Quantile Regression in Risk Calibration

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Dependence Risk

Risk Calibration

- Quantification of risk: Value-at-Risk and expected shortfall
- Drawbacks of VaR: Does not say much about dependence risk
- Need for macroprudential risk measures

Quantile Regression in VaR

- Parametric VaR: Chernozhukov and Umantsev (2001), Engle and Manganelli (2004)
- Kuester, Mittnik and Paolella (2006): Comparing VaR predictions from historical simulations, extreme value theory, GARCH model and CAViaR
- Nonparametric VaR (Double Kernel): Cai and Wang (2008), Taylor (2008) and Schaumburg (2010)

Risk Calibration

- Marginal Expected Shortfall (MES): Acharya et al. (2010)
- Distressed Insurance Premium (DIP): Huang et al. (2010)
- Adrian and Brunnermeier (2010)(AB): X_j and X_i are two asset returns,

$$\mathrm{P}\left\{X_j \leq \mathrm{CoVaR}_{j|i}(q) | C(X_i)\right\} = q.$$

where $C(X_i) = \{X_i = \mathrm{VaR}_q(X_i)\}$.

- Advantages:
  1. Cloning property
  2. Conservative property
  3. Adaptiveness

CoVaR Construction

$X_{j,t}$ and $X_{i,t}$ are two asset returns. Two linear quantile regressions:

$$X_{i,t} = \alpha_i + \gamma_i M_{t-1} + \varepsilon_{i,t}, \quad (1)$$

$$X_{j,t} = \alpha_{j|i} + \beta_{j|i} X_{i,t} + \gamma_{j|i} M_{t-1} + \varepsilon_{j,t}. \quad (2)$$

M_t: vector-valued state variables. $F^{-1}_{\varepsilon_{i,t}}(q|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j,t}}(q|M_{t-1}, X_{i,t}) = 0$.

$$VaR_{i,t} = \hat{\alpha}_i + \hat{\gamma}_i M_{t-1},$$

$$CoVaR_{j|i,t} = \hat{\alpha}_{j|i} + \hat{\beta}_{j|i} VaR_{i,t} + \hat{\gamma}_{j|i} M_{t-1}.$$

Nonlinear Dependence in Asset Returns

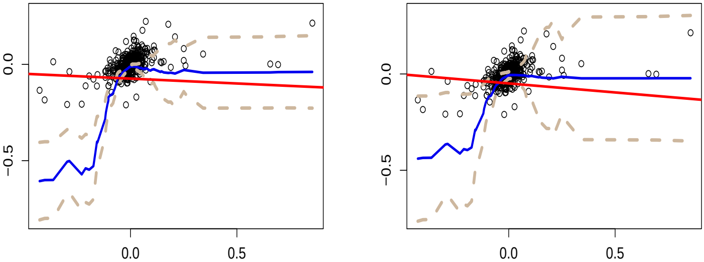

Figure 1: Goldman Sachs (GS) and Citigroup (C) weekly returns 0.05 (left) and 0.1(right) quantile functions. y-axis=GS returns; x-axis=C returns. LLQR lines. Linear parametric quantile regression line. Confidence band with level 5%. $N = 546$.

Semiparametric Specification

- More general, with functions f, g;

$$X_{i,t} = f(M_{t-1}) + \varepsilon_{i,t}; \quad (3)$$

$$X_{j,t} = g(X_{i,t}, M_{t-1}) + \varepsilon_{j,t}. \quad (4)$$

M_t: vector-valued state variables. $F^{-1}_{\varepsilon_{i,t}}(q|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j,t}}(q|M_{t-1}, X_{i,t}) = 0$.

- Challenge:
  1. The curse of dimensionality for f, g
  2. Numerical Calibration of (3) and (4)

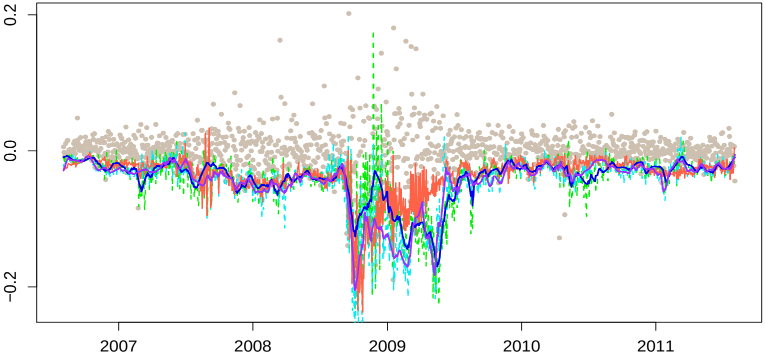

Figure 2: CoVaR of Goldman Sachs given the VaR of Citigroup. x-axis=time. y-axis= GS daily returns. PLM CoVaR . AB (2010) CoVaR . The linear QR VaR of GS.

