in their innate abilities, which are drawn randomly upon birth from a specific distribution function defined over $[\underline{a}, \overline{a}]$.

We let $i \in [0, 1]$ denote the individual index. Individual i born in period t is endowed with h_t^i units of human capital. Each individual will engage in human capital accumulation through her Youth. h_{t+1}^i denotes human capital level that she owns by the end of the Youth, and it also equals to the human capital level that is inherited by her child born in the time $t + 1$.

Individuals are classified into two classes according to their initial human capital levels. There exists a threshold human capital level $\overline{h}$. Individuals belong to the upper class if their initial human capital exceeds $\overline{h}$, otherwise they belong to the lower class. Individuals from two classes draw their innate abilities from two independent distributions. We denote $G(\cdot)$ as the distribution function if the individual belongs to upper class, and $F(\cdot)$ if the individual belongs to lower class. Moreover, we assume $G(\cdot)$ first-order stochastic dominates $F(\cdot)$ (FOSD). Specifically, we let $G(a)$ take the following form:

$$G(a) = \left(\frac{F(a) - \underline{a}}{\bar{a} - \underline{a}}\right)^{\frac{1}{\alpha}} \times (\bar{a} - \underline{a}) + \underline{a}, \quad \alpha > 1 \tag{1}$$

This captures the essence that individuals from the upper class face a "superior" distribution of talents.

Given the realization of innate ability a and initial human capital h_t^i, each individual i born in t then makes the human capital investment decision I_t^i. Specifically, the accumulation of human capital can be denoted by an increasing function as follows:

$$h_{t+1}^i = (1-\delta)h_t^i + a_t^i \left(I_t^i\right)^\gamma \tag{2}$$

where $\delta \in (0,1)$ is the human capital depreciation rate, and $\gamma \in (0,1)$ represents the elasticity of investment to human capital accumulation. Higher abilities or investment will result in higher human capital in the next period.

Individuals receive wage income both in the Youth and in the Old-age. Wage rate is paid at per unit of human capital. We abstract away from the analysis of labor market and assume that the wage rate is exogenous. Individuals obtain utilities from both consumption goods and housing services. We distinguish the housing tenure decision by allowing individuals to choose between renting and purchasing a house. Individuals gain higher utilities from owning a house than from renting one. We also assume individuals are altruistic and they value their children's initial human capital level. Specifically, the individual's utility function takes the following form:

$$u = \log c_t + \beta \log c_{t+1} + \nu \log h_{t+1} + e\Gamma_{\{i\in\{o,r\}\}} \tag{3}$$

where c_t and c_{t+1} represents the goods consumption of individuals in the Youth and the Old-age, respectively. $\beta \in (0,1)$ is the subjective discount rate, and $\nu > 0$ captures the degree of altruism. $e > 0$ governs the utility premium from owning the house. Following literatures (such as **?**), we assume the utility premium is positive. $\Gamma_{\{i\in\{o,r\}\}}$ is an indicator function and equals to 1 if the agent owns a house.

If an individual decides to rent the house, she solves the following utility maximiza-

tion problem [1]:

$$\begin{aligned} u^r &= \max\left(\log c_t^r + \beta \log c_{t+1}^r + \nu \log h_{t+1}^r\right) \\ s.t.\ c_t^r &= w_t h_t - R_t - I_t^r \\ c_{t+1}^r &= w_{t+1} h_{t+1}^r - R_{t+1} \\ h_{t+1}^r &= (1-\delta) h_t + a_t \left(I_t^r\right)^\gamma \end{aligned}$$

where R_t and R_{t+1} denote the rental price in t and $t+1$, respectively. We do not allow individuals to switch from renters to owners at the end of the Youth. Hence, the housing tenure decision should be made based upon the present value of life-time utilities.

If an individual decides to own the house, she solves the following utility maximization problem:

$$\begin{aligned} u^o &= \max\left(\log c_t^o + \beta \log c_{t+1}^o + \nu \log h_{t+1}^o + e\right) \\ s.t.\ c_t^o &= w_t h_t - P_t - I_t^o \\ c_{t+1}^o &= w_{t+1} h_{t+1}^o + P_{t+1} \\ h_{t+1}^o &= (1-\delta) h_t + a_t \left(I_t^o\right)^\gamma \end{aligned}$$

Individuals born in t purchase housing at price P_t by the end of their the Youth, and they sell their house at price P_{t+1} by the end of their Old-age. We do not consider the option of mortgages, and this implies individuals need to purchase the house at full price if they decide to own it.

To simplify the analysis, we also abstract away the supply side of the housing market. We let housing sales and the rental price evolve exogenously over time since examining housing price dynamics is not the major focus of the paper.

[1] We drop individual indexes in the remainder of the paper, except in parts where such choice may jeopardize the clarity of our exposition.

2.1 Equilibrium Characterization

In this subsection, we explicitly solve the individual's optimal decision over consumption, human capital investment and housing tenure.

If an individual decides to become a renter, solving the utility maximization problem gives following result:

$$\left(\frac{h^r_{t+1}-(1-\delta)h_t}{a_t}\right)^{\frac{1}{\gamma}}\left[1+\frac{h^r_{t+1}}{\gamma a_t}\frac{\left(\frac{h^r_{t+1}-(1-\delta)h_t}{a_t}\right)^{1-\gamma}}{\beta+\frac{\beta R_{t+1}}{\left(wh^r_{t+1}-R_{t+1}\right)}+\nu}\right]=w_t h_t-R_t \tag{4}$$

It is straightforward to obtain that h_{t+1} is increasing in both abilities a_t and initial human capital h_t. Moreover, the comparative statics show that $\partial h_{t+1}/\partial R_t < 0$ and $\partial h_{t+1}/\partial R_{t+1} > 0$. Therefore, higher future (current) rents tend to induce (encourage) more human capital investment. With given ability level and initial human capital, if the current rental is higher, the investment in education I_t will be crowded out. Lower I_t will lead to less human capital accumulation, then, $\partial h_{t+1}/\partial R_t < 0$. The reason why h_{t+1} increases with future rental R_{t+1} might not be intuitive. In our model, the information for the next period is available to agents. This means that, the agents who know that the rents are higher in the next period will save more to gain a higher total utility. In the basic model, we do not allow the individuals to save or to borrow. However, the investment in human capital accumulation could account for a kind of saving. Therefore, when rental goes up in the Old-age, agents will accumulate more human capital.

Similarly, if an individual decides to become an owner, solving the utility maximization problem gives following result:

$$\left(\frac{h^o_{t+1}-(1-\delta)h_t}{a_t}\right)^{\frac{1}{\gamma}}\left[1+\frac{h^o_{t+1}}{\gamma a_t}\frac{\left(\frac{h^o_{t+1}-(1-\delta)h_t}{a_t}\right)^{1-\gamma}}{\left[\beta+\nu-\frac{\beta P_{t+1}}{\left(wh^o_{t+1}+P_{t+1}\right)}\right]}\right]=w_t h_t-P_t \tag{5}$$

In contrast to the case of renters, the effects of innate abilities on human capital

investment become ambiguous. This is simply because housing can crowd out human capital investment by smoothing consumption between the Youth and the Old-age. Individuals do not necessarily need to invest in education in the Youth in order to enjoy higher consumption in the Old-age. Housing can play a similar role as they can be sold when individuals reach the end of the Old-age. The reason why human capital investment is not completely replaced by housing consumption is because individuals are altruistic.

