TENET: Tail-Event-Driven Network Risk

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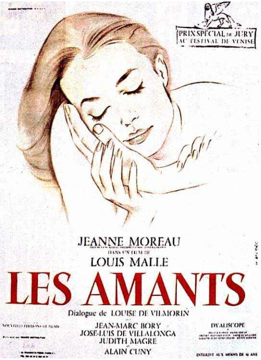
What is Systemic Risk?

"I know it when I see it".

Justice Potter Stewart, 1964.



What is Systemic Risk?

Systemic risk is a "risk of financial instability so widespread that it impairs the functioning of a financial system to the point where economic growth and welfare suffer materially".

ECB, Financial Network and Financial Stability, 2010.

"Financial institutions are **systemically important** if the failure of the firm to meet its obligations to creditors and customers would have significant adverse consequences for the financial system and the broader economy".

Daniel Tarullo, Regulatory Restructuring, 2009.

What is Systemic Risk?

Figure 1: Systemic Risk?

Bivariate CoVaR

The CoVaR of a financial institution j conditional on $X_i = \widehat{\text{VaR}}_i^{\tau}$ at $\tau \in (0,1)$ level is

$$\widehat{\text{CoVaR}}_{j|i,t}^{\tau} = \hat{\alpha}_{j|i} + \hat{\beta}_{j|i}\widehat{\text{VaR}}_{i,t}^{\tau} + \hat{\gamma}_{j|i}M_{t-1},$$
$$X_{j,t} = \alpha_{j|i} + \beta_{j|i}X_{i,t} + \gamma_{j|i}M_{t-1} + \varepsilon_{j|i,t},$$

where

- "co-" stands for "conditional, contagion, comovement" and VaR is the Value-at-Risk,
- M_{t-1} is a vector of macroprudential variables.

AB 2011 Model
Macroprudential variables

Challenges

- Linear tail behavior
  - White, Kim, and Manganelli (2010)
  - Acharya et al. (2010)
  - Brownlees and Engle (2011)
- Linear tail behavior in **high dimensions**
  - Hautsch, Schaumburg, and Schienle (2014)
- **Non-linear** tail behavior in **ultra-high dimensions**
  - Fan, Härdle, Wang, and Zhu (2013)

Non-Linearity

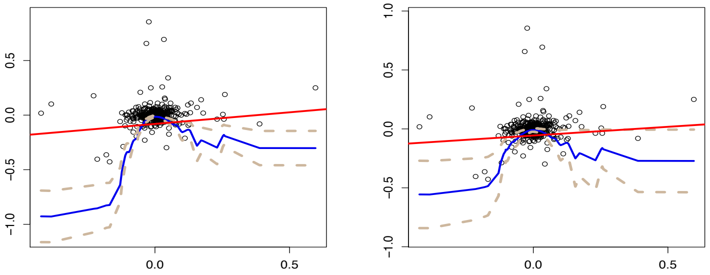


Figure 2: Bank of America (BOA) and Citi (C) weekly returns 0.05 (left) and 0.1 (right) quantile functions, y-axis = BOA returns, x-axis = C returns. Local linear quantile regression and Linear quantile regression. 95% confidence band, $T = 546$, weekly returns, 2005.01.31-2010.01.31. Chao, Härdle and Wang (2014).

Model Components

- **Non-Linearity:** Single-Index Model
- **Tail Behavior:** Generalized Quantile Regression
- **Ultra-High Dimensions:** Variable Selection

Linear Generalized Quantile Regression

Let $\{X_i, Y_i\}_{i=1}^n$ be independent r. v., $X \in \mathbb{R}^p$, $\tau \in (0, 1)$.

$$\begin{aligned} Y_i &= X_i^\top \theta + \varepsilon_i, \\ \hat{\theta} &= \operatorname*{arg\,min}_{\theta \in \mathbb{R}^p} \sum_{i=1}^n \rho_\tau(Y_i - X_i^\top \theta). \end{aligned}$$

Asymmetric loss function is

$$\rho_\tau(u) = |u|^\alpha |\mathbf{1}(u \leq 0) - \tau|,$$

where $\alpha = 1$ corresponds to quantile and $\alpha = 2$ corresponds to expectile regression.