Goal

- Computing CoVaR (i.e. two step quantile regression) in a nonparametric (or semiparametric) fashion
- Testing the effectiveness of the nonparametric CoVaR
- What can one learn from the semiparametric specification

Outline

1. Motivation ✓
2. Locally Linear Quantile Regression
3. A Semiparametric Model
4. Backtesting
5. Conclusions and Further Work

Locally Linear Quantile Estimation (LLQR)

- Locally Linear Quantile Regression (LLQR):

$$\underset{\{a_{0,0},a_{0,1}\}}{\operatorname{argmin}} \sum_{i=1}^{N} K\left(\frac{x_i - x_0}{h}\right) \rho_q \{y_i - a_{0,0} - a_{0,1}(x_i - x_0)\}. \quad (5)$$

- Choice of Bandwidth: Yu and Jones (1998)
- Asymptotic Confidence Band: Härdle and Song (2010)

Macroeconomic Drives

Component of M_t:

1. VIX
2. Short term liquidity spread
3. The daily change in the three-month treasury bill rate
4. The change in the slope of the yield curve
5. The change in the credit spread between BAA-rated bonds and the treasury rate
6. The daily S&P500 index returns
7. The daily Dow Jones U.S. Real Estate index returns

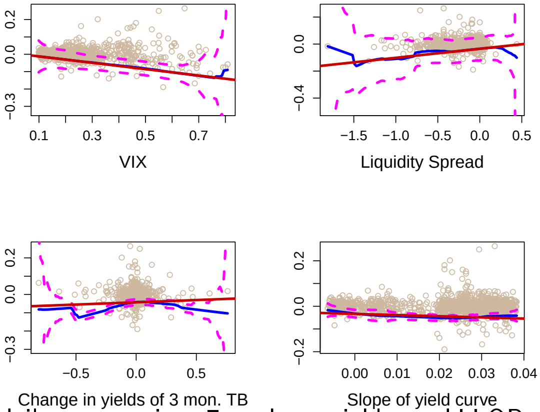

Figure 3: GS daily returns given 7 market variables and LLQR curves. Data 20060804-20110804. $N = 1260$. $\tau = 0.05$.

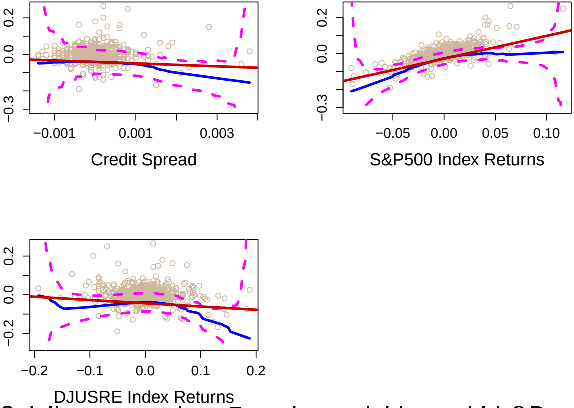

Figure 4: GS daily returns given 7 market variables and LLQR curves. Data 20060804-20110804. $N = 1260$. $\tau = 0.05$.

Partial Linear Model

- Consider

$$X_{i,t} = \alpha_i + \gamma_i M_{t-1} + \varepsilon_{i,t}; \quad (6)$$

$$X_{j,t} = \beta_j M_{t-1} + l(X_{i,t}) + \varepsilon_{j,t} \quad (7)$$

l: a general function. M_t: state variables. $F^{-1}_{\varepsilon_{i,t}}(q|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{j,t}}(q|M_{t-1}, X_{i,t}) = 0$.

- Advantage:
  1. Capturing nonlinear asset dependence
  2. Avoid curse of dimensionality

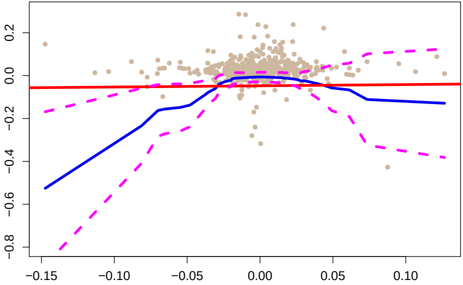

Figure 5: The nonparametric part of the PLM estimation. y-axis=GS daily returns. x-axis=C daily returns. The LLQR quantile curve. Linear parametric quantile line. Confidence band level 0.05. Data 20080625-20081223. N=126. h =0.2003. $q = 0.05$.