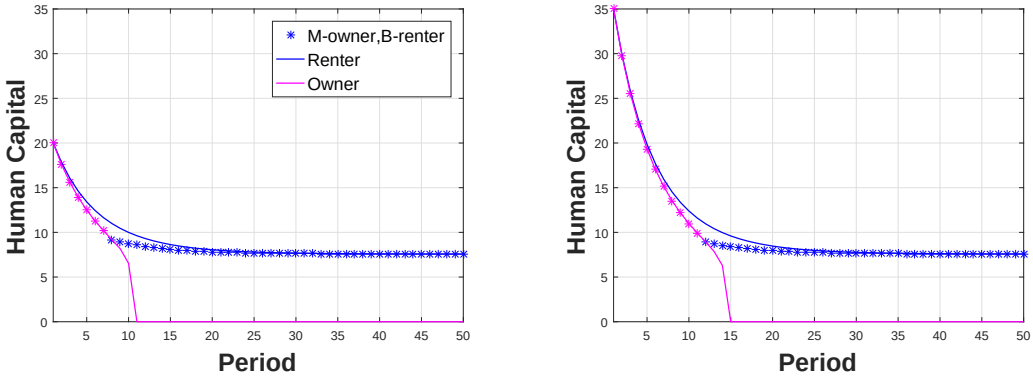
In the following, we also examine a special case where innate abilities can also be passed by from parent to child over time. In other words, individuals no longer need to draw their innate abilities from given distributions. This special case forces us to keep track of the dynamics of human capital accumulation for a given innate ability. It will also give us a clearer picture about how the agents end up choosing between renting and purchasing a house. In Figure 1, we present the results for an individual with a relatively-low innate ability ($\underline{a}$). The left and right panels depict human capital accumulation path over 50 periods if the individual is endowed with a relatively low or high initial human capital level, respectively.

The blue line and the pink line simulate the house renters' or owners' human capital level along time without the opportunity to make the tenure choices. The star denotes the tenure decision. If the star is blue, then the agent chooses to be a house renter.

The results suggest that the individual will eventually switch from owner to renter. A higher initial human capital level will delay the switching period. In the long run, human capital converges to a steady-state level, regardless initial human capital level. Mathematically, We define the steady-state human capital level for a renter (h^r) as follows:

$$\left(\frac{\delta h^r}{a}\right)^{\frac{1}{\gamma}} \left[\frac{1}{(wh^r - R)} + \frac{1}{\delta\gamma}\frac{1}{(\beta+\nu)\,wh^r - \nu R}\right] = 1 \tag{6}$$

Similarly, we simulate the dynamics of a dynasty with a relatively high ability. In Figure 2, we present the results for an individual with a relatively high innate ability.



(a) Low ability with low initial h (b) Low ability with high initial h

Figure 1: Dynamic Transition of Human Capital Given Fixed Low Ability Level

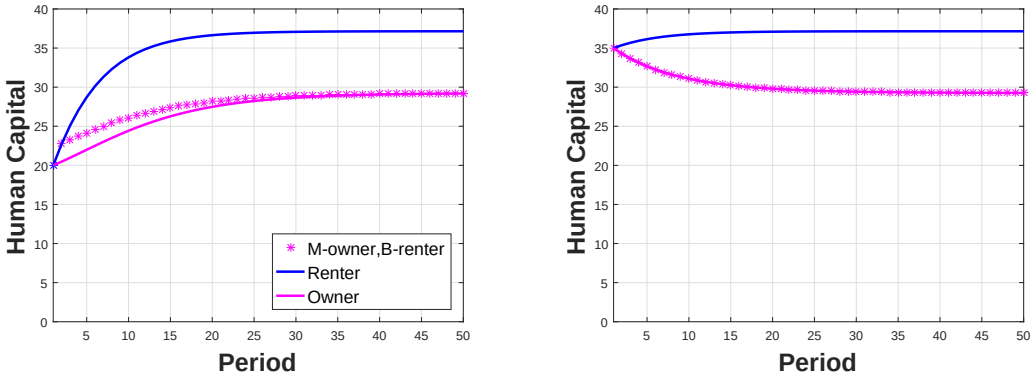
Note: Given P/R=2.8

The left and right panel depict the human capital accumulation path over 50 periods if the individual is endowed with a relatively low or high initial human capital level, respectively.

The results suggest that the individual will eventually be a house owner. A lower initial human capital level will delay the switching period. In the long run, human capital converges to a steady-state level. We formally define the steady-state human capital level for a house owner (h^o) as follows:

$$\left(\frac{\delta h^o}{a}\right)^{\frac{1}{\gamma}}\left[\frac{1}{wh^o-P}+\frac{1}{\delta\gamma}\frac{1}{(\beta+\nu)wh^o-(2\beta+\nu)P}\right]=1 \tag{7}$$

The following two panels in Figure 3 present the pattern of human capital accumulation if the dynasty is endowed with a high or low initial human capital level under the condition of a higher price-to-rent ratio (P/R). The left panel presents the result where in the beginning the individual chooses to be a renter and becomes an owner after 2 periods. The dynasty's human capital stocks at first then starts to decline as housing can already serve the purpose of smoothing consumption instead of the education investment. In period 4, the individual returns to be a renter till it is optimal for her offspring to be an owner again. Similar processes repeat. In the right panel,



(a) High ability with low initial h (b) High ability with high initial h

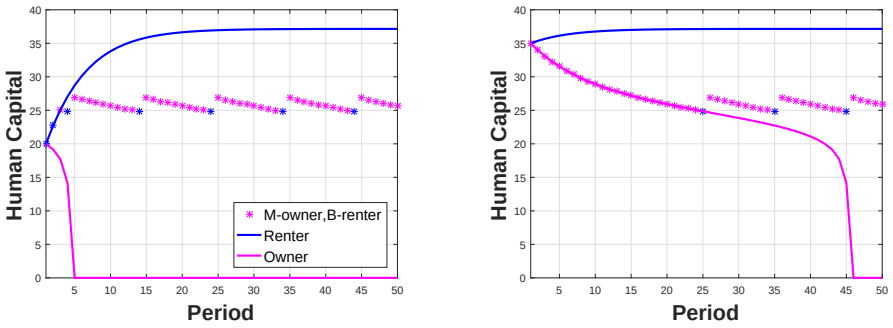
Figure 2: Dynamic Transition of Human Capital Given Fixed High Ability Level (Low Price-to-Rent Ratio)

Note: Given P/R=2.8

for a dynasty with high initial human capital and innate abilities, the individual starts to be an owner and remains to be owner over a long time. Meanwhile, for the same reason, human capital keeps declining until the offspring's optimal tenure choice is no longer owning the house, she switches to a renter status in a certain period. Then the processes are the same as in the left panel. Under both situations, there is no steady-state where the dynasty's housing tenure decision remains unchanged. They keep oscillating between being an owner and being a renter.

As depicted in the special case, there may not exist a steady-state at the dynasty level, if we define a steady-state as the state where human capital stock remains constant over time. This becomes more prominent when individuals within the same dynasty draw their innate abilities upon birth. In the following, we define a steady-state at the aggregate level instead. Define the set of dynasty in the upper class in t who remain in the upper class in $t+1$ as:

$$S_{t,t+1} = \left\{ i : h_{t+1}(i) > \bar{h} \mid h_t(i) > \bar{h} \right\} \tag{8}$$



(a) High ability with low initial h (b) High ability with high initial h

Figure 3: Dynamic Transition of Human Capital Given Fixed High Ability Level (High Price-to-Rent Ratio)

Note: Given P/R=6.2

Define the set of dynasty in lower type in t who upgrade to upper class in $t+1$ as:

$$J_{t,t+1} = \left\{ i : h_{t+1}(i) > \bar{h} \mid h_t(i) \leq \bar{h} \right\} \tag{9}$$

The fraction of dynasty in the upper class is:

$$Q_{t+1} = \sum_{i \in S_{t,t+1}} + \sum_{i \in J_{t,t+1}} \tag{10}$$

When the fraction of the upper class remains stable, we claim that this economy asymptotically converges to the steady-state.