Asymmetric Loss Functions

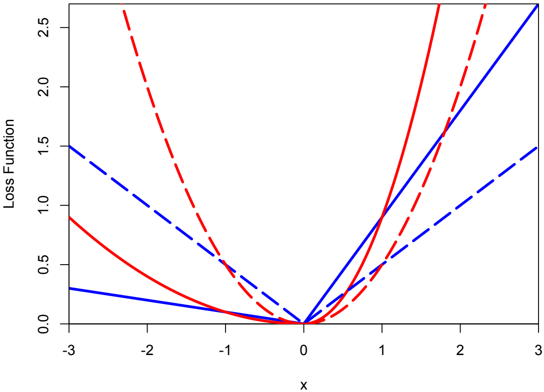


Figure 3: Asymmetric Loss Functions for Quantile and Expectile, $\tau = 0.9$: a solid line, $\tau = 0.5$: a dashed line.

Linear Quantile and Expectile Curves

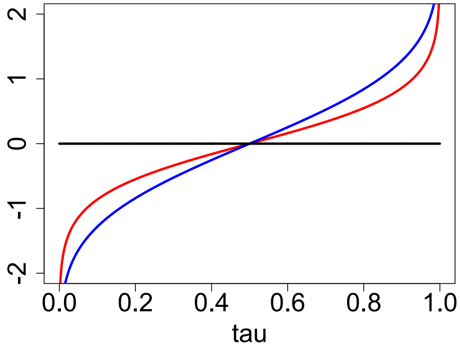


Figure 4: Quantile and Expectile for N(0, 1).

Expectile-Quantile Correspondence

Local Linear Generalized Quantile Regression

Let $\{X_i, Y_i\}_{i=1}^n$ be independent r. v. following $Y_i = l(X_i) + \varepsilon_i$, $X \in \mathbb{R}^p$, $\tau \in (0, 1)$.

$$\hat{\gamma} = \underset{\gamma \in \mathbb{R}^{p+1}}{\arg\min}\, n^{-1} \sum_{i=1}^{n} \rho_\tau\{Y_i - \gamma_0 - \gamma_1(X_i - x_0)\} K_h(X_i - x_0)$$

where $K_h(.) = K(./h)/h$ with $K(.)$ a kernel and h a bandwidth.

Härdle, Spokoiny, and Wang (2013)

Single-Index Model

Let $\{X_i, Y_i\}_{i=1}^n$ be independent r. v., $X \in \mathbb{R}^p$.

$$Y_i = g(\beta^\top X_i) + \varepsilon_i, \tag{1}$$

where

- $g(\cdot)$ is the link function,
- $\beta \in \mathbb{R}^p$ is the vector of index parameters,
- $p = \mathcal{O}(\exp(n^\alpha))$ for some $\alpha \in (0,1)$.

Estimation

Recall (1):

$$Y_i = g(\beta^\top X_i) + \varepsilon_i$$

A quasi-likelihood approach under assumption $F_{\varepsilon_i}^{-1}(\tau|X) = 0$

$$\min_{\beta \in \mathbb{R}^p} \mathsf{E}\, \rho\{Y - g(\beta^\top X)\} \tag{2}$$

Further assumptions:
$\|\beta\|_2 = 1$ and first component of β is positive.

Estimation

Taylor approximation:

$$g(\beta^\top X_i) \approx g(\beta^\top x) + g'(\beta^\top x)\beta^\top (X_i - x) \tag{3}$$

Theoretically:

$$\begin{aligned} L_x(\beta) &\stackrel{\text{def}}{=} \mathsf{E}\,\rho\{Y - g(\beta^\top x) - g'(\beta^\top x)\beta^\top (X - x)\} \\ &\quad K_h\{\beta^\top (X - x)\} \end{aligned} \tag{4}$$

Empirically:

$$\begin{aligned} L_{n,x}(\beta) &\stackrel{\text{def}}{=} n^{-1}\sum_{i=1}^{n} \rho\{Y_i - g(\beta^\top x) - g'(\beta^\top x)\beta^\top (X_i - x)\} \\ &\quad K_h\{\beta^\top (X_i - x)\} \end{aligned} \tag{5}$$

where $K_h(.) = K(./h)/h$ with $K(.)$ a kernel and h a bandwidth.

