Statistical Analysis of Neuroeconomic Data

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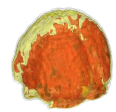
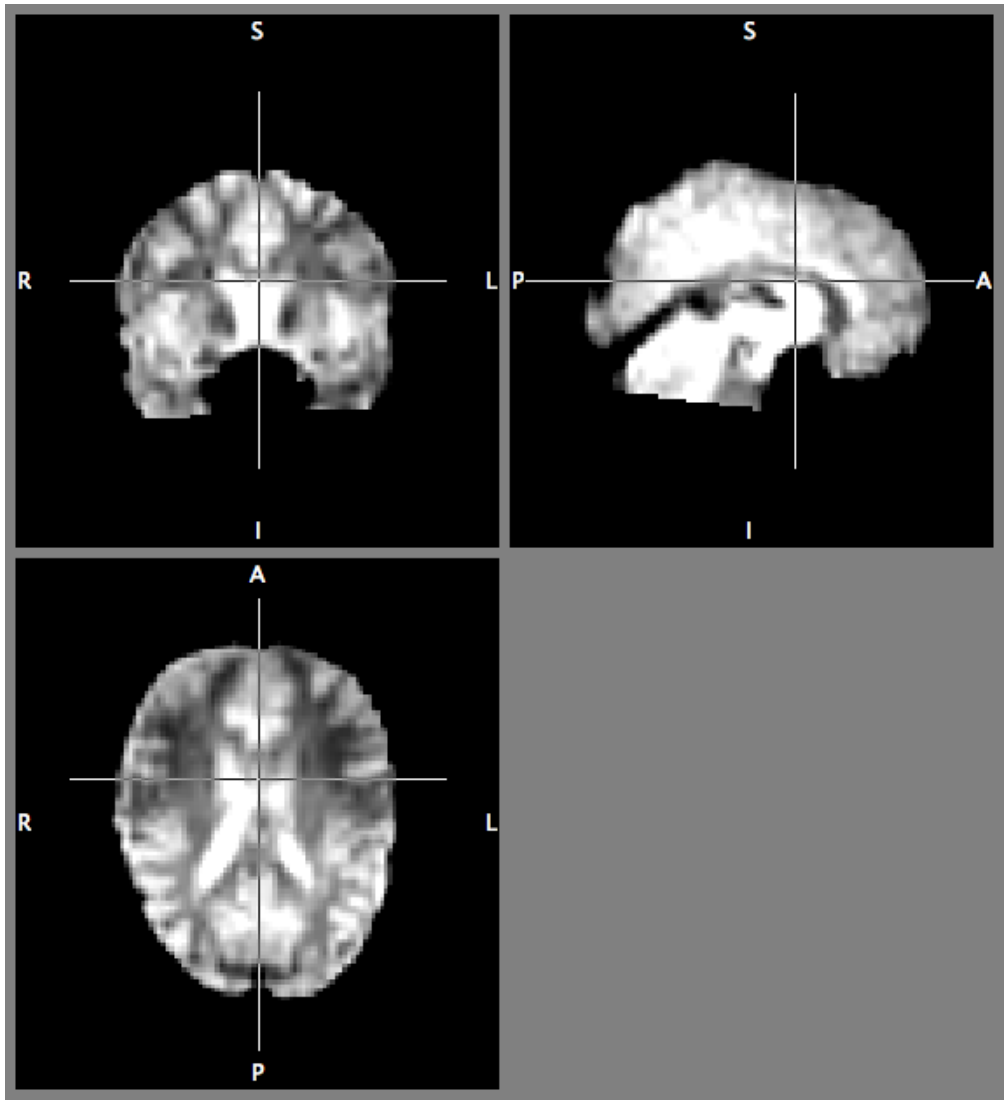
Motivation

- Which part of our brain is activated during *risky decisions* ?
- Can statistical analysis help to detect this area without any a priori information?
- Can we provide an analysis of the *whole* brain?

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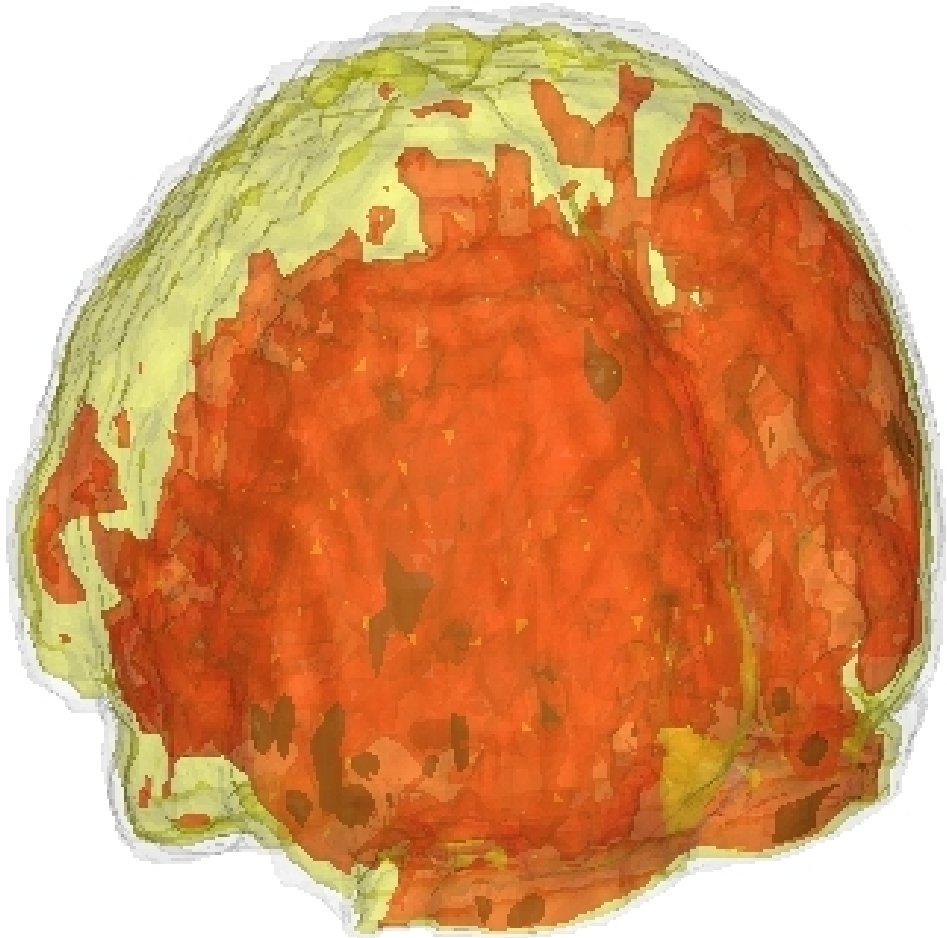
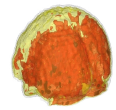
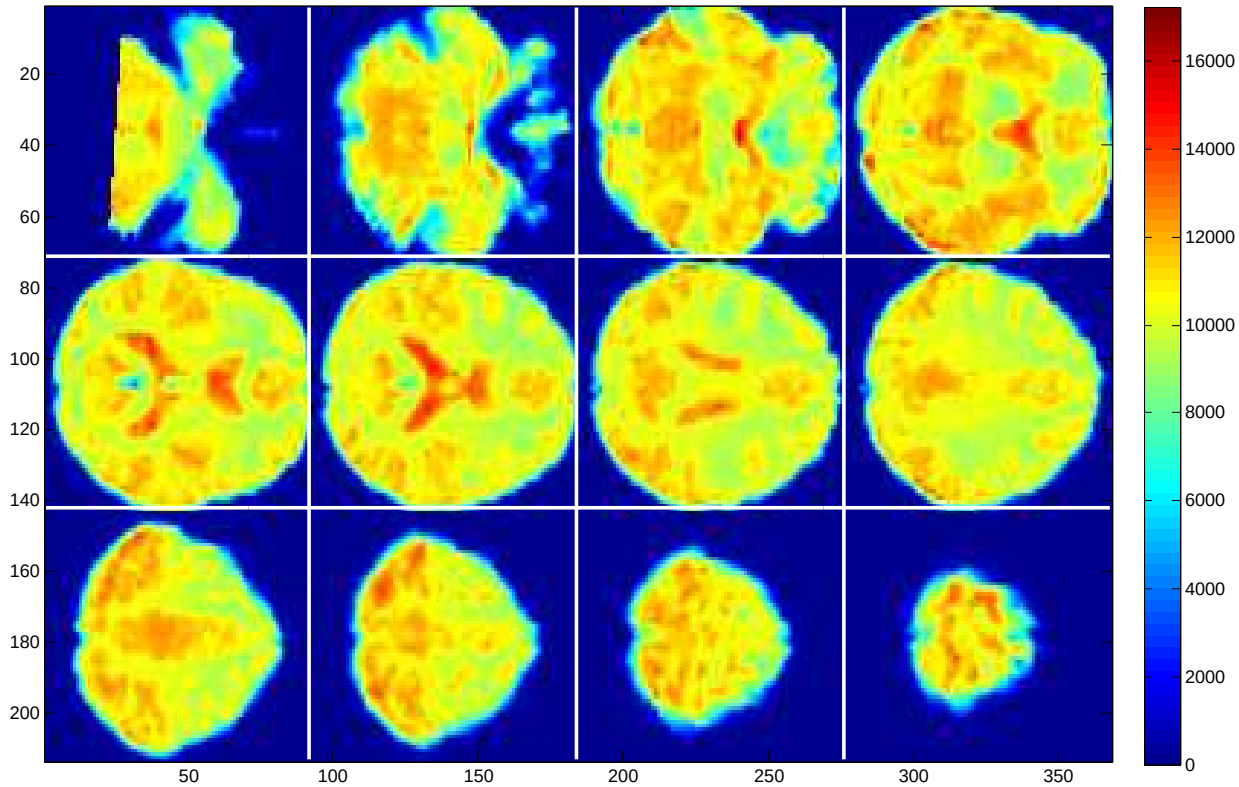
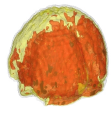
Different brain visualization II





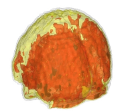
Different brain visualization III





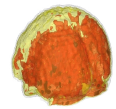
Motivation

- search for neuro-physiological analogue to the specification of risk type
- include complete brain data for all experiment participants
- massive data set from experiments
  - statistical analysis necessary
  - dimension reduction keeping the data structure
  - **time** consideration (DSFM)



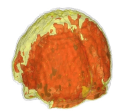
Outline

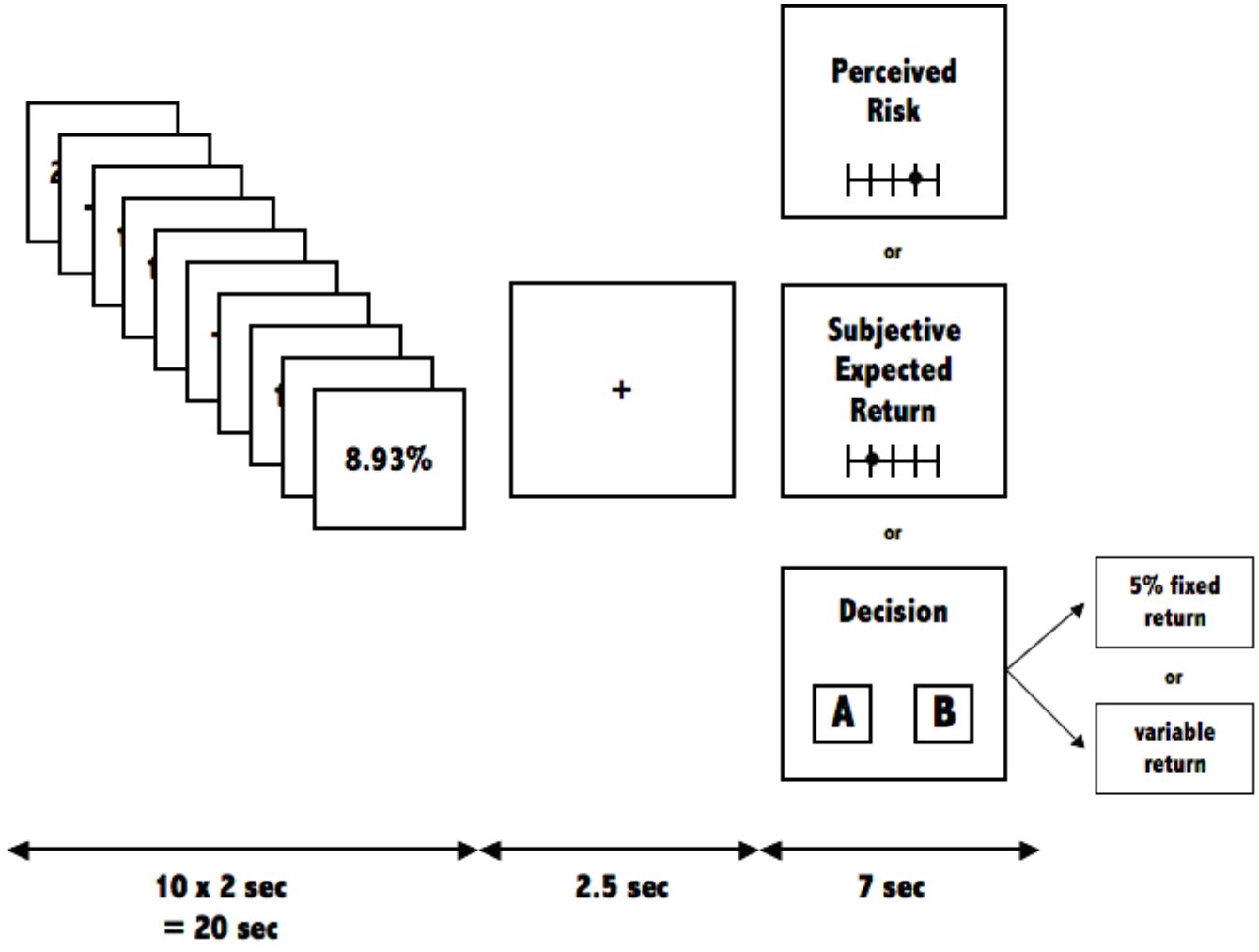
1. Motivation ✓
2. Experiment
3. Statistical Model
4. Results
5. Future Perspectives



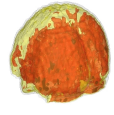
Experiment participants

- 20 volunteers (age 18-35 years)
- 11 females, 9 males
- native German speakers, right-handed (according to the Edinburgh Handedness Inventory)
- no history of neurological or psychiatric diseases
- flat payment (10 EUR) $\pm$ outcome resulting from the participant's decision
- 2 participants excluded due to extensive head motion and modeling problems



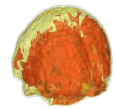


Risk Perception and Investment Decision (RPID)



RPID task in each trial

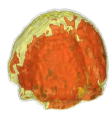
1. presentation of a return stream (rs)
   - ten returns from an investment (each for 2 sec)
   - each rs independent of the others
   - 9 different combinations of means (6%, 9%, 12%) and standard deviations (1%, 5%, 9%)



RPID task

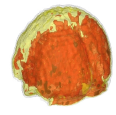
2. decision *or* subjective judgment task (chosen randomly)
   - choice between an investment with 5% fixed return (safe investment) and the investment represented by the rs (risky investment)
   - subjective expected return judge with range: -5% – 15%
   - perceived risk judge on scale: 0 (no risk) – 100 (maximum risk)

Altogether: 81 trials (3 tasks 27 times) in 57 mins.



fMRI Acquisition

- fMRI = functional Magnetic Resonance Imaging
- noninvasive technique of recording brain's signals
- BOLD (blood oxygenation level dependent)-sensitive imaging
- 1.5 T Magnetom Sonata MRI system (Siemens)
- 26 axial slices of 4mm thickness

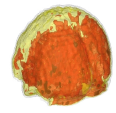


Data Set

Series of 3-dim images

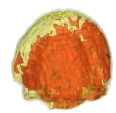
- each scan transformed on the resolution $2 \times 2 \times 2mm^3$
- 91 slices
- observed every 2.5 seconds
- data set: series of 1360 images with $91 \times 109 \times 91$ voxels

High-dimensional, high frequency data.