3 Quantitative Analysis

In the quantitative analysis, we assume that the Price-to-Rent ratio varies. Then, we focus on the changes at the social mobility level and the shifts of inequality. In order to explain the mechanism, we also examine detailedly how the fraction of individuals of upper type, investment in education and welfare etc. respond when the economy converges to steady-state. We set the value for a group of parameters and use it as our

benchmark. Then, we also do a counterfactual experiments on the different elasticity of human capital investment γ and extra utility gain as house buyers e.

3.1 Parameterization

Each period, the Youth or the Old-age, in our economy is set to be 20 years. Since the individual's behavior in the adolescent years would not affect our analysis, we do not take this period into account. This means that, the period of Youth starts from the age of 20 years and lasts until the age of 40 years, while agents in the Old-age are from 40 years old to 60.

The parameter space in our model contains $\{\beta, \delta, \alpha, w, v, \gamma, e, \underline{a}, \overline{a}, \overline{h}\}$ (Table 1). β is the subjective discount rate and we set it to be 0.6, so it corresponds to an annual discount rate of 0.97. δ is human capital depreciation rate, and we let it be 0.2 so it is equivalent to an annual depreciation rate of 1.1 percent. α is the parameter which controls the ability distribution differences between the upper class and the lower class. It was calibrated to ensure that the upper income tail follows the Pareto's law with the tail index equal to 1.2 (**?**). We normalize the wage rate w to be 1 and set it constant. Then, we let the rental equal to one third of the average income. The housing price-to-rent ratio is exogenous, and we set it to be 6.0 in the benchmark calibration, which matches the weighted house value-to-rental ratio in 1985 in the U.S. (Benabou, 2000). e is the utility premium from owning a house, and we calibrate it to match an average homeownership rate of 66 percent. γ is the elasticity of human capital investment and we solve it to match the elasticity of return to education from a standard Mincer regression commonly found in the literature. $\overline{h}$ denotes the threshold human capital level to identify upper and lower classes. It was solved by the model to match the social classes distribution in the U.S. in 1985. We also normalize the lower bound of the ability distribution($\underline{a}$) to be 1, and calibrate the upper bound of the uniform distribution ($\bar{a}$) such that the resulting 90-10 income ratio in U.S. equals to 5.0, which is commonly documented in the literature. v captures the bequest motive and measures

Parameter	Value	Source (Targets)
β	0.6	discount rate
δ	0.2	human capital depreciation rate
α	2	parameter to control ability distributions
v	5.0	bequest motive
γ	0.3	elasticity of human capital investment
e	1.0	extra utility gain as house owners
$\underline{a}$	1	lower bound of ability
$\bar{a}$	3	upper bound of ability
$\bar{h}$	26	threshold human capital level to identify social class
w	1	wage rate

Table 1: Parameters

the relative importance of the decendent's human capital level to the individual's own consumption. We calibrate the number to match a 30-percent average expenditure share on education.

3.2 Social Mobility and Inequality

We follow Kennickell and Starr-Mccluer (1997) to compute the Shorrocks Index in this subsection. They calculated the six-year transition matrix ($A_{KS,6}$) from 1983 to 1986 for quantiles and top percentile ranges. The seven states were: bottom 20 %, 20-39%, 40 - 59%, 60 - 79%, 80 - 89%, 90 - 94%; 95 - 99%, top 1% percentile.

$$A_{KS,6}= \begin{bmatrix} 0.672 & 0.246 & 0.063 & 0.018 & 0.001 & 0.000 & 0.000 \\ 0.246 & 0.495 & 0.190 & 0.042 & 0.019 & 0.007 & 0.000 \\ 0.066 & 0.192 & 0.480 & 0.208 & 0.037 & 0.016 & 0.000 \\ 0.021 & 0.082 & 0.329 & 0.418 & 0.113 & 0.036 & 0.002 \\ 0.011 & 0.071 & 0.212 & 0.301 & 0.225 & 0.177 & 0.004 \\ 0.000 & 0.028 & 0.164 & 0.104 & 0.180 & 0.430 & 0.094 \\ 0.000 & 0.031 & 0.024 & 0.061 & 0.045 & 0.247 & 0.593 \end{bmatrix}$$

In the transition matrix, the number in the i_{th} row and j_{th} column of the matrix represents the fraction of individuals who belong to income group i in the initial period and switch to income group j in the next period. Hence, the sum of each row equals to 1. With a square mobility transition matrix A of dimension m, the Shorrocks index $s(A)$ is given by:

$$s(A) = \frac{m - \sum_i a_{ii}}{m - 1}$$

where m denotes the number of income groups and a_{ii} is the diagonal element of the i_{th} row. The Shorrocks Index $S_{KS,6}$ from Kennickell and Starr-Mccluer (1997) was equal to 0.61450. When the social mobility in one economy is harder, more families will be trapped in the same income group, which makes a_{ii} relatively large. It also results in a larger negative part in the Shorrocks index calculation. Therefore, the harder the changes of social class are, the smaller the Shorrocks index will be.

At the beginning of our simulation period, we divide all the individuals into 7 groups according to their life-time income. Specifically, similar to the study of Kennickell and Starr-Mccluer (1997), the seven income groups include bottom 20 %, 20-39%, 40 - 59%, 60 - 79%, 80 - 89%, 90 - 94%; 95 - 99%, and the top 1% percentile, respectively. We then track all the individuals throughout the entire simulation period until the economy reaches its steady-state. By comparing the state to which each individual initially belongs to with the state at the last period, we get the transition matrix for the whole period $T(A_S^T)$. By comparing the state which each individual/ family belongs at t with the state at t+1, we could get the transition matrix for each period $t(A_S^t)$.

Therefore, for each given Price-to-Rent ratio, there exist one whole period transition matrix $T(A_S^T)$ and $(T-1)$ each period transition matrix $t(A_S^t)$.

In Benabou (2000), the weighted house value-to-rental ratio in 1985 in the U.S. is close to the Price-to-Rent ratio 6 in our model. Given this ratio, the simulated transition matrix A_S^T is given as,

$$A_S^T = \begin{bmatrix} 0.764 & 0.232 & 0.004 & 0.000 & 0.000 & 0.000 & 0.000 \\ 0.212 & 0.536 & 0.240 & 0.012 & 0.000 & 0.000 & 0.000 \\ 0.024 & 0.216 & 0.552 & 0.208 & 0.000 & 0.000 & 0.000 \\ 0.000 & 0.027 & 0.313 & 0.480 & 0.153 & 0.027 & 0.000 \\ 0.000 & 0.000 & 0.020 & 0.360 & 0.320 & 0.300 & 0.000 \\ 0.000 & 0.000 & 0.050 & 0.075 & 0.250 & 0.500 & 0.125 \\ 0.000 & 0.000 & 0.100 & 0.200 & 0.100 & 0.100 & 0.500 \end{bmatrix}$$

The transition matrix of our study is similar to $A_{KS,6}$ shown previously. After calculation, the Shorrocks Index S_S^T of our benchmark equals 0.6294, which bears a close parallel to Kennickell and Starr-Mccluer's $S_{KS,6}$ as well. From the matrix $A_{KS,6}$ we can observe that updating the social class to a level higher than 3 is almost impossible. It is also less likely for the top classes and the bottom classes to change their level. The upper middle classes (60-79% and 80-89%) have more opportunities to jump to another social class. However, it is harder to upgrade than to downgrade. These features all match the real data.