Estimation of Partial Linear Model

- Method: Liang, Härdle and Carroll (1999) and Härdle, Ritov and Song (2011)
- Estimation of l: LLQR
- j: GS daily returns,
  i: C daily returns
  Data 20060804-20110804

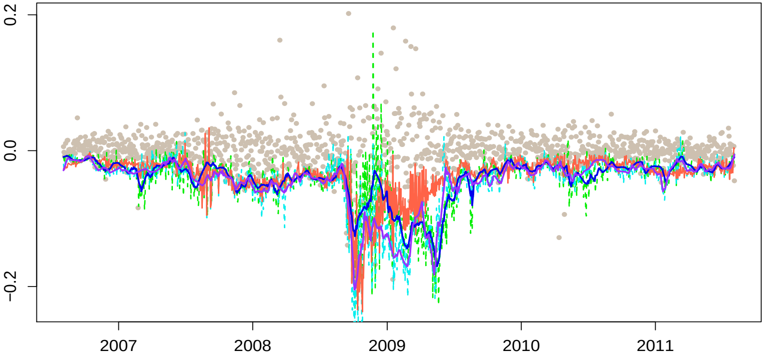

Figure 6: CoVaR of Goldman Sachs given the VaR of Citigroup. The x-axis is time. The y-axis is the GS daily returns. PLM CoVaR . AB (2010) CoVaR . The linear QR VaR of GS.

Backtesting Tests

- Berkowitz, Christoffersen and Pelletier (2011): If the VaR algorithm is correct, violations should be unpredictable

$$I_t = \begin{cases} 1, & \text{if } R_t < VaR_{t-1}(q) \\ 0, & \text{otherwise.} \end{cases}$$

- Formally, violations I_t form a sequence of martingale difference (M.D.)

Box Tests

- $\hat{\rho}_k$ be the estimated autocorrelation of lag k of violation $\{I_t\}$ and N be the length of the time series.
- Ljung-Box test:

$$\mathrm{LB}(m) = N(N+2)\sum_{k=1}^{m} \frac{\hat{\rho}_k^2}{N-k} \tag{8}$$

- Lobato test:

$$\mathrm{L}(m) = N\sum_{k=1}^{m} \frac{\hat{\rho}_k^2}{\hat{v}_{kk}} \tag{9}$$

CaViaR Test

- Engle and Manganelli (2004)
- Berkowitz, Christoffersen and Pelletier (2011): CaViaR performs best overall
- $$\begin{aligned} I_t =& \alpha + \sum_{k=1}^{n} \beta_{1k} I_{t-k} + \\ & \sum_{k=1}^{n} \beta_{2k} g(I_{t-k}, I_{t-k-1}, ..., R_{t-k}, R_{t-k-1}, ...) + u_t \end{aligned}$$

  $g(I_{t-k}, I_{t-k-1}, ..., R_{t-k}, R_{t-k-1}, ...) = VaR_{t-k+1}$.
  $u_t \sim$Logistic distribution. $n = 1$. Testing the nested model and $\mathrm{P}\{I_t = 1\} = e^{\alpha}/(1 + e^{\alpha}) = q$ by Wald test.

Summary of Backtesting Tests

- LB(1): i.i.d. test
- LB(5): i.i.d. test
- L(1): Testing first one lag autocorrelation = 0
- L(5): Testing first five lags autocorrelation = 0
- CaViaR-overall: all data 20060804-20110804
- CaViaR-crisis: data 20080804-20090804

Table 1: PLM CoVaR backtesting p-value.

i	LB(1)	LB(5)	L(1)	L(5)	CaViaR-overall	CaViaR-crisis
S&P500	0.0518	0.0006***	0.0999	0.0117*	2.2×10^{-16}***	0.0019**
C	0.8109	0.0251*	0.8162	0.2306	2.946×10^{-9}***	0.0535

Table 2: Linear CoVaR backtesting p-value.

i	LB(1)	LB(5)	L(1)	L(5)	CaViaR-overall	CaViaR-crisis
S&P500	0.0869	0.2059	0.2684	0.6586	8.716×10^{-7}***	0.0424*
C	0.0489*	0.2143	0.1201	0.4335	3.378×10^{-9}***	0.0001***

Table 3: VaR backtesting p-value.

i	LB(1)	LB(5)	L(1)	L(5)	CaViaR-overall	CaViaR-crisis
GS	0.3449	0.0253*	0.3931	0.1310	1.265×10^{-6}***	0.0024**

Conclusions and Further Work

- Semiparametric model may capture risk better than linear model during financial crisis
- Multivariate nonlinear part in PLM
- Other assets returns

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Macorprudential Risk Measures

- Marginal Expected Shortfall (MES): $R = \sum_i y_i R_i$, y_i: weights, R_i: asset return

$$\text{MES}_\alpha^i = \frac{\partial ES_\alpha(R)}{\partial y_i} = -\mathsf{E}\left[R_i | R \leq -VaR_\alpha\right]$$

- Distressed Insurance Premium (DIP): Huang et al. (2010) $L = \sum_{i=1}^N L_i$ total loss of a portfolio

$$DIP = \mathsf{E}^Q\left[L | L \geq L_{min}\right]$$

Advantages of CoVaR

- Cloning Property: if dividing X_i into several clones, then the value of CoVaR conditioning on the individual large firm does not differ from the one conditioning on one of the clones
- Conservative Property: CoVaR conditioning on some bad event, the value would be more conservative than VaR
- Adaptive to the changing market conditions

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