Panel Dynamic Semiparametric Factor Model (Panel DSFM)

$X_{t,j} = (X_{t,1}, \ldots, X_{t,J})^\top$	observable covariates defined on $\mathbb{R}^d$
$Y_{t,j} = (Y_{t,1}, \ldots, Y_{t,J})^\top$	observable random vector on $\mathbb{R}^d$
$Z_{t,j} = (Z_{t,1}, \ldots, Z_{t,L})^\top$	unobservable L-dimensional process
$(\overline{m}_0, \ldots, \overline{m}_L)$	unknown real-valued functions defined on a subset of $\mathbb{R}^d$
$\varepsilon_{t,j} \sim (0, \sigma^2_{t,j})$	errors with $\sigma^2_{t,j} < \infty$



Panel DSFM

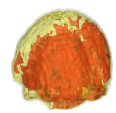
- assume *fixed effects* α_i for individual i with $\sum_{i=1}^{n} \alpha_i := 0$
- thus for the "average brain":

$$\overline{Y}_{t,j} = \overline{m}_0(X_{t,j}) + \sum_{l=1}^{L} \overline{Z}_{t,l} \overline{m}_l(X_{t,j}) + \varepsilon_{t,j}\ , \quad 1 \le j \le J \qquad \text{(DSFM)}$$

- for individual i is then:

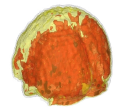
$$Y^i_{t,j} = \overline{m}_0(X_{t,j}) + \sum_{l=1}^{L} Z^i_{t,l} \overline{m}_l(X_{t,j}) + \varepsilon^i_{t,j} \qquad \text{(LS)}$$

with the general basis functions $\overline{m}_l$



Fitting fMRI Data

- cut off parts of images without brain scan
- reduction of the original data by taking every second slice in each direction and the first part of experiment only
- voxel's index (i_1, i_2, i_3) as covariate X_j
- BOLD signal as $Y_{t,j}$
- then $J = 36 \times 46 \times 46$ and $T = 722$

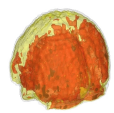


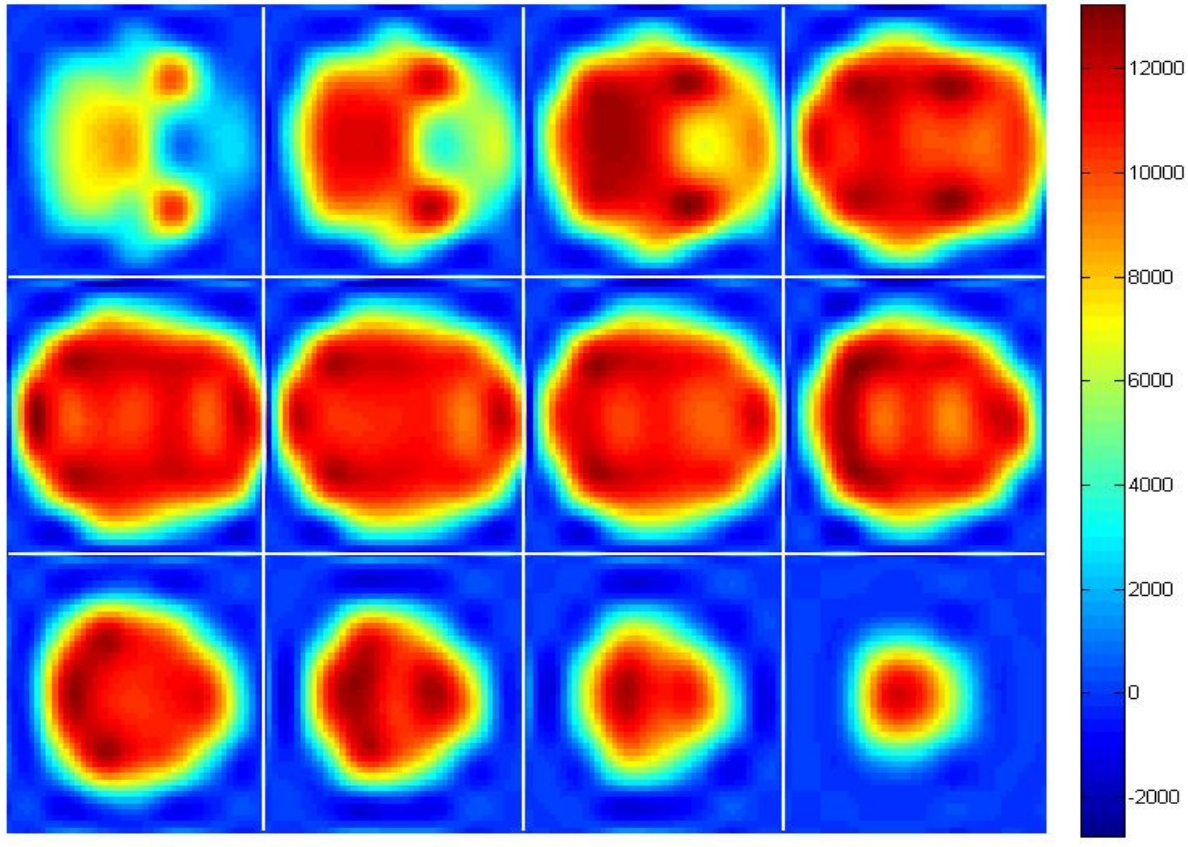
Estimation of DSFM

- choose $K = 7 \times 8 \times 8 = 448$ parabolic tensor B-splines to estimate $\widehat{m}$
- set $L = 2$

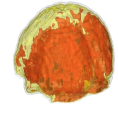
$$1 - RV(L) = \frac{\sum_t^T \sum_j^J \{Y_{t,j} - \widehat{m}_0(X_{t,j}) - \sum_l^L \widehat{Z}_{t,l} \widehat{m}_l(X_{t,j})\}^2}{\sum_t^T \sum_j^J (Y_{t,j} - \overline{Y})^2}$$

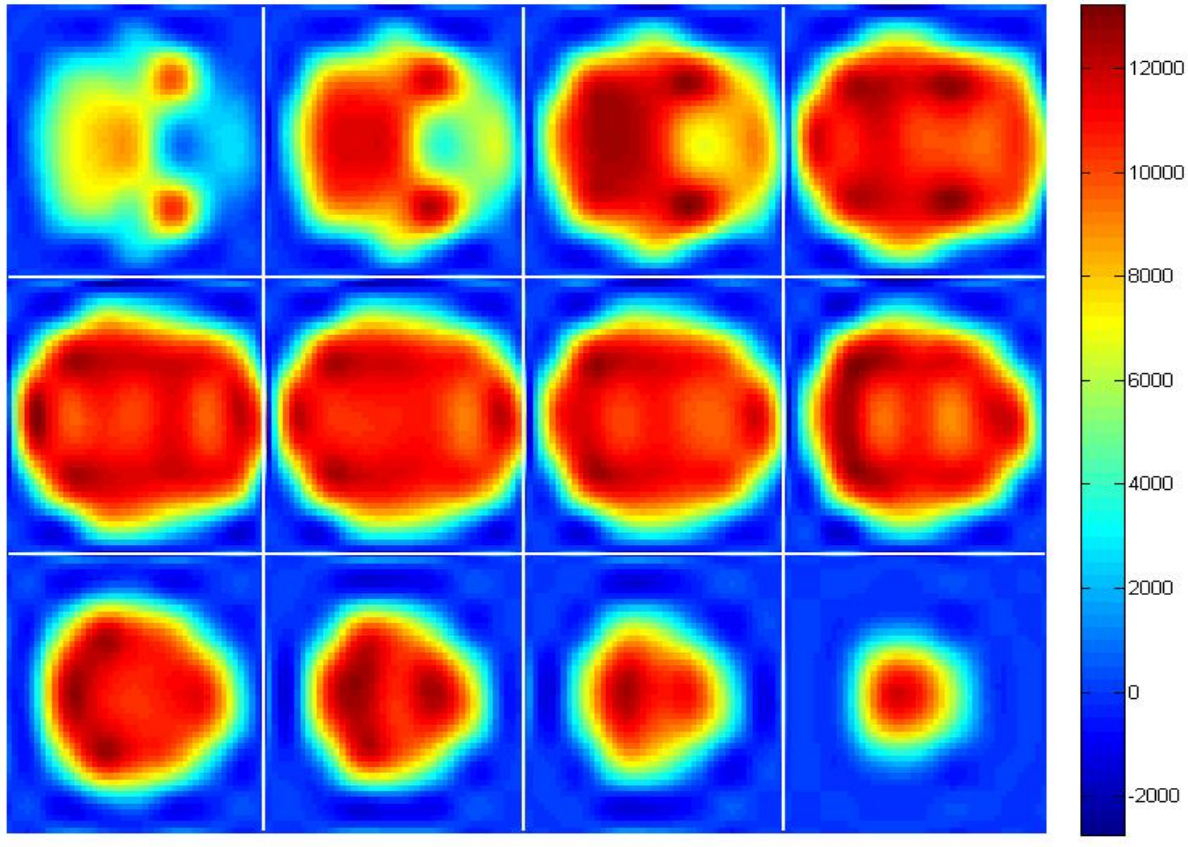
No.of factors	$L = 2$	$L = 3$	$L = 4$
$1 - \overline{RV}(L)$ in %	88.85	88.88	88.91



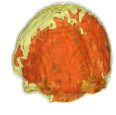


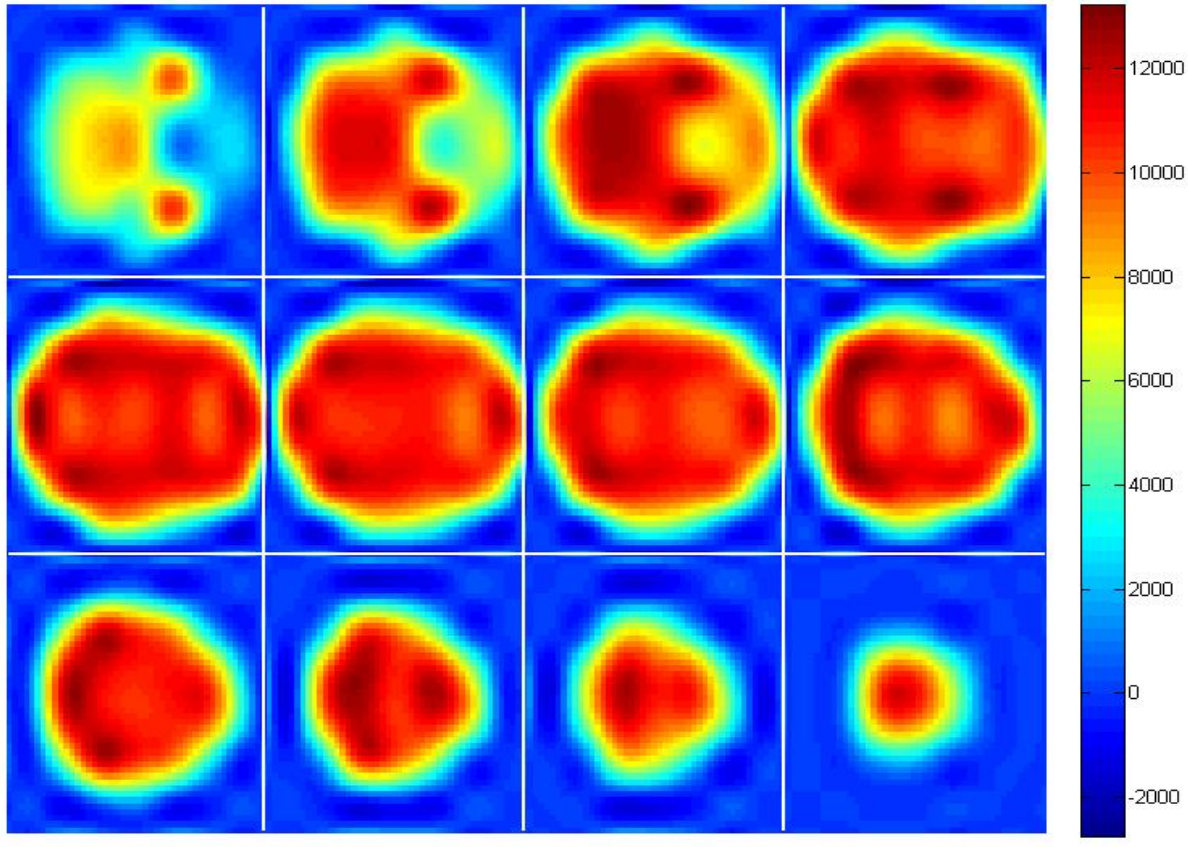
Estimated factor loading $\hat{m}_0$ with $L = 2$.



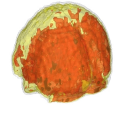


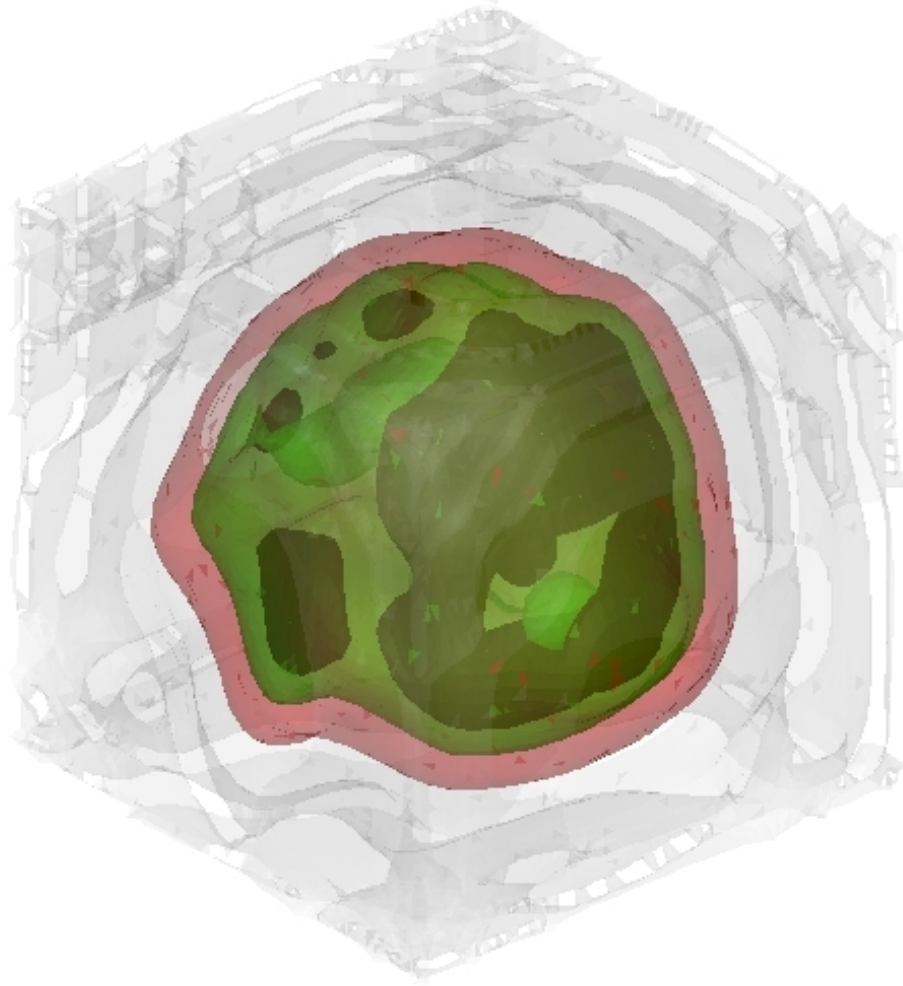
Estimated factor loading $\hat{m}_0$ with $L = 3$.