Figure 4 displays the plot for the social mobility (Shorrocks Index S_S^T, blue) and inequality (90/10 ratio, red) with respect to the changes of Price-to-Rent ratio. Until $\frac{P}{R} = 8$, the Shorrocks Index at first descends from 0.60, hit the bottom of 0.35 at $\frac{P}{R} = 4$ and then climbs up to 0.81. In contrast, the inequality, the 90-to-10 ratio, rises from 1.24, reaches the peak of 1.8 when $\frac{P}{R} = 3.8$ and then falls to the lowest level at 1.2. While inequality demonstrates an inverted U-shape, the social mobility presents a U-shaped curve. After $\frac{P}{R}$ equals to 8, inequality increases marginally and social mobility drops quickly. A negative correlation between the social mobility and inequality can

Figure 4: Social Mobility and Inequality

Note: negative correlation between social mobility and inequality.

Price to Rent	Correlation
$1.0 - 2.8$	-0.4932
$3.0 - 5.2$	-0.7529
$5.4 - 12.0$	-0.4502

Table 2: Correlation between Inequality and Social Mobility

be observed from the figure.

To further test the relationship between social mobility and inequality, we divide the X-Axis into three parts according to the intersection points of these two curves. Table 2 exhibits the correlations between social mobility and inequality corresponding to certain Price-to-Rent ratio ranges. In general, the Shorrocks Index negatively correlates with the 90/10 ratio. Before and after the intersection, the correlation is less strong than in the middle.

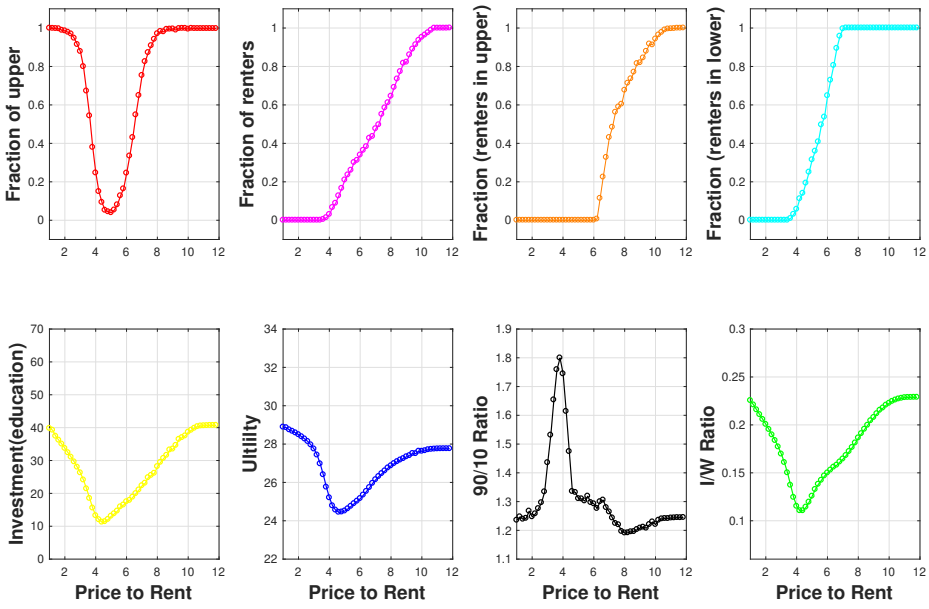


Figure 5: Effects of Price-to-Rent Ratio Changes

3.3 Effects of Price-to-Rent Changes

In order to have a clearer picture of the story behind the movement of housing, social mobility and inequality, in Figure 5 we present simulation results regarding the fraction of upper class, the investment in education, the average utility, inequality and the investment-to-wealth ratio with various housing price-to-rent ratios ranged from 1.0 to 12.0. We also show the fraction of renters in the whole economy, as well as the renters' proportions among upper class and among lower class. We only report the steady-state result. We consider the economy has reached its steady-state if the percentage of change in the fraction of individuals in the upper class within several consecutive periods has been sufficiently small ($< 0.01\%$).

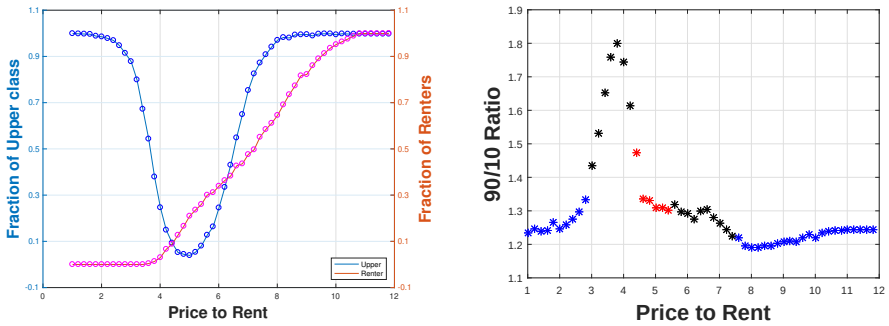
When owning a house becomes more expensive than renting, the fraction of renters in both classes will undoubtedly increase as shown in our results (Figure 5). There is also a "U-shape" pattern characterizing the relation between the fraction of individuals in upper class and the Price-to-Rent ratio. In contrast, an inverted "U-shape" pattern is observed in the 90-to-10 wage ratio against the Price-to-Rent ratio. The intuition

captured in these results is illustrated as follows: when housing price is equal to or just slightly above the rental price, individuals can manage to own housing as well as invest in human capital simultaneously. This explains why there is a high fraction of individuals in the high class and low wage inequality when housing Price-to-Rent ratio is close to 1. When it becomes more expensive to own a house, individuals start to trade off human capital investment against housing purchase. When human capital gets deaccmulated, the fraction of individuals in the upper class declines and wage inequality rises. Finally, when owning a housing becomes too expensive such that the utility premium from owning a house cannot compensate for the cost, individuals will decide to become renters and invest in human capital instead. Therefore, the fraction of individuals in the upper class starts to recover and inequality declines when the price-to-rent ratio becomes too high. A similar pattern emerges if we measure inequality using a wage 99-to-01 ratio.

3.4 Mechanism

There exist two key driving forces for the movement of social mobility and inequality in our study: the trade-off between owning a house and investing in education; and the difference of the ability distribution between upper and lower class agents. With fixed income, the higher the costs of the tenure choice are, the less amount of budget can be used for the education investment. Then, the human capital accumulation will descends. Meanwhile, if the agents are from the upper class, they have better chance to draw a high ability. The higher the ability is, with fixed investment in education, the more human capital accumulates.

With the help of by Figure 6, we illustrate the interaction between these two driving forces and the effects of them together. The left panel displays the fraction of renters and the fraction of upper type and the diagram on the right shows to which group, upper or lower, the 90th and 10th quantile, the individuals belong. When the individuals, who are ranked 90th and 10th in wealth, are both upper type or both from



(a) Fractions of renters and upper class (b) Inequality

Figure 6: Fractions of Rents and the Upper Class and Inequality

lower type, the scatter is blue or red respectively. When the 10th richest is from the upper group but the 90th is from the lower one, the scatter is black.