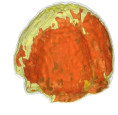


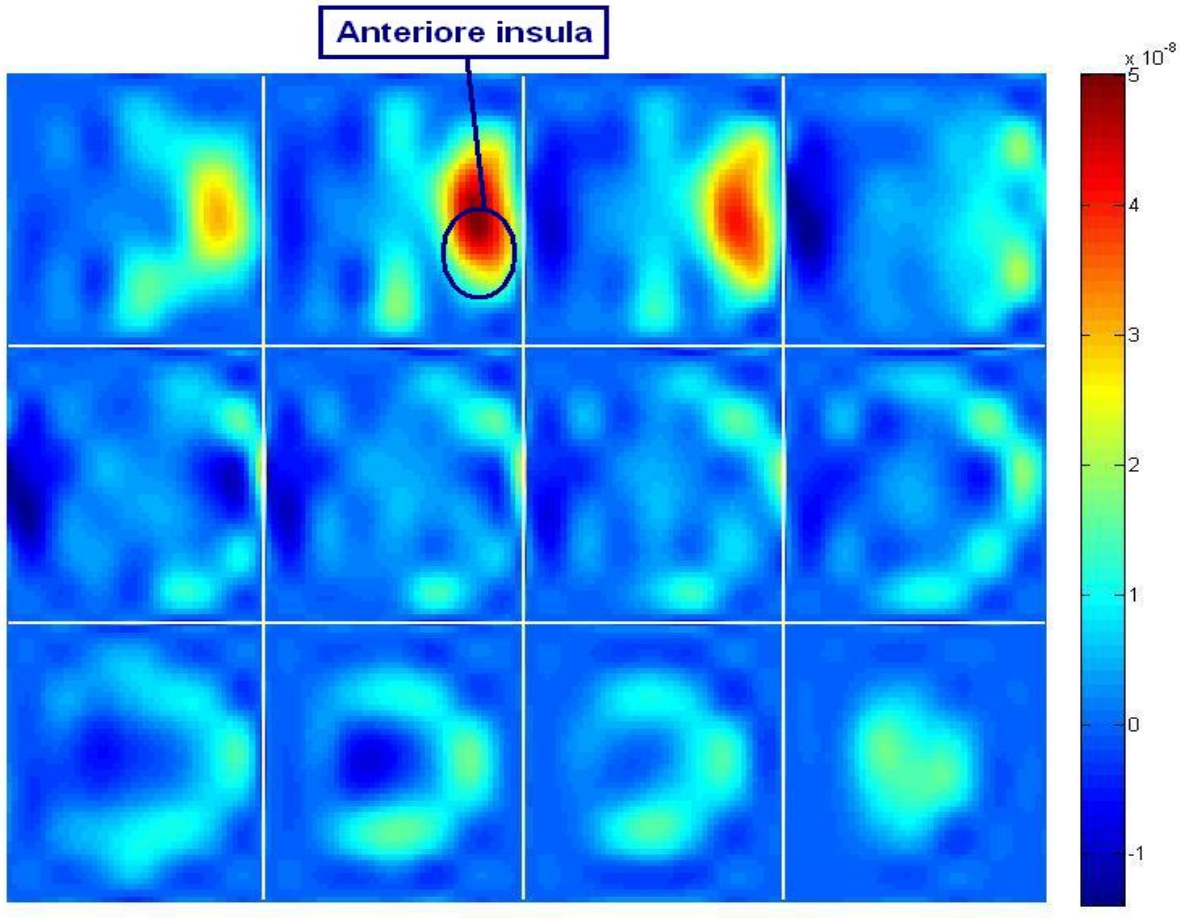
Estimated factor loading $\hat{m}_0$ with $L = 4$.



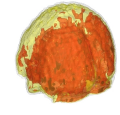


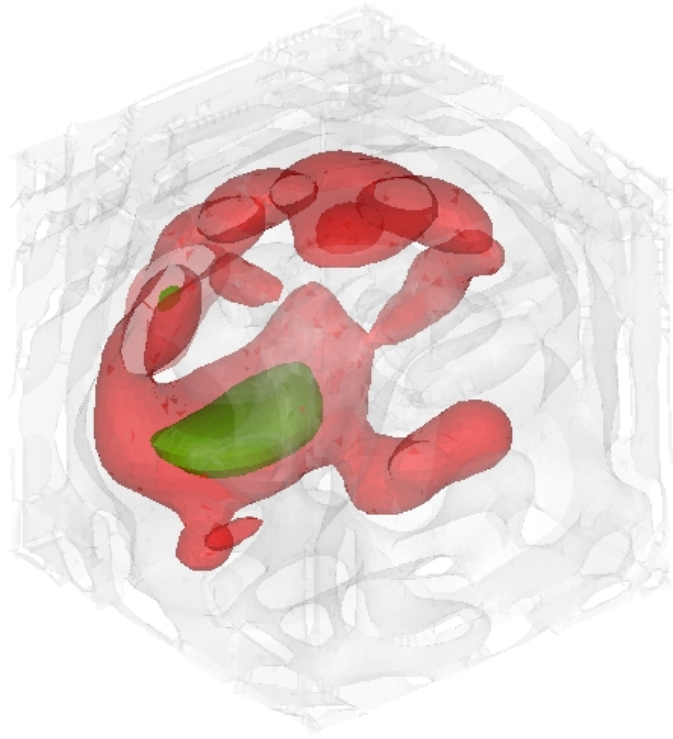
Estimated factor loading $\hat{m}_0$ with $L = 2$, rear view.



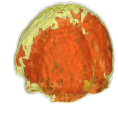


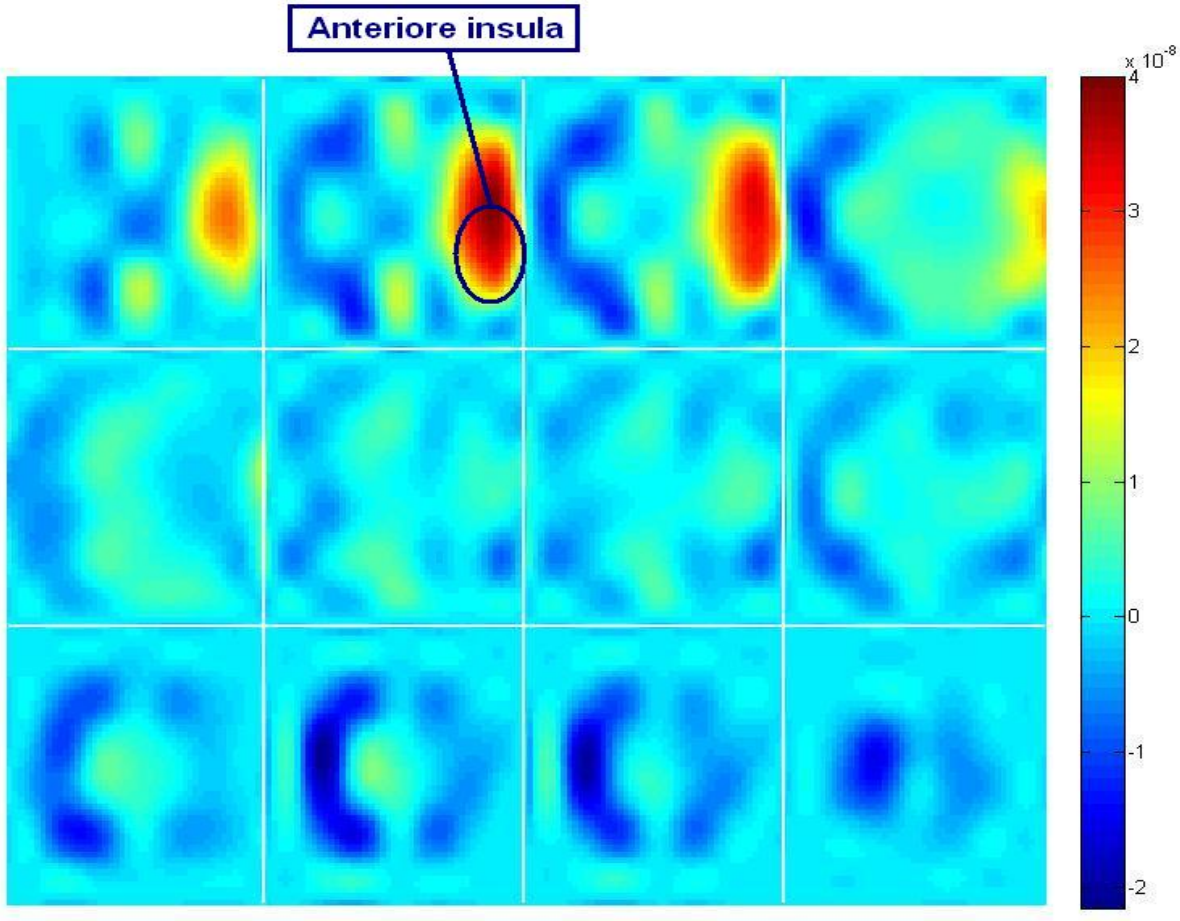
Estimated factor loading $\hat{m}_1$ with $L = 2$.



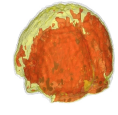


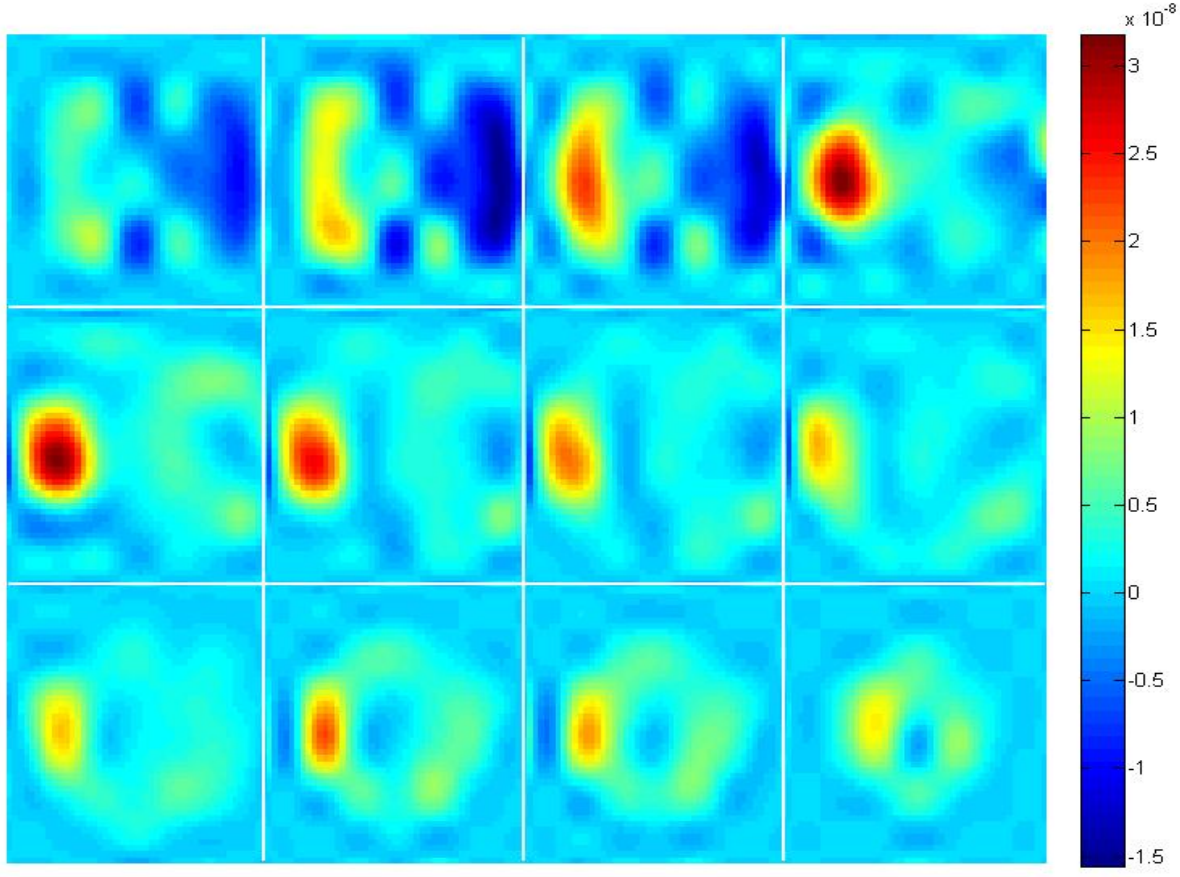
Estimated factor loading $\hat{m}_1$ with $L = 2$, rear view.



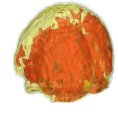


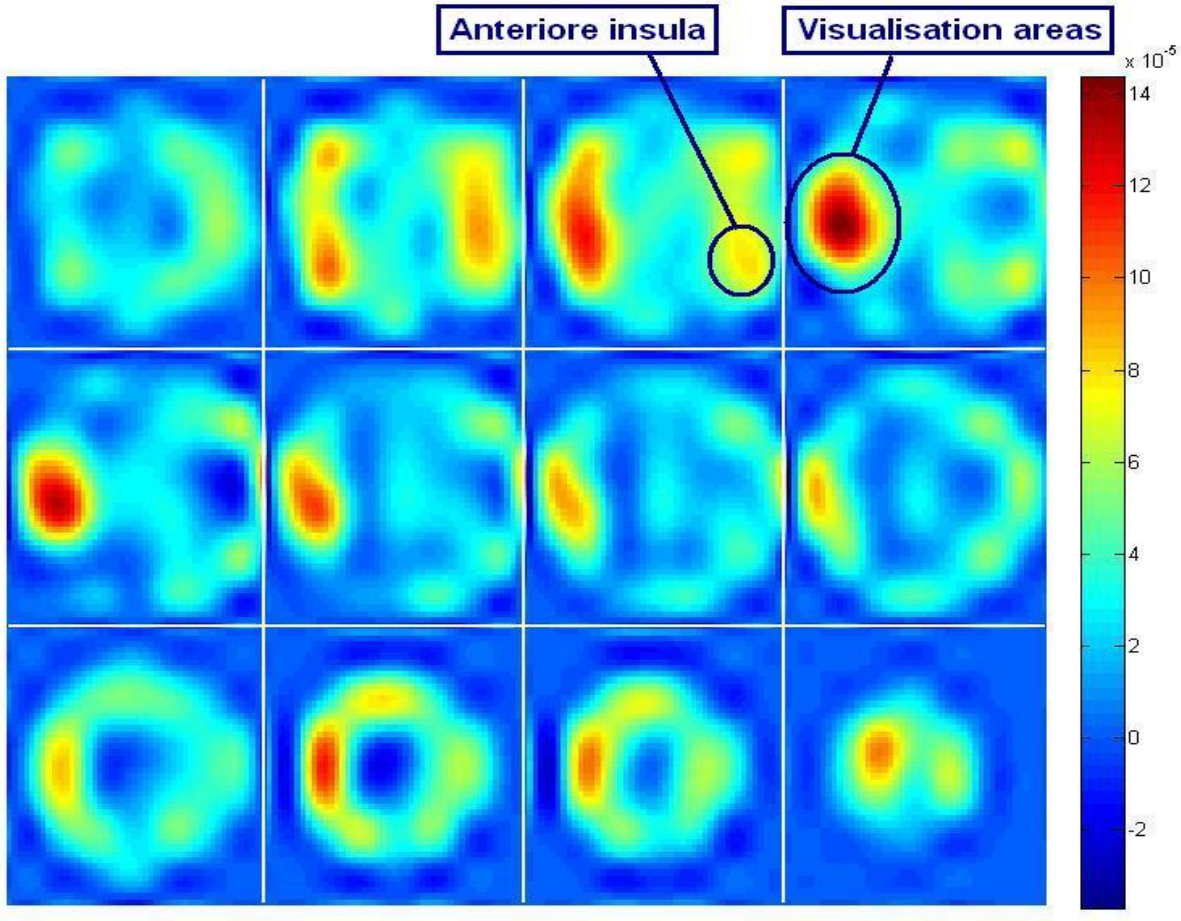
Estimated factor loading $\hat{m}_1$ with $L = 3$.



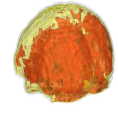


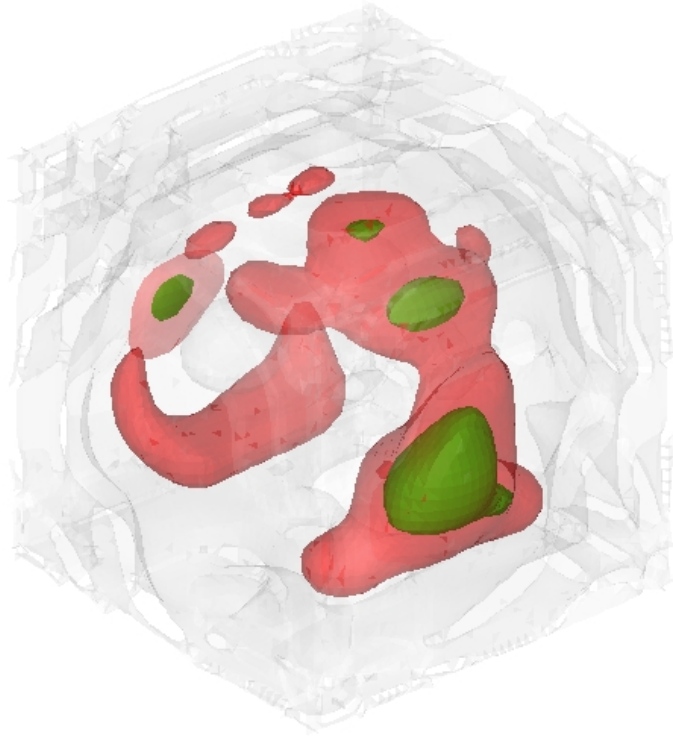
Estimated factor loading $\hat{m}_1$ with $L = 4$.



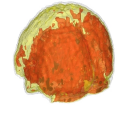


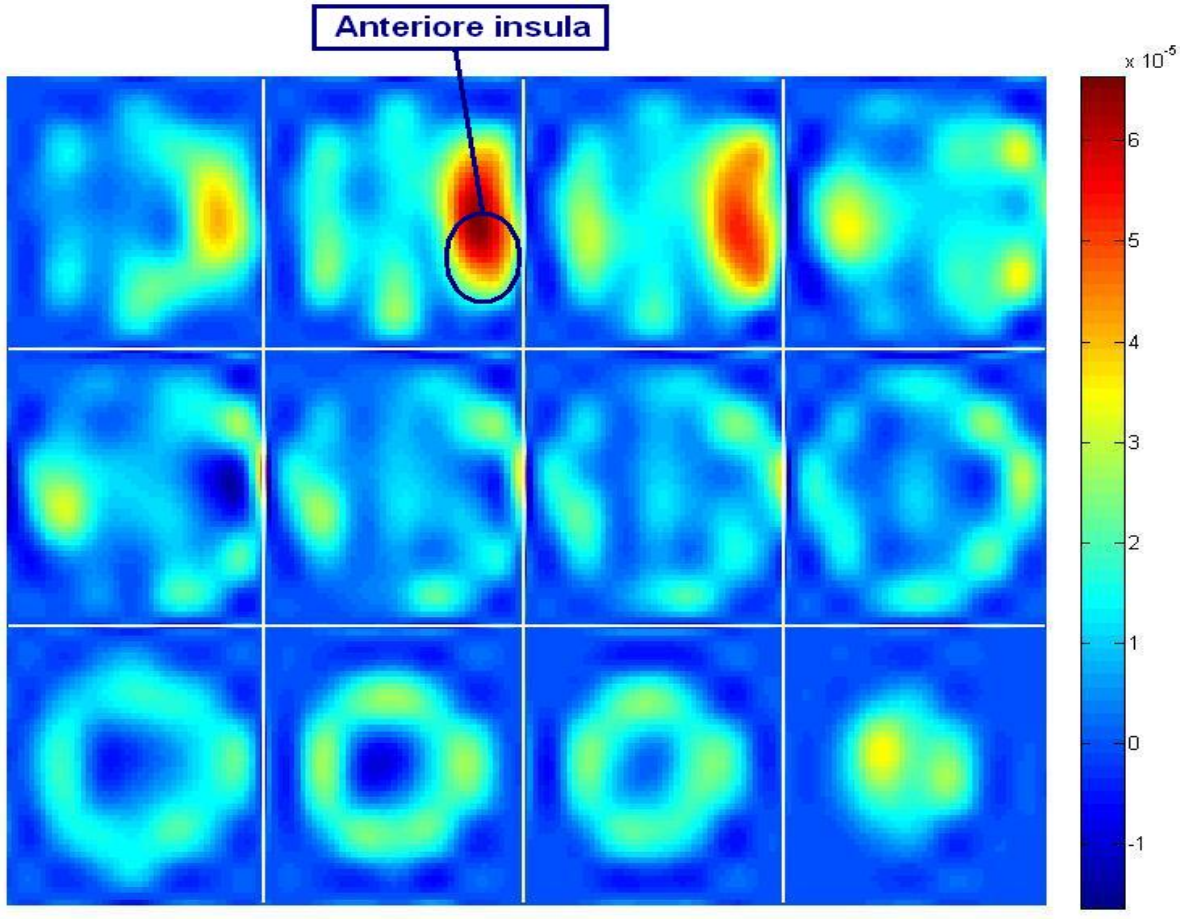
Estimated factor loading $\hat{m}_2$ with $L = 2$.



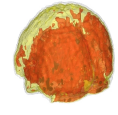


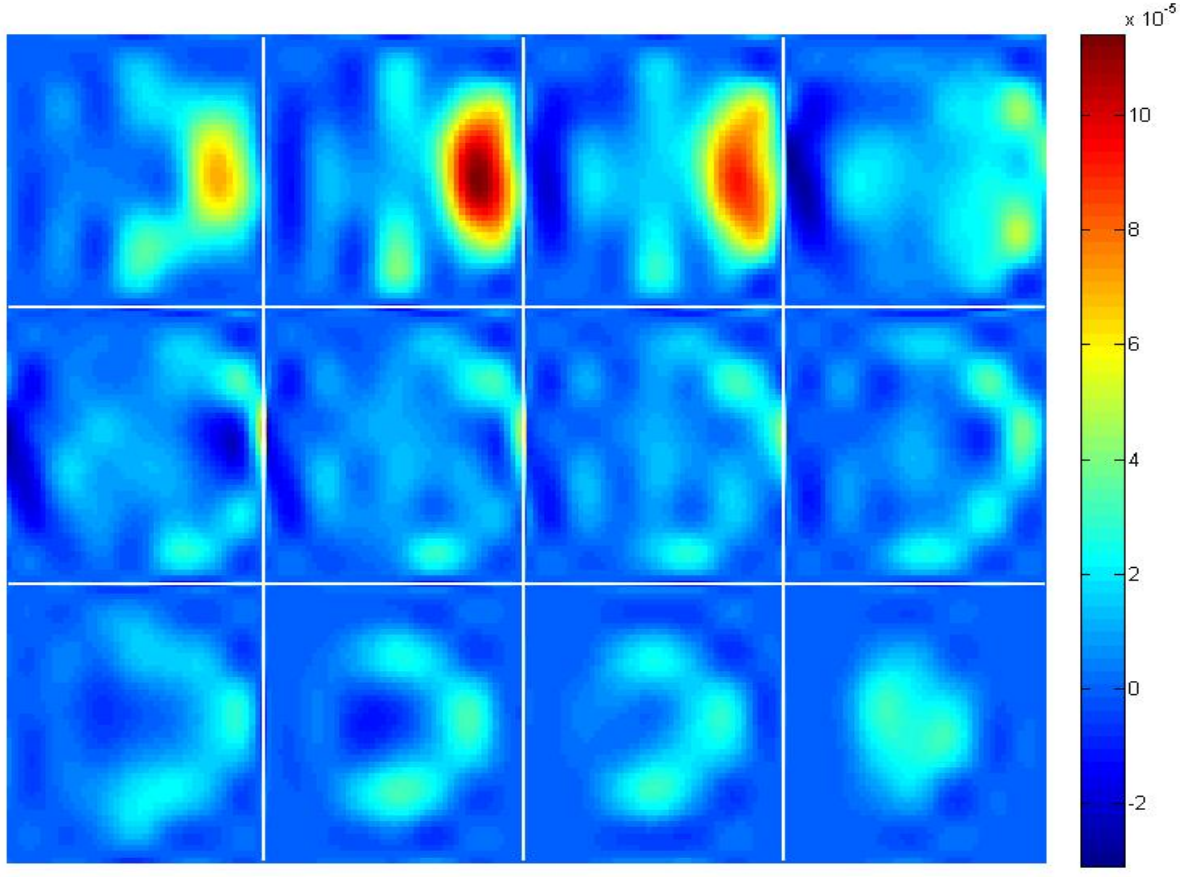
Estimated factor loading $\hat{m}_2$ with $L = 2$, rear view.



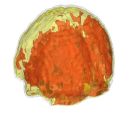


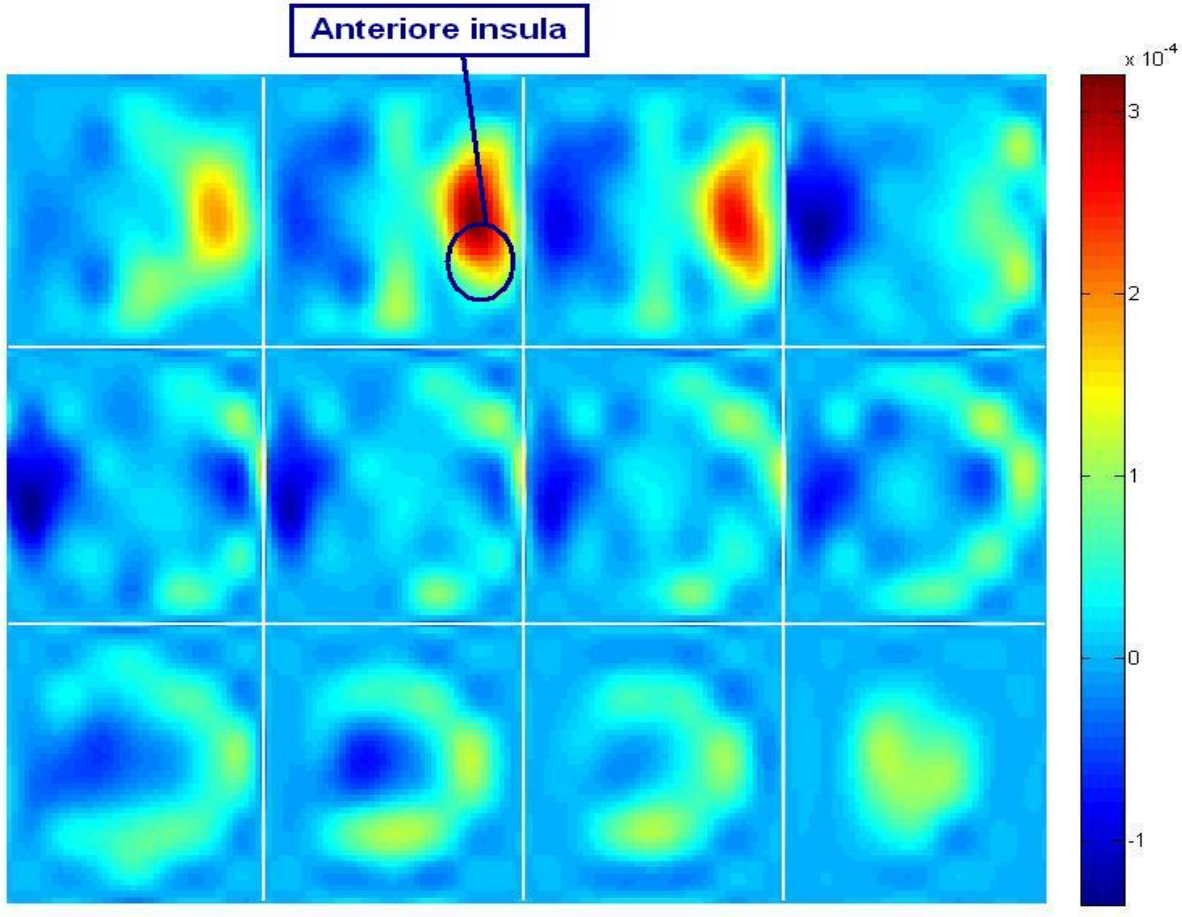
Estimated factor loading $\hat{m}_2$ with $L = 3$.



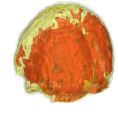


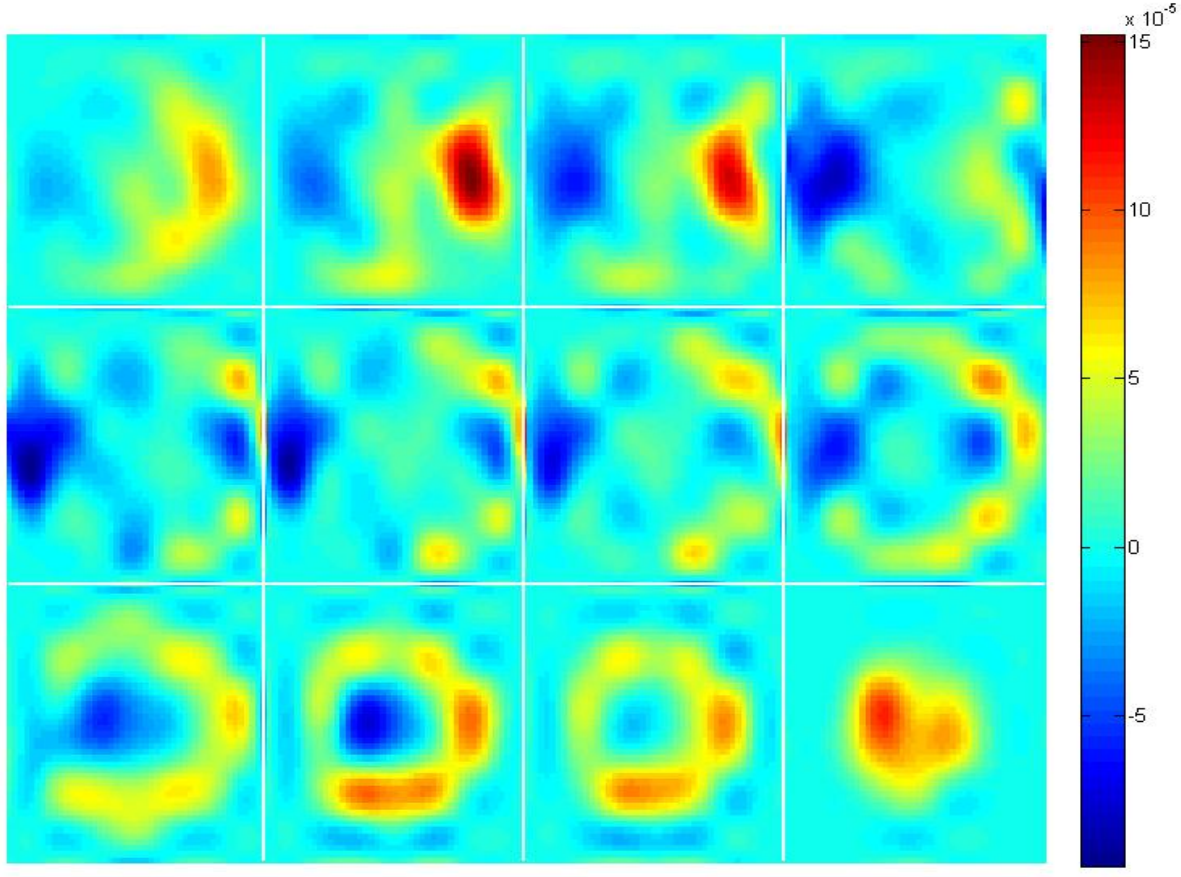
Estimated factor loading $\hat{m}_2$ with $L = 4$.