From the left panel we can see that, the main force that draws down the fraction of the upper type and pulls up the inequality level is the remaining unchanged fraction of renters. Until $\frac{P}{R}$ comes to near 4, all households own houses. The increased housing price is a barrier to human capital accumulation. Meanwhile, inequality reaches the peak. After that, with more households give up owning houses, inequality begins to drop. Although the fraction of upper type (the blue line) keeps going down for a while then turns to increase again, but the speed of falling is slower.

The right panel of Figure 6 exhibits whether the representative agents (10th and 90th richest) draw the ability from the same distribution. When the Price-to-Rent ratio is relatively low, the 90th and 10th richest individuals are both from the upper type (blue stars). This means that both the poor and the rich draw the ability from the distribution $G(\cdot)$ which has a higher average. Agents have equal chances of human capital accumulation. Inequality is relatively low. When the 90th comes from the lower group but the 10th belongs to the upper type (black stars), they draw the ability from different distributions $G(\cdot)$ and $F(\cdot)$. This means that even investing same amount in the education, the accumulated human capital is lower for the poor. Inequality rises fast and reaches a peak. As the 90th and the 10th are both "poor", when they are both from the lower class (red stars), they have equal "bad luck" and inequality decreases.

With the increase of the fraction of the upper class, the 90th and 10th again come from different groups and eventually both from the upper class.

In the simulation results, the point where Price-to-Rent ratio $\frac{P}{R}$ equals to 4 is the turning value for most of the curves. At that point, the fraction of renters starts to increase from 0 and part of individuals begin to rent instead of owning houses, while inequality and social mobility hit the ceiling and the floor respectively (Figure 4). Meanwhile, the investment in education, the utility and the investment-over-wealth ratio turn decrease into increase (Figure 5). However, only the fraction of the upper type keeps going down after $\frac{P}{R} = 4$. The reason is easy to be understand: the fraction of the upper type house renters remain unchanged until $\frac{P}{R} = 6$ (third diagram in the first row in Figure 5). The housing price is not high enough to force any household in the upper type to give up owning a house but to invest more in education. Then the higher housing price will draw human capital accumulation down in the upper type. Some of the initial upper class agents cannot accumulate enough (higher than the threshold $\overline{h}$), and then they fall to the lower class. Because of that, the fraction of the upper type continues to drop.

The mechanism is as follows: when the housing price is low, almost every individual can afford both owning house and human capital investment. With housing price increase, at first the tenure choice among the whole population remain unchanged (pink line in the left panel in Figure 6). Individuals with relatively low human capital may choose owning house rather rather than investing in human capital. Social mobility declines, mainly because the poor remain poor. The rich, however, invest in their education, accumulate human capital, enjoy higher income along time and simultaneously can afford owning a house. They still remain in the upper class. Since the initially poor individuals partially give up the human capital accumulation but the rich ones keep increasing their income, the upgrade of social class gets harder and inequality grows. When housing prices become even higher, individuals with high initial human capital may also have to purchase a house instead of investing in human capital. The poor invest in education while the initially richer ones consume expensive housing services.

The dispersion of the poor and the rich get smaller. Then, the social upgrade becomes easier and then social mobility increases. Income inequality starts to decline. Eventually, when housing prices become "unaffordable", every individual chooses to rent a house and engage in human capital accumulation, so inequality declines again.

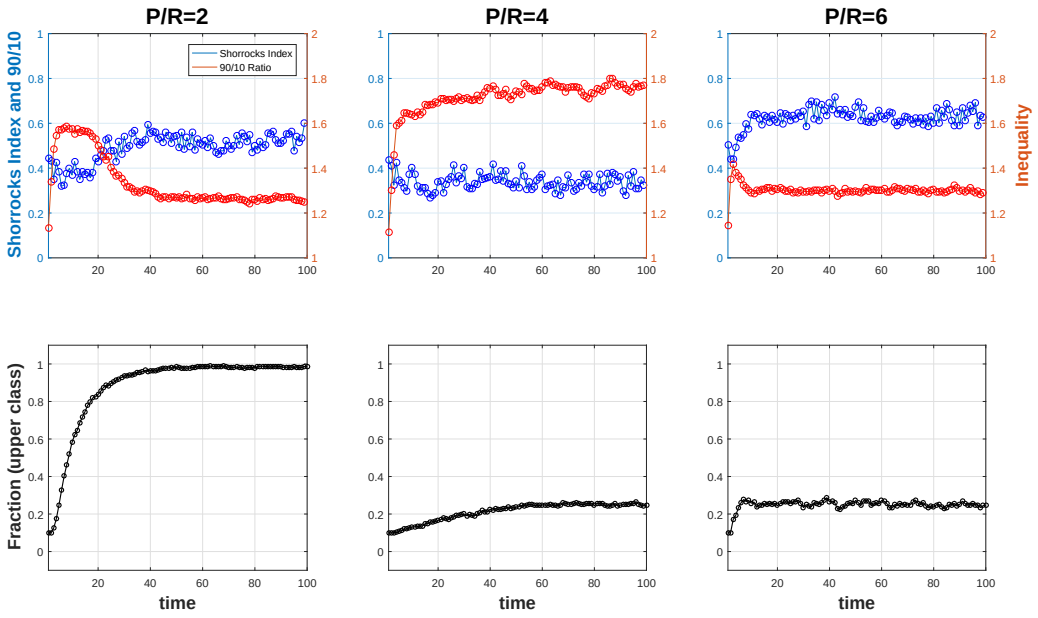


Figure 7: Transition Dynamics of Social Mobility, Inequality and Fraction of Upper Class given Different Price-to-Rent Ratio

Figure 7 shows the transition dynamics of inequality (red), social mobility measurement, Shorrocks Index (blue), and the fraction of upper type (black) when the Price-to-Rent ratio equals to 2, 4 and 6 respectively. It is clear that the relationship between social mobility and inequality are negative for every given $\frac{P}{R}$ on the process to the steady state. For the fraction of upper type (black), no matter how high or low the value in the equilibrium is, it quickly converges to the level close to the equilibrium state. When the black line becomes stable, the red and blue, the lines of inequality and of social mobility, come to a relatively stable situation.

There exists an uncertainty in our model, namely the ability. In order to rule out the possibility that one or few households keep drawing extremely low or high ability in a row, we do the bootstrapping (Figure 8). We repeat the simulation 100 times.

Each time, we have the same setting for everything except a different ability draw. Then we take the average of each variables and plot the figure. We can see, the trend for every curve is the same, meaning that our simulation result is not a special case of our model.