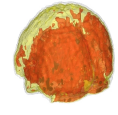


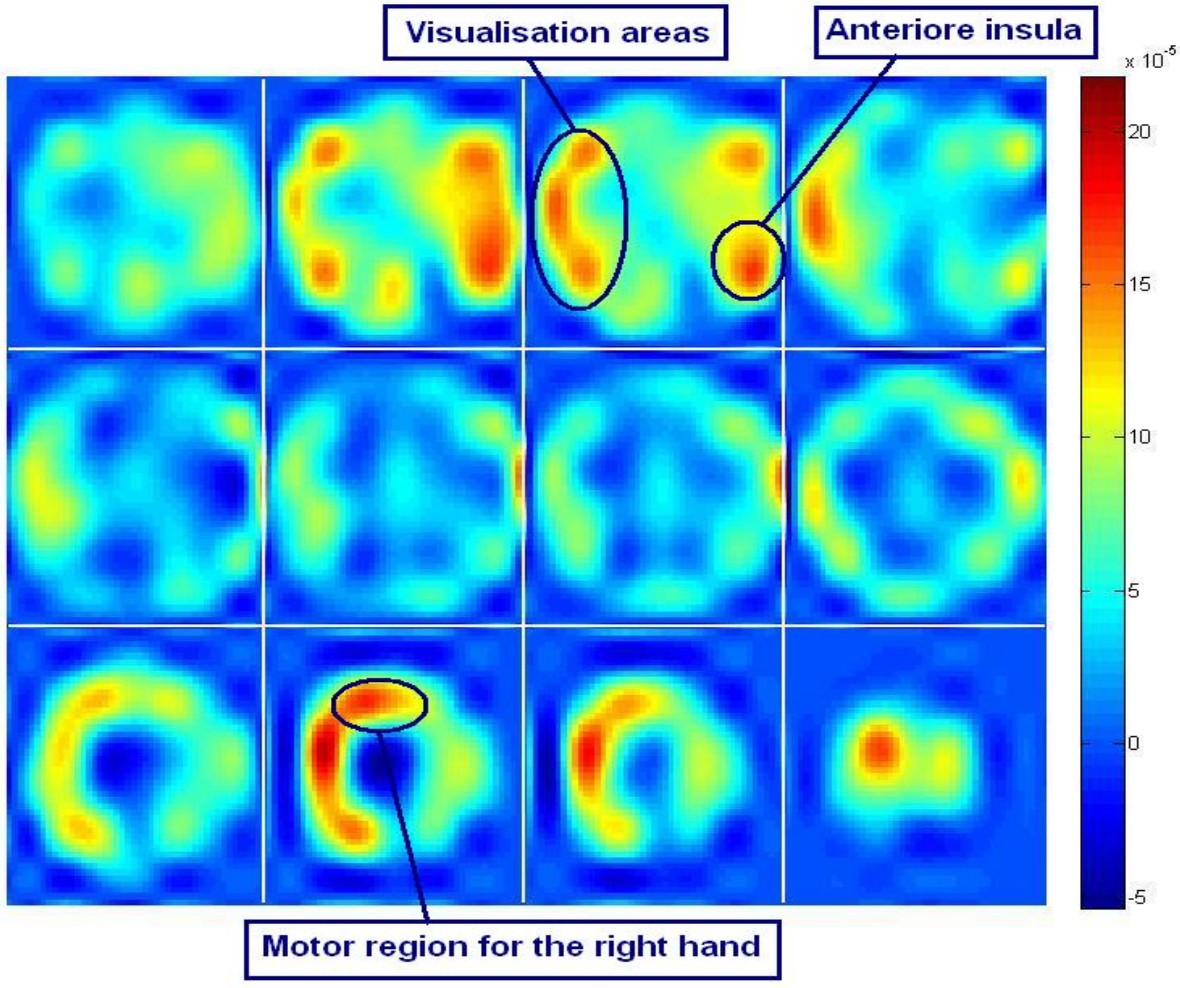
Estimated factor loading $\hat{m}_3$ with $L = 3$.



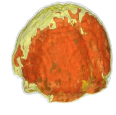


Estimated factor loading $\hat{m}_3$ with $L = 4$.

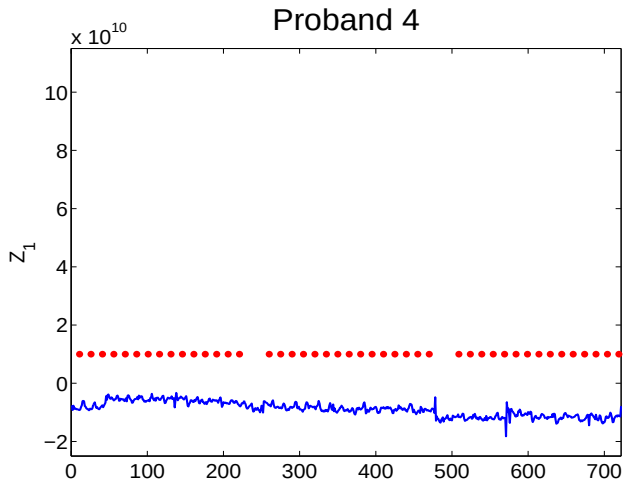


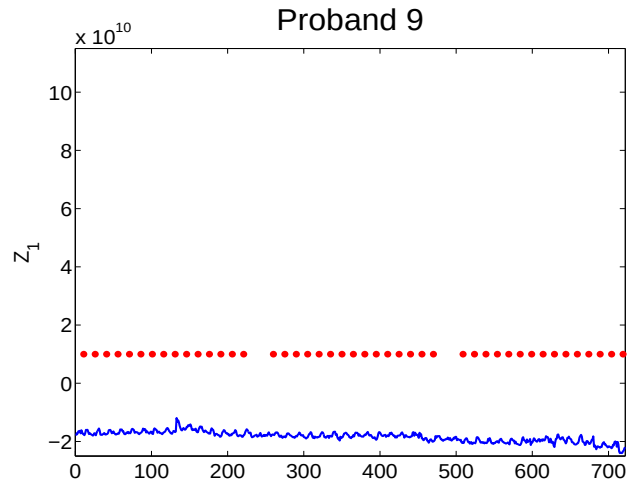


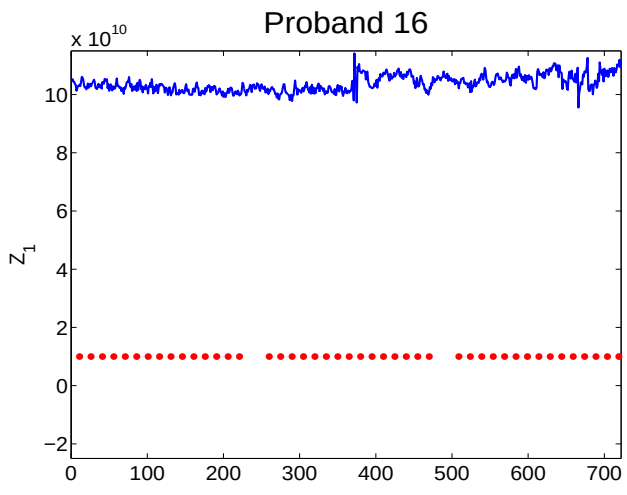
Estimated factor loading $\hat{m}_4$ with $L = 4$.

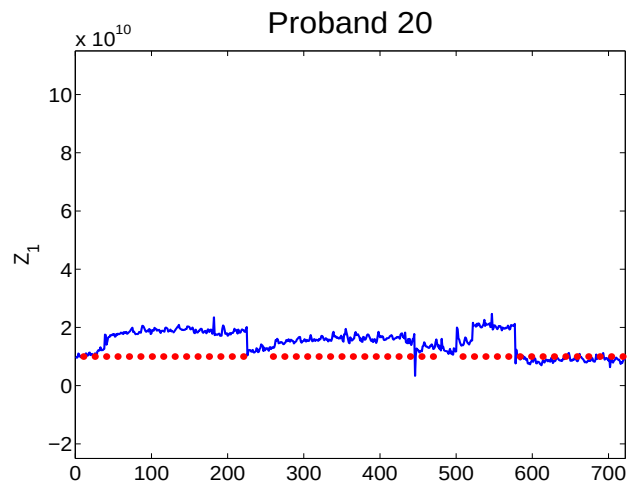


Factor $\widehat{Z}_1$

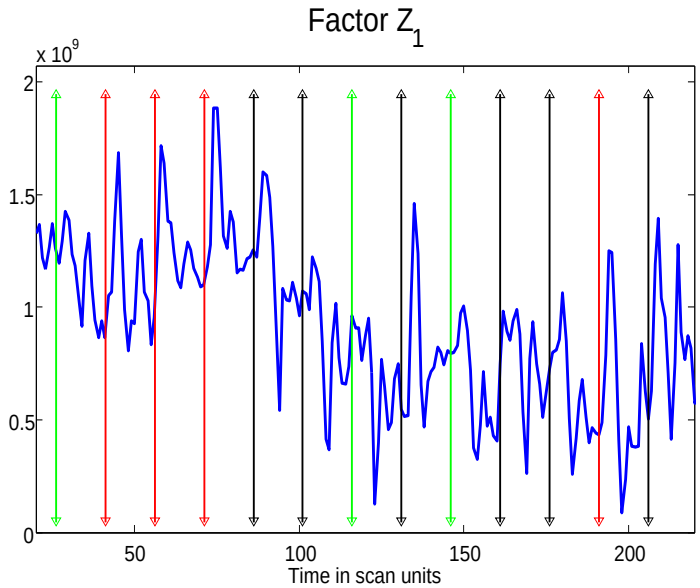


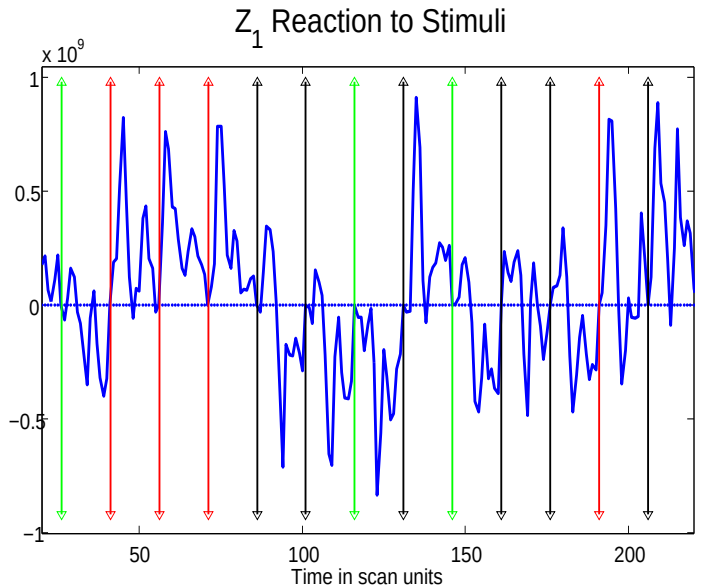




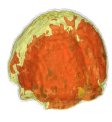


Reaction to stimulus in factor $\widehat{Z}_1$

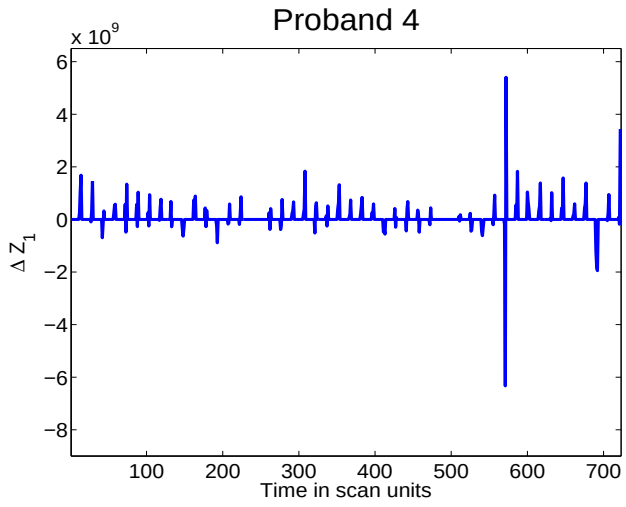


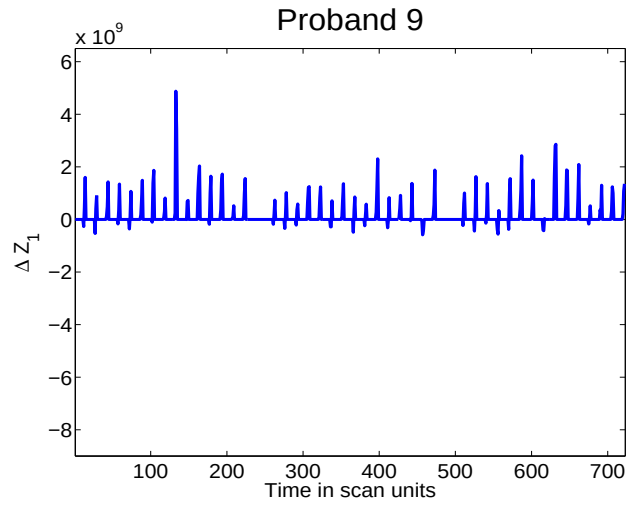


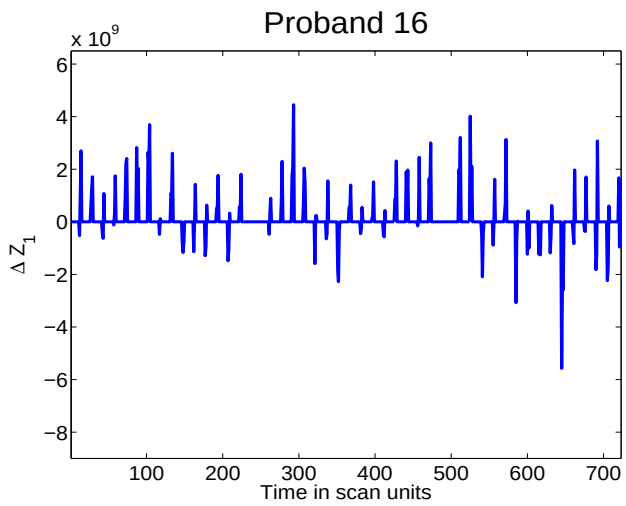
Lines correspond to the time points of judgement tasks: decision, return, risk.

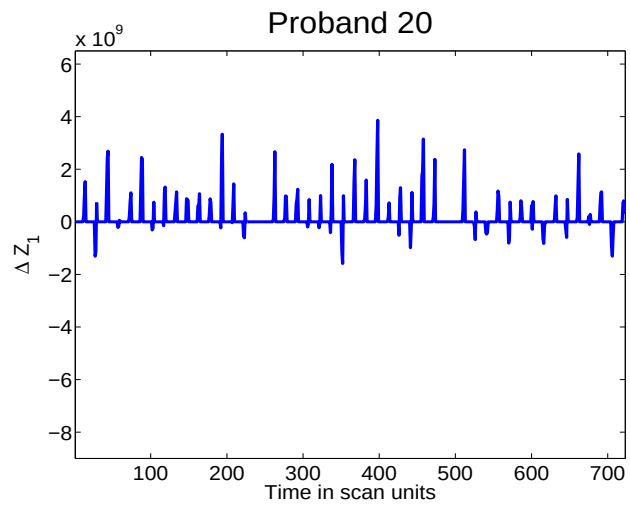


Reaction to stimuli in factor $\widehat{Z}_1$

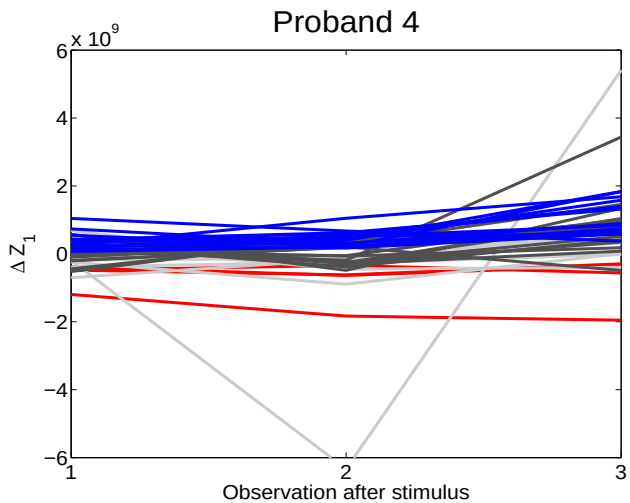


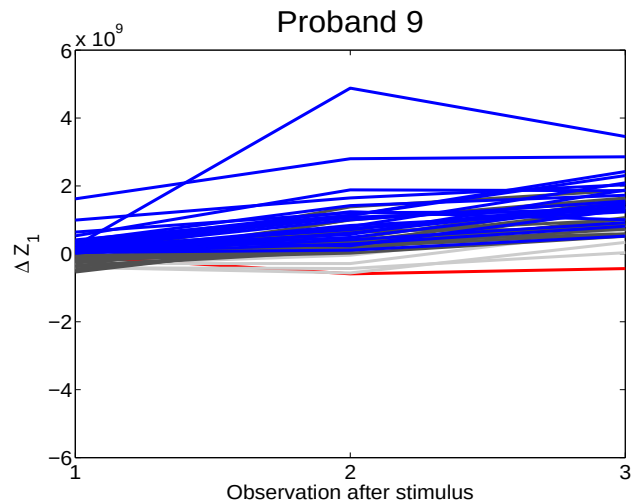


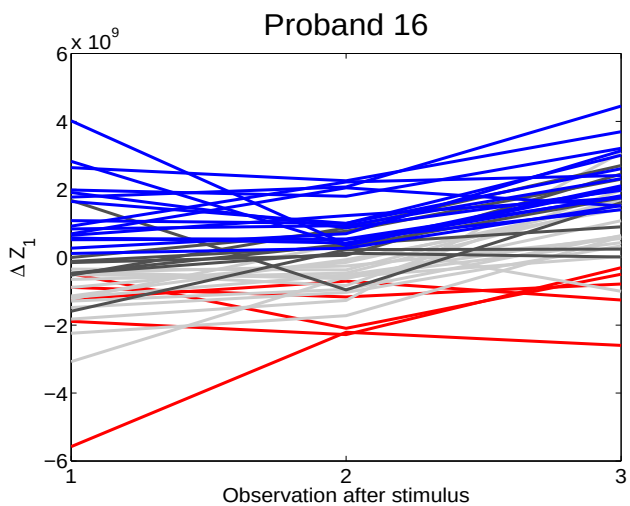


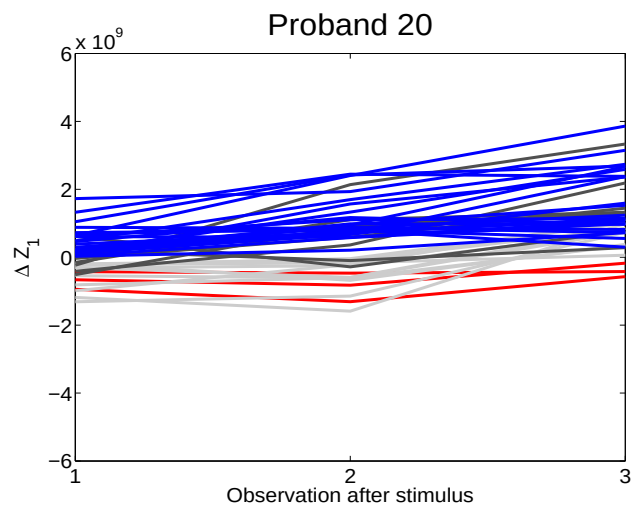


Reaction to stimuli in factor $\widehat{Z}_1$



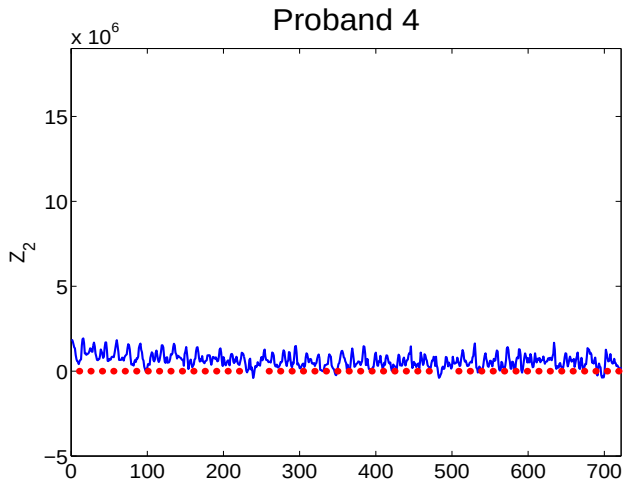


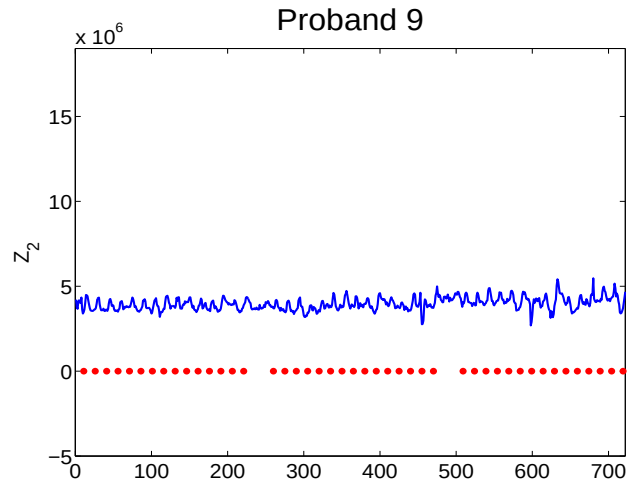


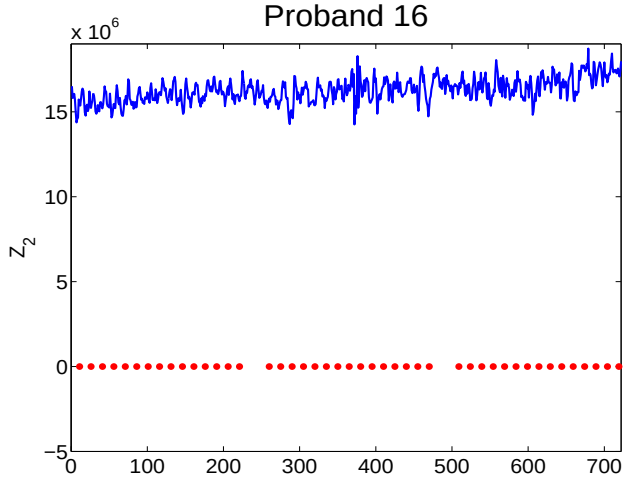


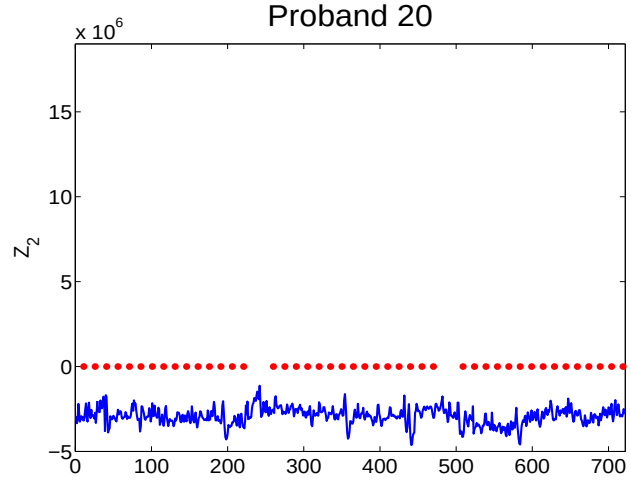
blue: all 3 points > 0, light gray: 2 points > 0, gray: 1 point > 0, red: all 3 points ≤ 0.