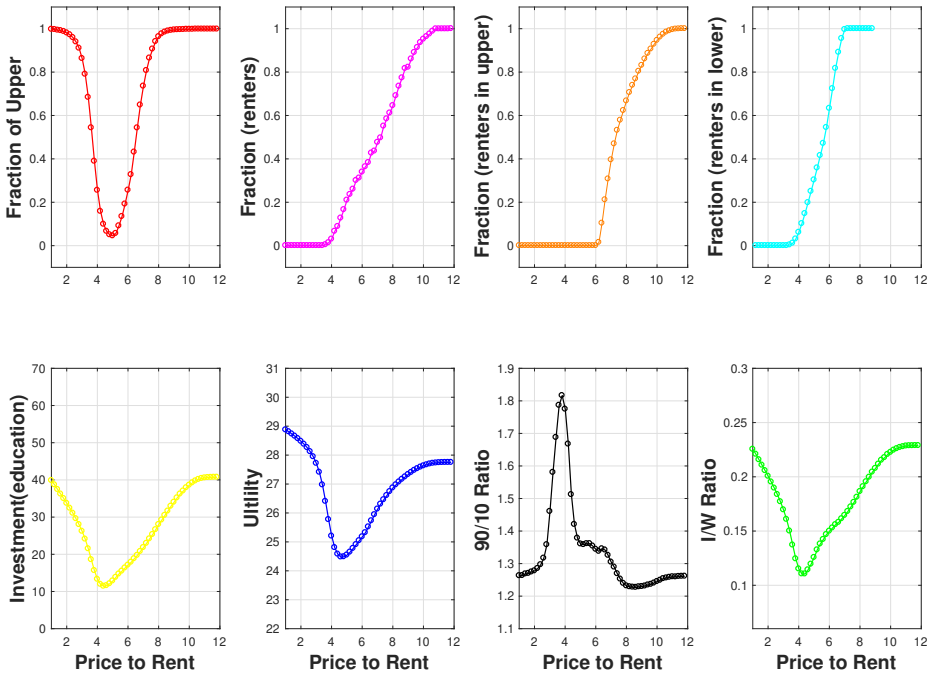


Figure 8: Bootstrapping Results

Note: 100 times.

3.5 Different Elasticities of Human Capital Investment

Previously, the elasticity of the human capital investment has been set to 0.3. In this subsection, we release this setting and show the movement of the key variables in the model: the fraction of upper type, the fraction of renters, social mobility and inequality. From Figure 9, we can see that the trend of each line remains the same: inequality(red) and social mobility (dark blue) are negatively correlated; generally, the inequality line shows an inverted U-shape; as the Price-to-Rent ratio increases, the fraction of renters

rises from 0 up to 1. Meanwhile, the fraction of upper type (light blue) still presents a U-shape. They only differ in degree: the U-shape becomes shallower and the difference between peak and crest of the inverted U is smaller.

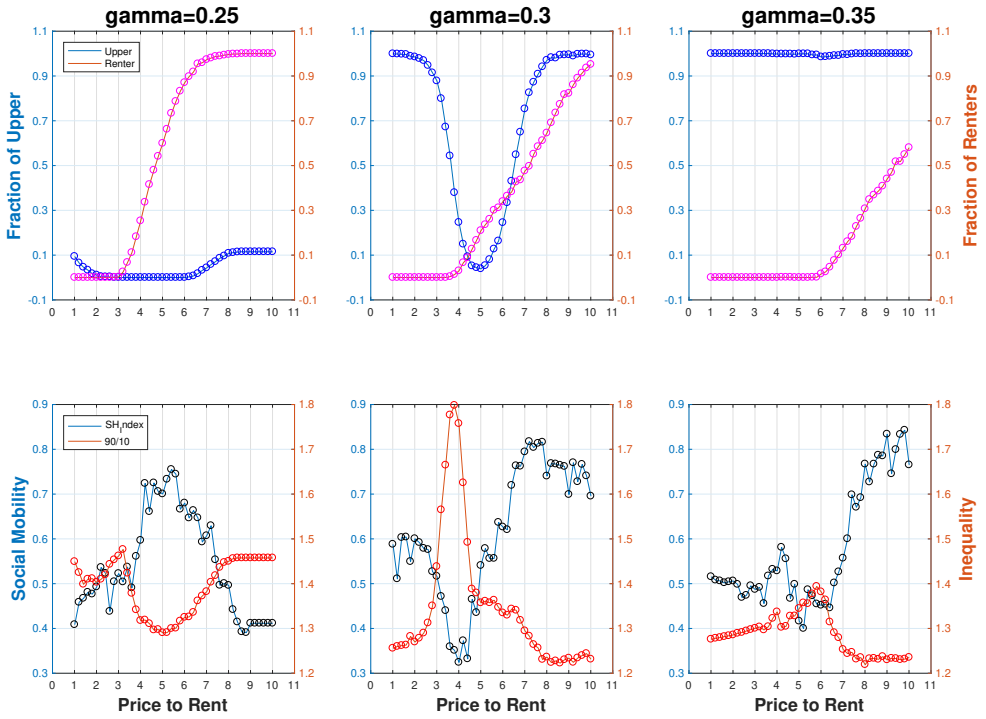


Figure 9: Different Elasticities of Investment to Human Capital Accumulation

When γ is smaller, the same investment in education turns to less human capital accumulation. Then, compared to purchasing and owning a house, investment is a less attractive option for households. Since there is not enough incentive for education, only few initially rich individuals could invest enough to reach the threshold for upper type and a larger proportion of households is stuck in the cycle: less investment, lower human capital, worse ability draw. Every household suffers poverty, but it does not mean that the social mobility is low. In contrary, the Shorrocks Index climbs to the top. This could explain why there is a U-shape in the equality curve instead of an inverted U. In addition, since the average of human capital is low, the income must be low. Then, with the same increase in the housing price, individuals facing lower elasticity

of investment in human capital start to give up the choice of owning a house sooner than in the benchmark. The fraction of renters hits the upper limit more quickly.

On the other hand, when the elasticity of human capital investment is high, education brings more rewards, then accumulation of human capital becomes easier, higher income follows and more investment in education will be made. The accumulation of human capital is also high. That's why no matter how much the housing price rises, the fraction of upper type is always near 1. Everyone enjoys the virtuous circle. Social mobility is high and inequality is low. One thing needs to be pointed out: when γ equals to 0.35, the inflection point of the inverted U-shaped inequality curve happens at the spot where the individuals start to rent a house.

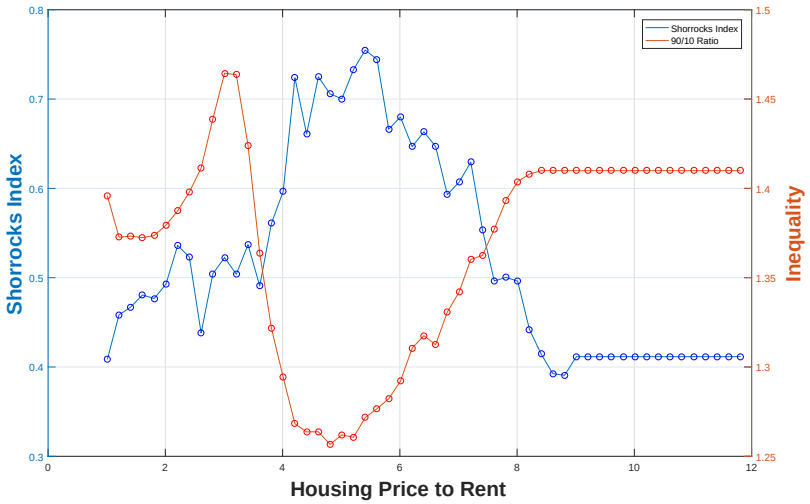


Figure 10: Social mobility and Inequality when $\gamma = 0.25$

It is interesting that the inequality curve looks more like a U-Shape rather than an inverted U shape, when $\gamma = 0.25$ (Figure 10, red line). The inequality curve is cut into four stages: increase, decrease, rise again and remain unchanged by the peak point ($P/R = 3.4$), the bottom point ($P/R = 5$) and the point where $P/R = 8$ (the inflection point from increasing to staying stable). It seems that in the first stage, the results shown in Figure 10 go against what we find before: both inequality and social mobility increase. It is necessary to analyze the transition dynamics to see how this

happens.