Factor $\widehat{Z}_2$

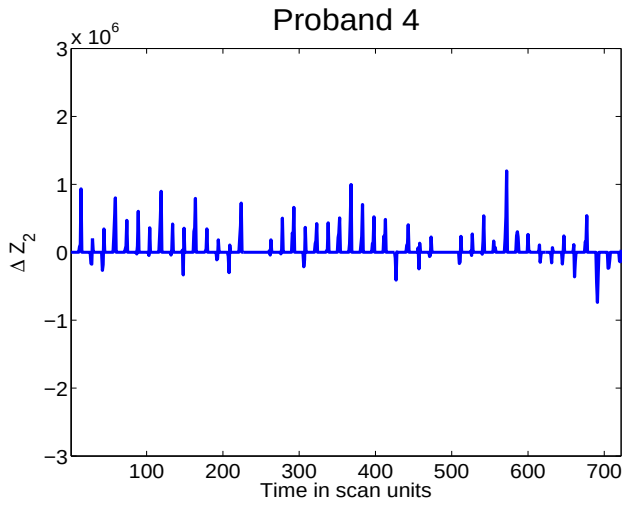


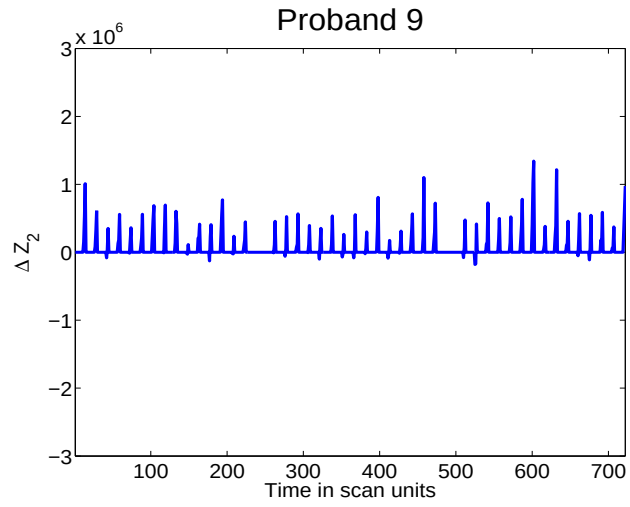


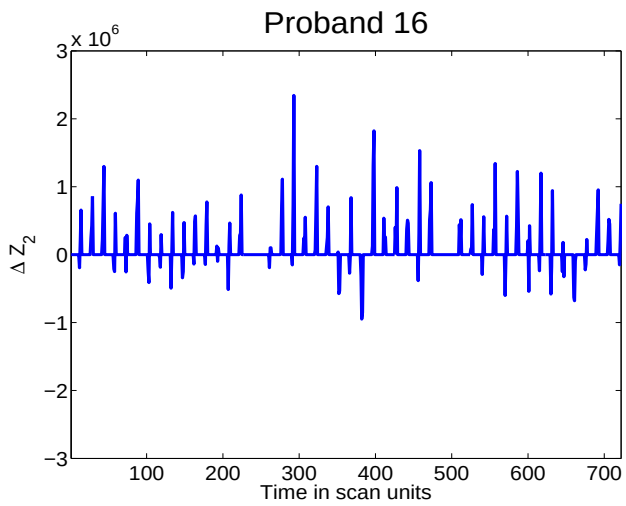


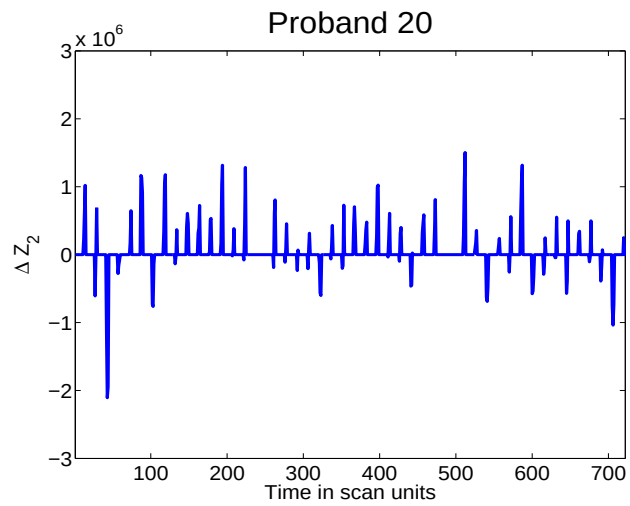


Reaction to stimuli in factor $\widehat{Z}_2$

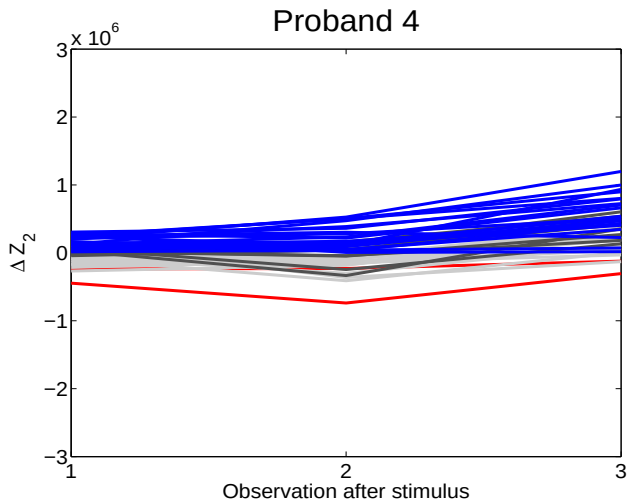


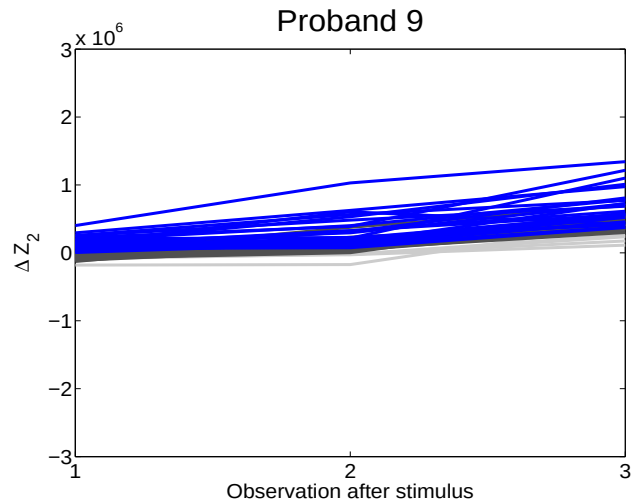


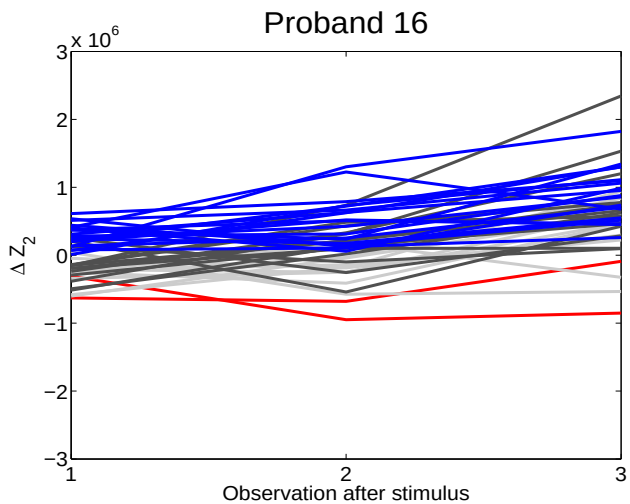


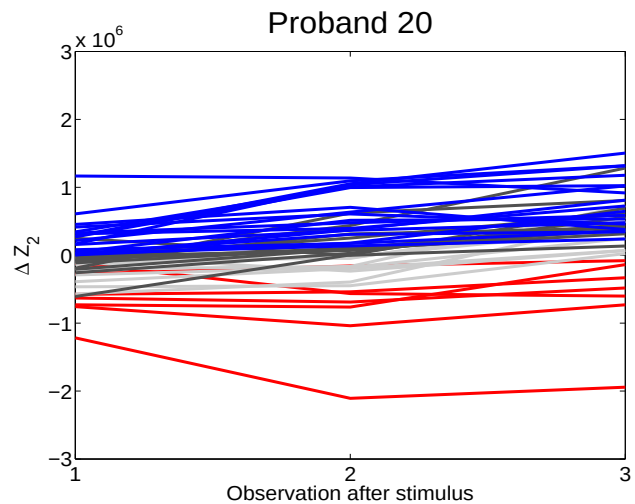


Reaction to stimuli in factor $\widehat{Z}_2$



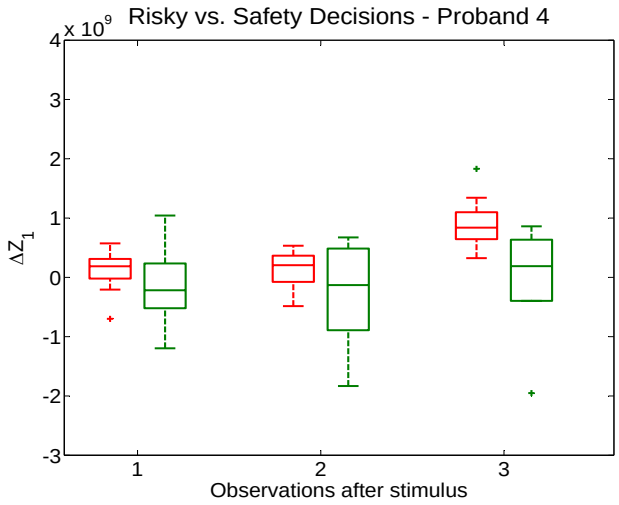


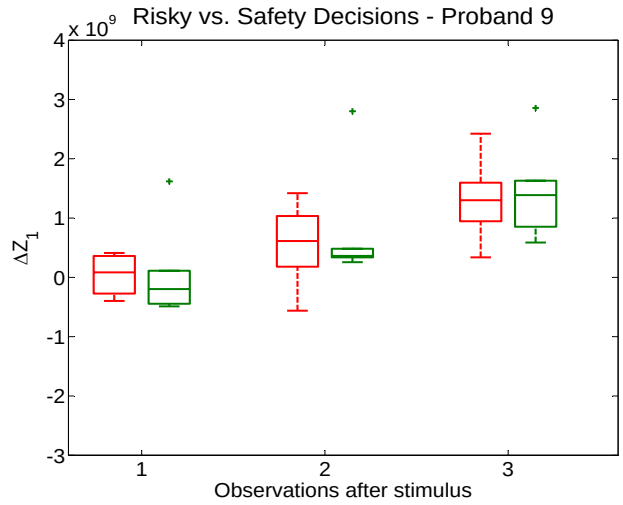


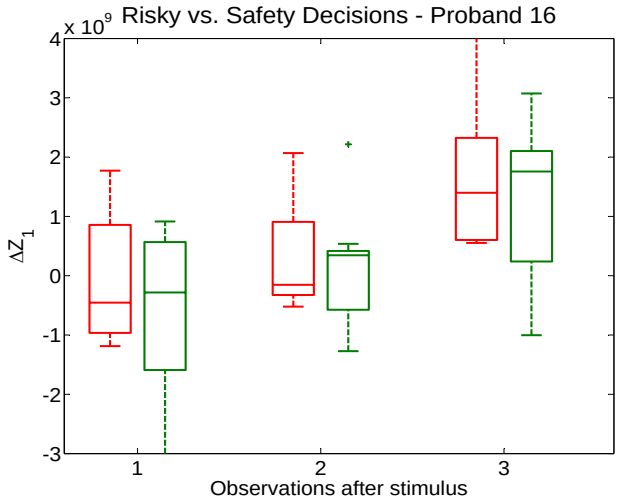


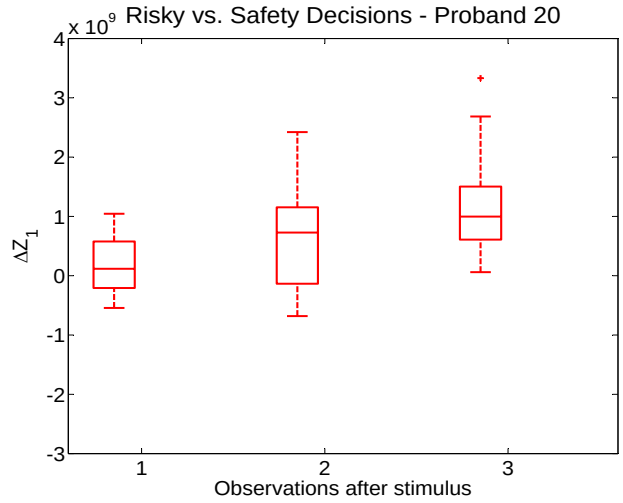
blue: all 3 points > 0, light gray: 2 points > 0, gray: 1 point > 0, red: all 3 points ≤ 0.

Reaction after decision tasks in factor $\widehat{Z}_1$



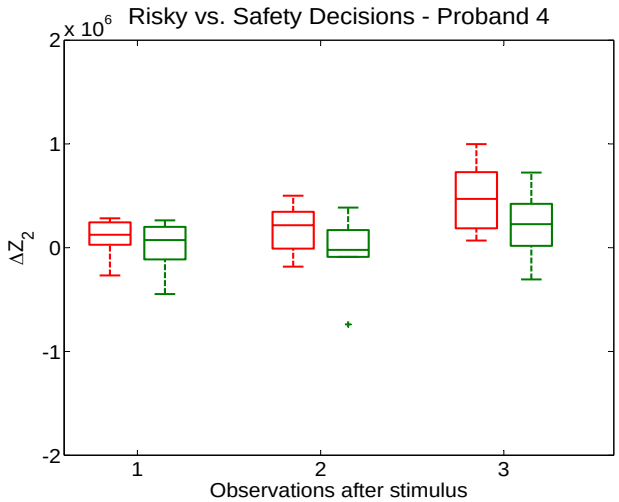


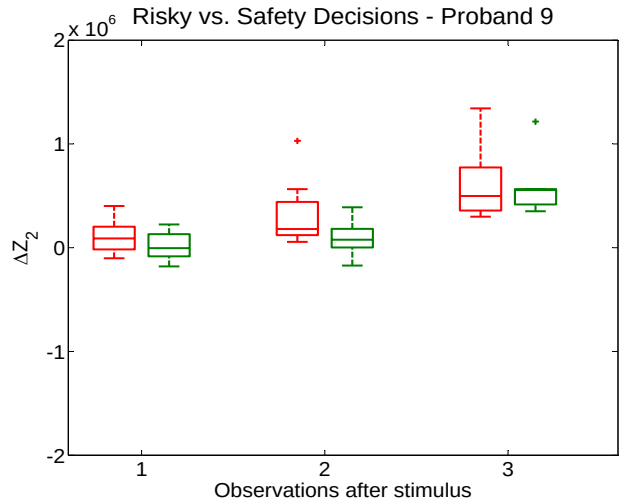


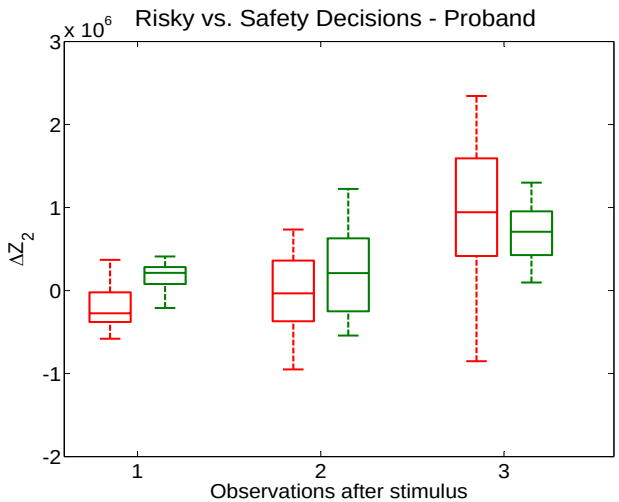


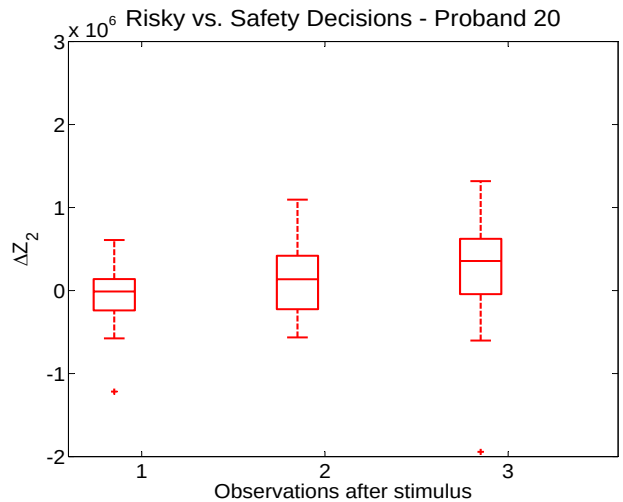
Red boxplots correspond to risky decisions, green boxplots to safety decisions.

Reaction after decision tasks in factor $\widehat{Z}_2$



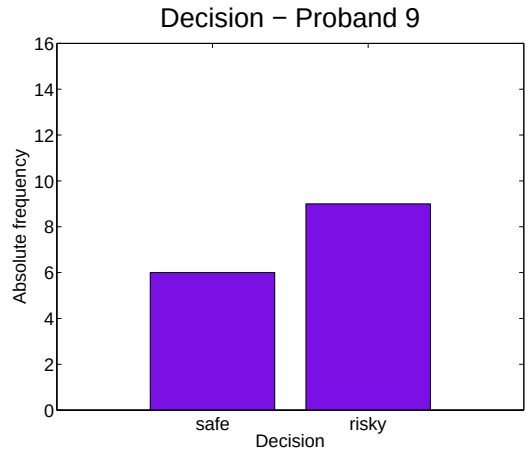


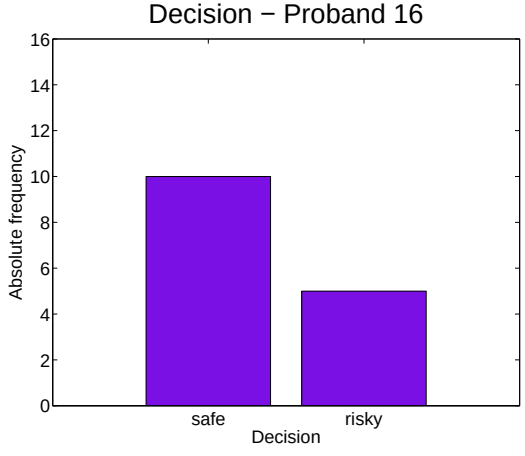


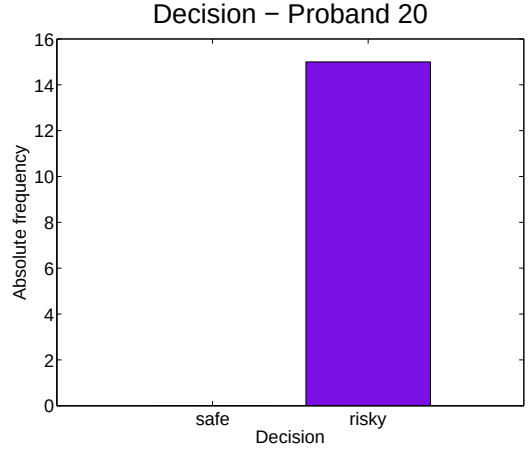


Red boxplots correspond to risky decisions, green boxplots to safety decisions.

Decision

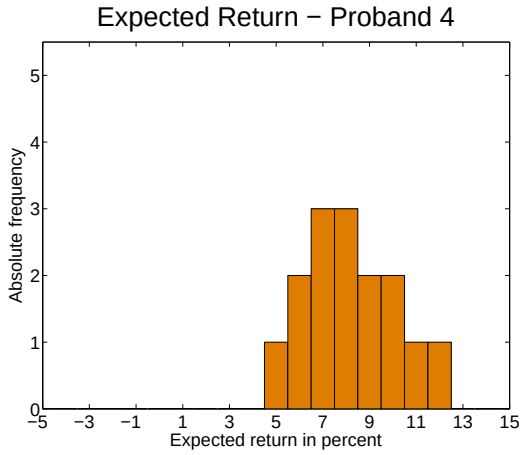


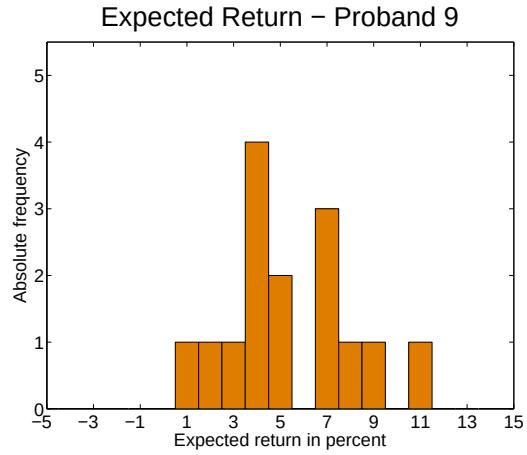


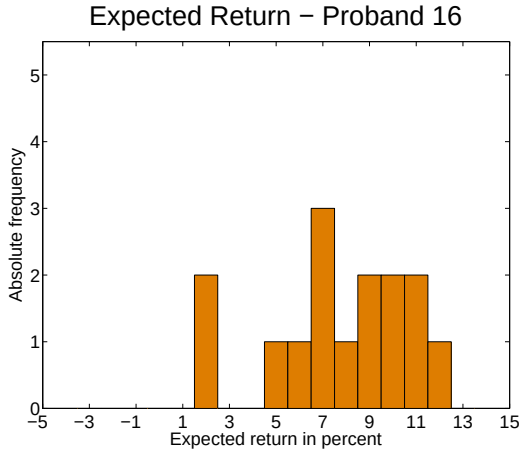


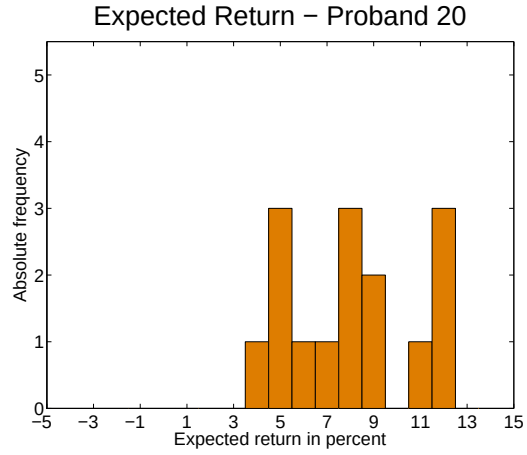
Choice between an investment with 5% fixed return (safe investment) and the investment represented by the return stream (risky investment)

Expected Return



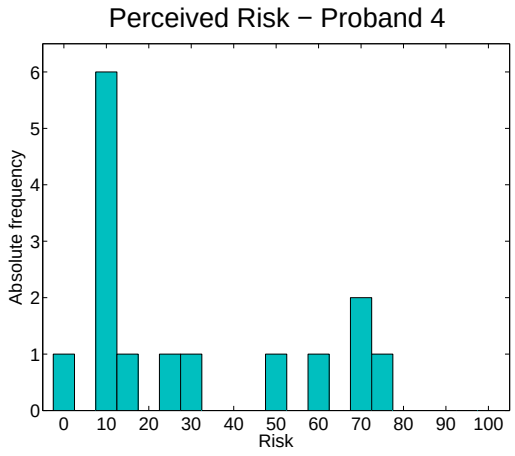


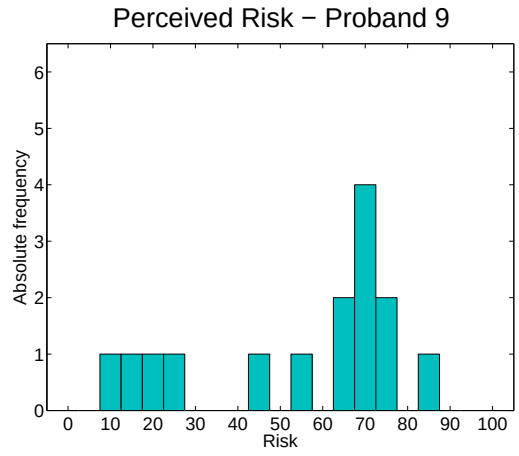


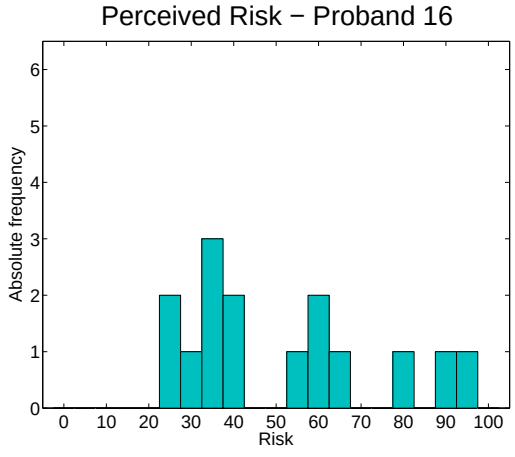


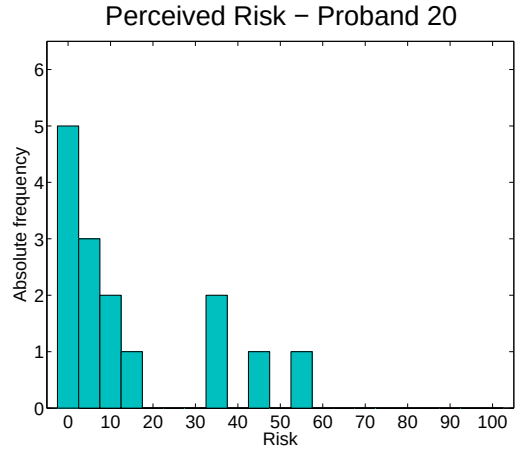
Subjective expected return judge in range (-5% – 15%) of the investment represented by the return stream

Perceived Risk



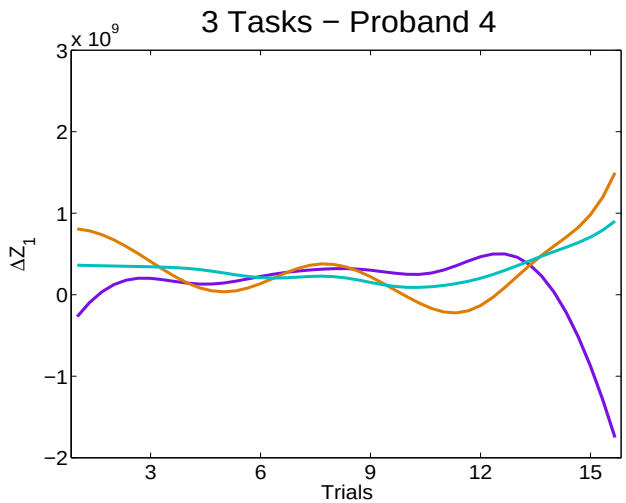


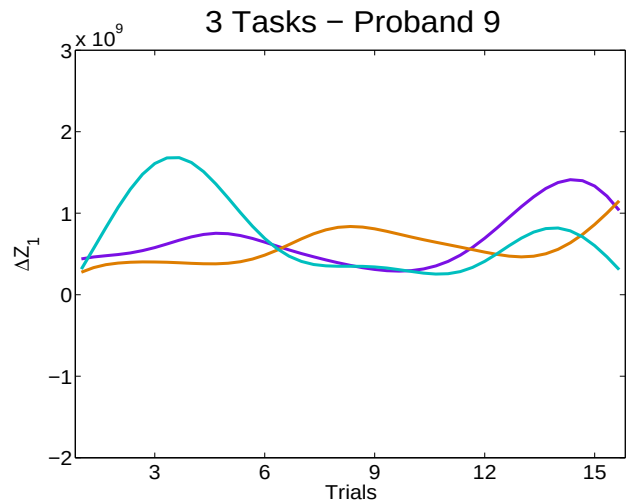


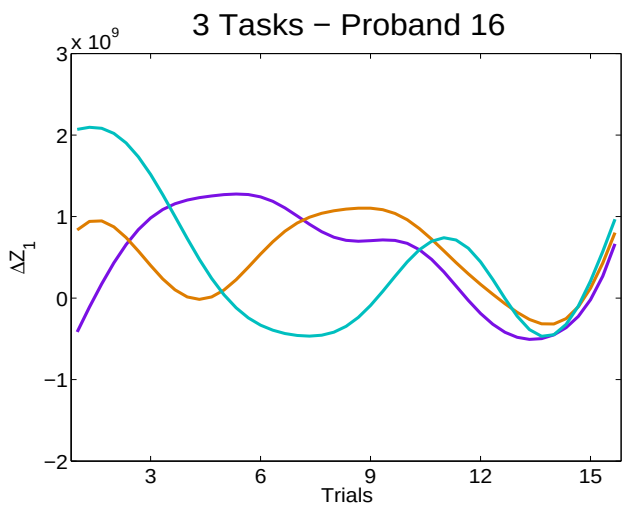


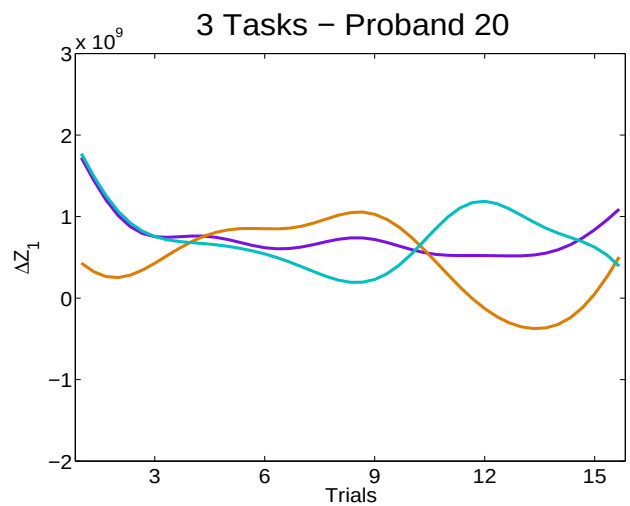
Perceived risk judge on scale: 0 (no risk) – 100 (maximum risk)

Local linear smoother for reactions in factor $\widehat{Z}_1$



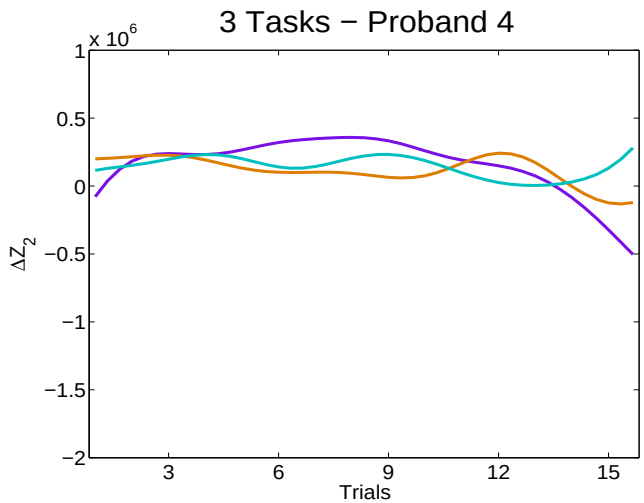


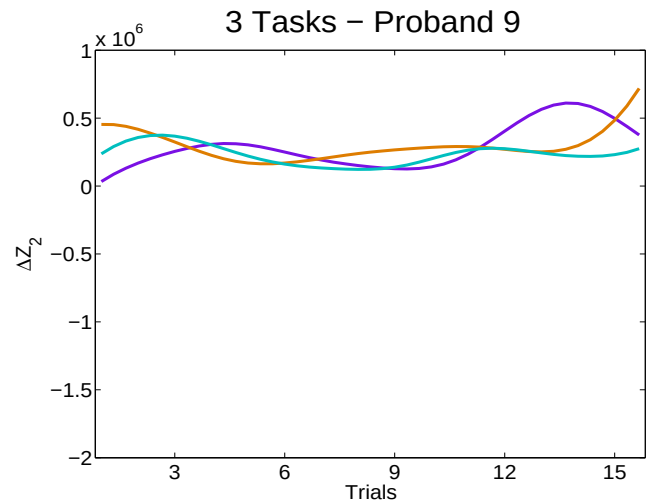


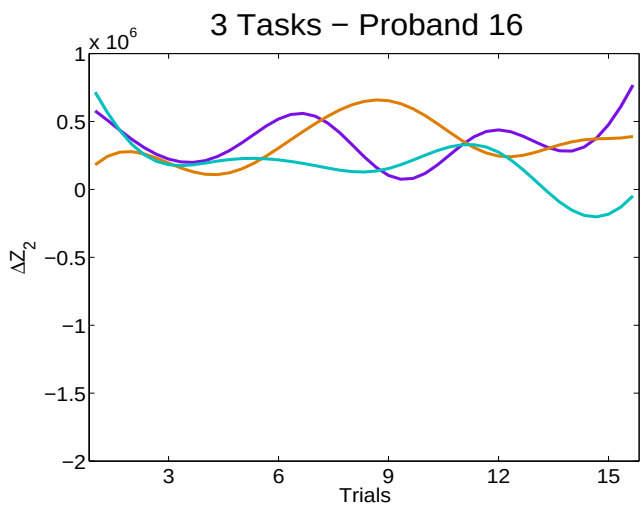


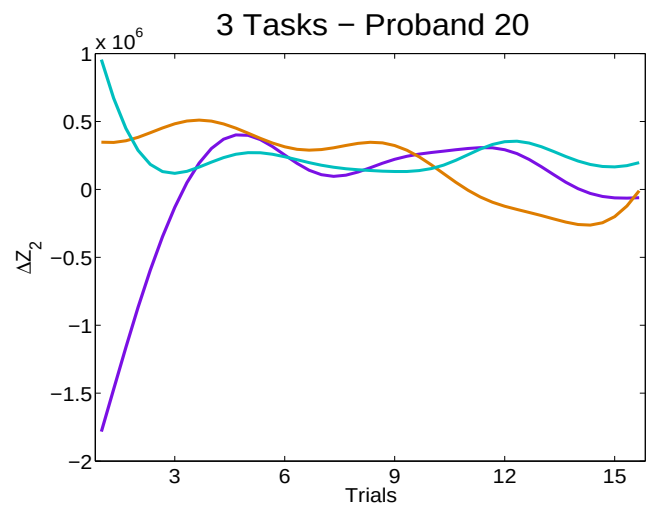
Violet line corresponds to decision tasks, orange line to expected return, blue line to the perceived risk.

Local linear smoother for reactions in factor $\widehat{Z}_2$



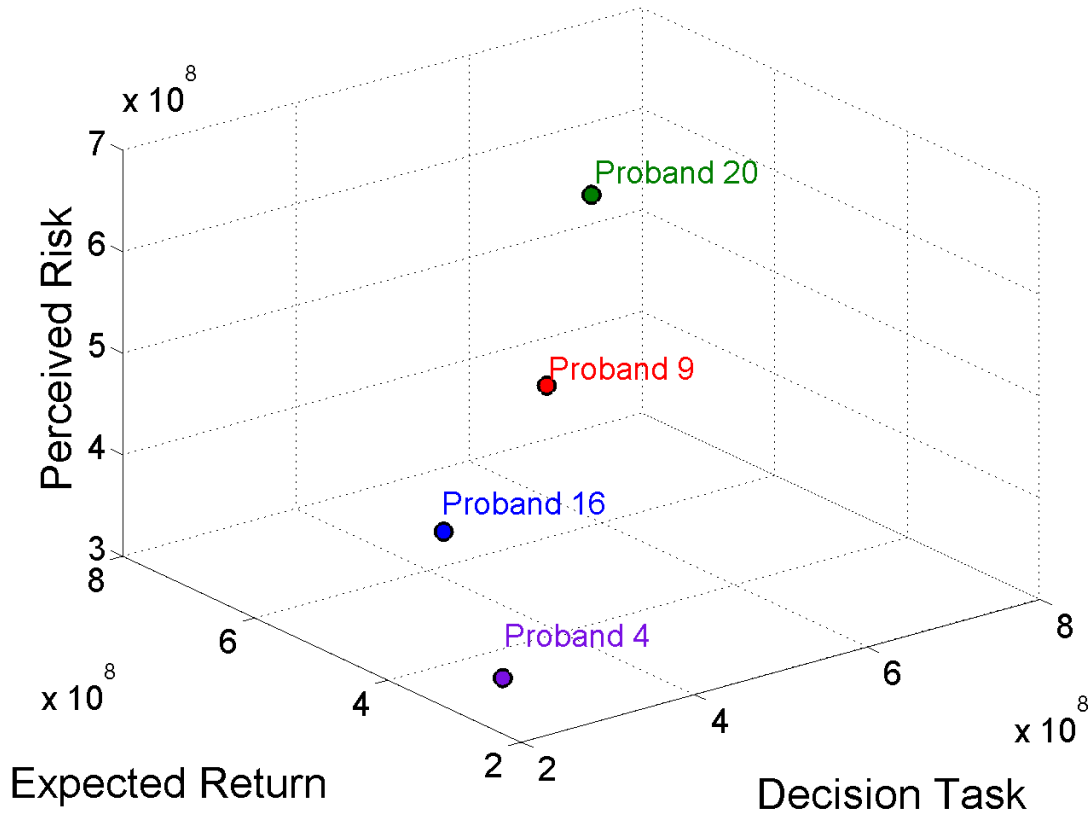


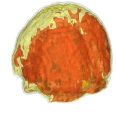




Violet line corresponds to decision tasks, orange line to expected return, blue line to the perceived risk.

Median of Reactions to Stimuli in Z_1





Median of Reactions to Stimuli in Z_2