From the Figure 11, we can see that in the dynamics, inequality and social mobility are still negative correlated. Indeed, if we only trace the last point, both inequality and social mobility increase. In the cases of $\frac{P}{R} < 3.4$ (1.4, 2.0, 2.6 and 3.2 respectively), for the first 4 periods, inequality directly jumps to as high as 1.5 and then declines with the increase of social mobility. But when $\frac{P}{R}$ is smaller, the inequality decreases more along the time.

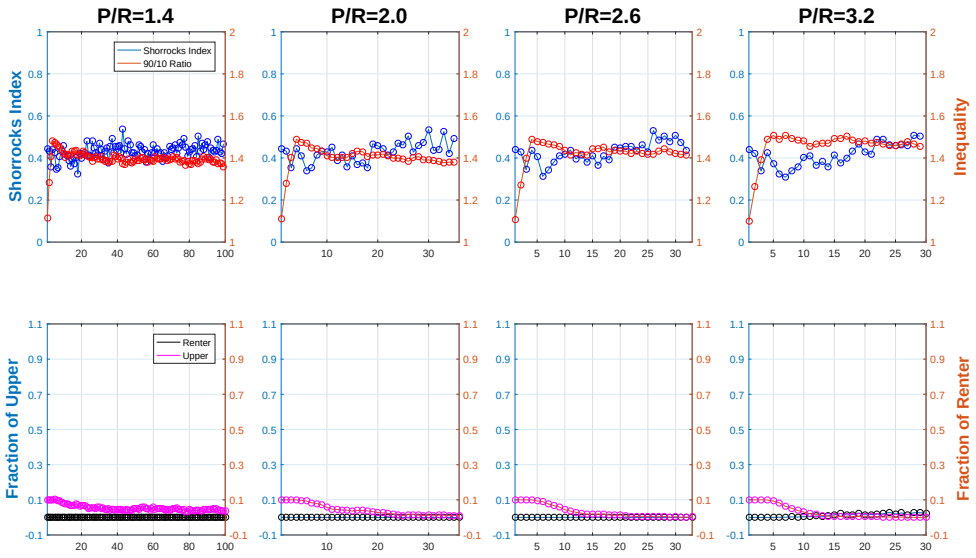


Figure 11: Transition Dynamics when $\gamma = 0.25$

Given the elasticity of human capital investment γ equals to 0.25, before P/R reaches 3.4, the fraction of renters (blue) in the dynamics remains zero, or almost zero along the whole periods (Figure 11). All individuals purchase houses instead of investing in education. Inequality increases. In Figure Figure 10, when P/R is between 3.4 to 5, the "poor" individuals invest in education and accumulate human capital, while the "rich" ones can still afford to own a house. This will eventually narrow the gap between the "poor" and the "rich". Then the inequality level starts to decrease. When P/R is between 5 to 8, the fraction of renters no longer increases from 0. The differences of initial accumulation of human capital force the inequality level

to increase. After $P/R = 8$, all individuals rent a house for the whole periods. The increase of Price-to-Rent no longer affects investment in education (human capital). The inequality level remains stable.

We could observe that a very small difference in γ could change the shift of social mobility and inequality dramatically. Therefore, a small improvement of the transition from education to human capital would strongly contribute to the control of the increase of inequality. This change could be reached by, for example, improving the efficiency of education, or reallocating the education resources. This finding insight gives the government a new political option to solve the social mobility and the inequality problems.

3.6 Different Preference of Owning a House

In the previous analysis, the preference of owning a house (the extra utility gain from owning a house, e) has been set to 1 for all cases. Here, we release this parameter to the range between 0.1 and 4 (Figure 12).

Similar to the results with different elasticity of investment to human capital accumulation, the trends of social mobility, inequality, the fraction of the upper class and the faction of renters match the results when we control the parameter e. With the increase of the Price-to-Rent ratio, the Shorrocks index takes a U-shape while inequality and the fraction of upper class show an inverted U-shape.

However, when the individuals have very low preference for owning a house, the effect of housing price changes is quite weak (graphs in the first column in Figure 12). The fraction of renters will quickly jump from 0 to 1, while the fraction of upper type will have a minor change along the increase of the Price-to-Rent ratio. The social mobility and inequality will be affected but quickly become stable. When the preference increases, the role played by housing gets more important. The U-shape of the fraction of the upper type gets longer and deeper. The bottom level of social mobility becomes lower. This results in that the peak level of inequality hits a new height.

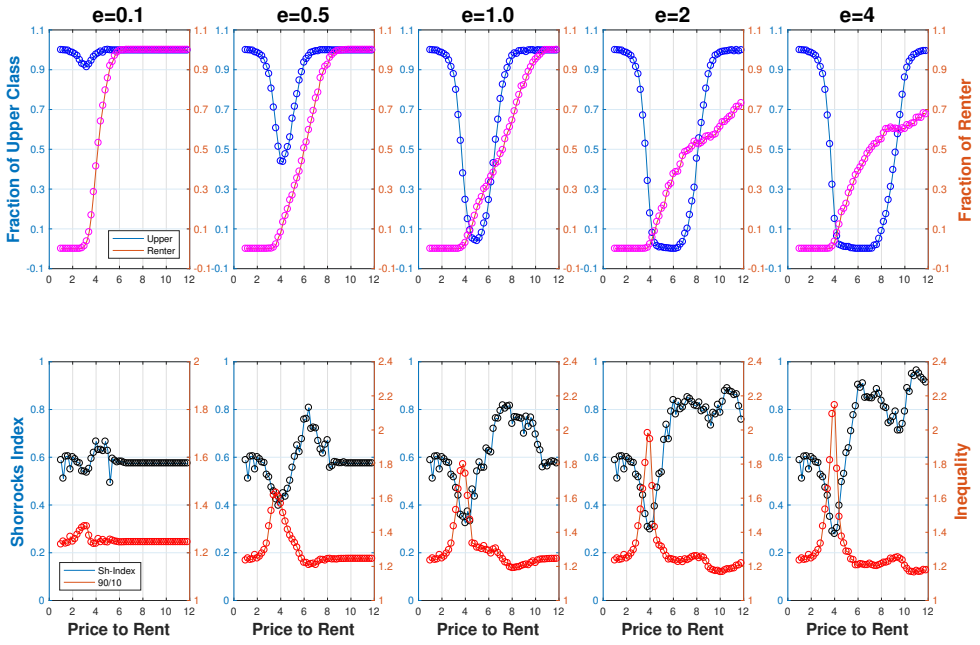


Figure 12: Different Preference of Owning a House

This suggests that when the government can "teach" individuals to be less persistent in owning their own house, the inequality problem brought by the housing market will be automatically mitigated. In the reality, the preference of owning a house is indeed different from country to country. Some argue that the preference is related to culture, which is different to be changed. Indeed, in certain culture, people (e.g. Chinese) view owning a house as the foundation of building a family, and as a popular investment tool. They have a higher utility premium from this point. However, even among OECD countries, which have distinguished rent control level and policies to protect renters, the homeownership is significantly different. It means that the government can change the magnitude of the utility gain from owning a house. Take 2004 for example, the ratio of owning a flat or a house was 83.2% in Spain, 70.7% in the U.K., 68.7% in the U.S., 54.8% in France and 41% in Germany respectively (**?**). In Germany, the homeownership is quite stable and remains around 40% after the Second World War. In **?**, they show that Germany has the third strictest rent control in the private rental market. Besides, Germany has a good supply of high quality standard of social housing

for the rental market which makes the rent could be long and stable enough for the tenants (**?**). In the private rental market, the tenants' right is also well protected. For example, German law accepts only several grounds for termination from the landlords' side. However, even though house owners have valid reason and informed tenants three-month in advance, tenants still can object to the termination and require continuation of the lease if the termination causes undue hardship. With the protection, the benefit earned from owning a house becomes less, it will lower the utility premium e and will further weak the impact of housing on inequality.

4 Conclusion

Housing typically takes up a major proportion of household expenditure and thus it certainly plays a critical role in shaping the pattern of income inequality and social mobility. Whether a high housing relative to rental price will amplify inequality and inhibit social class upgrading is still a controversial issue in the existing literature. In this paper, we develop a partial equilibrium lifecycle framework to address those issues.

The agents in our economy are divided into two social classes according to the initial human capital level inherited from their parents. Those who belong to upper class will draw their innate abilities from a distribution that first order stochastically dominates those from lower class. Throughout the entire lifecycle, agents make endogenous human capital investment and housing tenure decisions. We calibrate the model into mimic some steady state in the the real world counter.

Our simulation results indicate an inverted-U pattern between the housing Price-to-Rent ratio and measures of income inequality, and a U-shape pattern between the Price-to-Rent ratio and social mobility measured by the Shorrocks Index. Social mobility is negatively correlated with inequality in all the cases we analyzed in this paper.

The mechanism is clear: when the housing price is low, almost every individual can afford both owning house and human capital investment. With housing price increase, individuals with relatively low human capital may choose owning house rather rather

than investing in human capital. On the other hand, the rich invest in their education, accumulate human capital, enjoy higher income along time and simultaneously can afford owning a house. Since the initially poor individuals partially give up the human capital accumulation but the rich ones keep increasing their income, the upgrade of social class gets harder and inequality grows. When housing price becomes even higher, individuals endowed with high initial human capital need to choose between purchasing a house and investing in human capital. The poor who can no longer afford owning a house, invest in education while the initially richer ones consume expensive housing services. The dispersion of the poor and the rich get smaller. Then, the social upgrade becomes easier and then social mobility increases. Income inequality starts to decline. Eventually, when housing prices become "unaffordable", every individual has to rent a house and therefore engage in human capital accumulation. Inequality declines again.

Moreover, our results also suggest that better quality of education induced by a higher elasticity of human capital investment against added human capital tends to dampen the role of housing. Similarly, if the government can "teach" individuals to be less persistent in owning their own house, the inequality problem brought by the housing market will be automatically mitigated.

References

Aaberge, Rolf, Anders Björklund, Markus Jäntti, Mårten Palme, Peder J Pedersen, Nina Smith, and Tom Wennemo, "Income inequality and income mobility in the Scandinavian countries compared to the United States," *Review of Income and Wealth*, 2002, *48* (4), 443–469.

Andrews, Dan and Andrew Leigh, "More inequality, less social mobility," *Applied Economics Letters*, 2009, *16* (15), 1489–1492.

Belsky, Eric, Nicolas Retsinas, and Mark Duda, *The financial returns to low-income homeownership*, Ithaca and London: Cornell University Press, 2007.

Benabou, Roland, "Human capital, inequality, and growth: A local perspective," *European Economic Review*, 1994, *38* (3-4), 817–826.

_ , "Unequal societies: Income distribution and the social contract," *American Economic Review*, 2000, *90* (1), 96–129.

Bluestone, Barry and Bennett Harrison, "The great U-turn: Corporate restructuring and the polarizing of America," 1988.

Bound, John and George E Johnson, "Changes in the Structure of Wages during the 1980's: An Evaluation of Alternative Explanations," Technical Report, National Bureau of Economic Research 1989.

Campbell, Sean D, Morris A Davis, Joshua Gallin, and Robert F Martin, "What moves housing markets: A variance decomposition of the rent–price ratio," *Journal of Urban Economics*, 2009, *66* (2), 90–102.

Cantó, Olga et al., "Income mobility in Spain: How much is there?," *Review of Income and Wealth*, 2000, *46* (1), 85–102.

Charles, Kerwin K and Erik Hurst, "Intergenerational wealth correlations," *Journal of Political Economy*, 2003, *111* (6), 1155–1182.

Chi, Peter SK and Joseph Laquatra, "Profiles of housing cost burden in the United States," *Journal of Family and Economic Issues*, 1998, *19* (2), 175–193.

Di, Zhu Xiao, "Does housing wealth contribute to or temper the widening wealth gap in America?," *Housing Policy Debate*, 2005, *16* (2), 281–296.

Harrison, Bennett and Barry Bluestone, *The great U-turn: Corporate restructuring and the polarizing of America*, Basic Books, 1990.

Immergluck, Dan, *Foreclosed: High-risk lending, deregulation, and the undermining of America's mortgage market*, Cornell University Press, 2011.

Jarvis, Sarah and Stephen Jenkins, "How much income mobility is there in Britain?," *The Economic Journal*, 1998, *108* (447), 428–443.

Keister, Lisa A and Stephanie Moller, "Wealth inequality in the United States," *Annual Review of Sociology*, 2000, pp. 63–81.

Kennickell, Arthur B and Martha Starr-Mccluer, "Household saving and portfolio change: evidence from the 1983–89 SCF panel," *Review of Income and Wealth*, 1997, *43* (4), 381–399.

Klevmarken, N Anders, Joseph P Lupton, and Frank P Stafford, "Wealth Dynamics in the 1980s and 1990s Sweden and the United States," *Journal of Human Resources*, 2003, *38* (2), 322–353.

Levy, Frank and Peter Temin, "Inequality and institutions in 20th century America," 2007.

Maasoumi, Esfandiar and Sourushe Zandvakili, "A class of generalized measures of mobility with applications," *Economics Letters*, 1986, *22* (1), 97–102.

Mimura, Yoko, "Housing cost burden, poverty status, and economic hardship among low-income families," *Journal of Family and Economic Issues*, 2008, *29* (1), 152–165.

Morris, Martina and Bruce Western, "Inequality in Earnings at the Close of the Twentieth Century," *Annual review of sociology*, 1999, pp. 623–657.

Nichols, Austin and Melissa Favreault, "A Detailed Picture of Intergenerational Transmission of Human Capital.," *Urban Institute (NJ1)*, 2009.

Piketty, Thomas, "Capital in the 21st Century," *Cambridge: Harvard Uni*, 2014.

_ **and Emmanuel Saez**, "The evolution of top incomes: a historical and international perspective," Technical Report, National Bureau of Economic Research 2006.

Quigley, John M and Steven Raphael, "Is housing unaffordable? Why isn't it more affordable?," *The Journal of Economic Perspectives*, 2004, *18* (1), 191–214.

Rognlie, Matthew, "A note on Piketty and diminishing returns to capital," *Tillgänglig:¡ http://www. mit. edu/˜ mrognlie/piketty_diminishing_returns. pdf*, 2014.

Saez, Emmanuel, "Striking it richer: The evolution of top incomes in the United States (update with 2007 estimates)," 2009.

Schluter, Christian and Mark Trede, "Local versus global assessment of mobility," *International Economic Review*, 2003, *44* (4), 1313–1335.

Sherman, Arloc and Chad Stone, "Income gaps between very rich and everyone else more than tripled in last three decades, new data show," *Center on Budget and Policy Priorities*, 2010, *25.*

Shorrocks, Anthony F, "The measurement of mobility," *Econometrica: Journal of the Econometric Society*, 1978, pp. 1013–1024.

Stegman, M, R Quercia, Walter Davis, W Rohe, and H Watson, "The wealth-creating potential of home ownership," *Chasing the American Dream: New perspectives on affordable home ownership*, 2007, pp. 171–190.

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