Minimum Average Contrast Estimation

$$
\begin{aligned}
L_n(\beta) &\stackrel{\text{def}}{=} n^{-1}\sum_{j=1}^{n} L_{n,X_j}(\beta) \\
&= n^{-2}\sum_{j=1}^{n}\sum_{i=1}^{n}\rho\left\{Y_i - g(\beta^\top X_j) - g'(\beta^\top X_j)\beta^\top(X_i - X_j)\right\} \\
&\quad K_h\{\beta^\top(X_i - X_j)\} \qquad (6)
\end{aligned}
$$

$$
\widehat{\beta} \approx \arg\min_{\beta} L_n(\beta) \qquad (7)
$$

Variable Selection

$$\widehat{\beta} = \underset{g,g',\beta}{\arg\min}\, n^{-1} \sum_{j=1}^{n} \sum_{i=1}^{n} \rho \left\{ Y_i - g(\beta^\top X_j) - g'(\beta^\top X_j)\beta^\top (X_i - X_j) \right\} K_h\{\beta^\top (X_i - X_j)\} + \sum_{l=1}^{p} \gamma_\lambda(\widehat{\beta}_l^{(0)})|\beta_l|$$

where

- $\widehat{\beta}^{(0)}$ is an initial estimator of β^*,
- $\gamma_\lambda(t)$ is some non-negative function.

More on Variable Selection

Numerical procedure

1. Given $\widehat{\beta}^{(t)}$, standardize $\widehat{\beta}^{(t)}$ so that $\|\widehat{\beta}^{(t)}\| = 1$, $\widehat{\beta}_1^{(t)} > 0$. Then compute

$$(\widehat{a}_j^{(t)}, \widehat{b}_j^{(t)}) \stackrel{\text{def}}{=} \arg\min_{(a_j, b_j)'\text{s}} \sum_{i=1}^{n} \rho\big(Y_i - a_j - b_j X_{ij}^\top \widehat{\beta}^{(t)}\big)\omega_{ij}(\widehat{\beta}^{(t)}),$$

where

- $\widehat{\beta}_0$ initial estimator of β^*,
- $X_{ij} = X_i - X_j$,
- $a_j = g(\beta^\top X_j)$,
- $b_j = g'(\beta^\top X_j)$,
- $\omega_{ij}(\widehat{\beta}_0^{(t)}) \stackrel{\text{def}}{=} \dfrac{K_h(X_{ij}^\top \beta_0^{(t)})}{\sum_{i=1}^n K_h(X_{ij}^\top \beta_0^{(t)})}$,
- $t = 1, 2, \ldots$ are iterations.

Numerical procedure

2. Given $(\widehat{a}_j^{(t)}, \widehat{b}_j^{(t)})$, solve

$$\widehat{\beta}^{(t+1)} = \arg\min_{\beta} n^{-1} \sum_{j=1}^{n} \sum_{i=1}^{n} \rho\big(Y_i - \widehat{a}_j^{(t)} - \widehat{b}_j^{(t)} X_{ij}^{\top} \beta\big) \omega_{ij}(\widehat{\beta}^{(t)}),$$

$$+ \textstyle\sum_{l=1}^{p} \widehat{d_l}^{(t)} |\beta_l|.$$

where

- $\widehat{d}_l^{(t)} = \gamma_\lambda(|\widehat{\beta}_l^{(t)}|)$,
- $\omega_{ij}(.)$ are from the step before.

Theory

Denote $\widehat{\beta}$ as the final estimate of β^*.

Theorem

Under A 1-5, the estimators $\widehat{\beta}^0$ *and* $\widehat{\beta}$ *exist and* $\mathrm{P}(\widehat{\beta}^0 = \widehat{\beta}) \to 1$. *Moreover,*

$$\mathrm{P}(\widehat{\beta}^0 = \widehat{\beta}) \geq 1 - (p - q)\exp(-C' n^{\alpha}). \tag{8}$$

Assumptions

Theory

Theorem

Under A 1-5, $\widehat{\beta}_{(1)} \stackrel{\text{def}}{=} (\widehat{\beta}_l)_{l \in \mathcal{M}_*}$, $b \in \mathbb{R}^q$, $\|b\| = 1$*:*

$$\|\widehat{\beta}_{(1)} - \beta^*_{(1)}\| = \mathcal{O}_p\{(\lambda D_n + n^{-1/2})\sqrt{q}\} \tag{9}$$

$$\boldsymbol{b}^\top C_{0(1)}^{-1}\sqrt{n}(\widehat{\beta}_{(1)} - \beta^*_{(1)}) \xrightarrow{\mathcal{L}} \mathrm{N}(0,\ \sigma^2) \tag{10}$$

where $\sigma^2 = \mathsf{E}[\psi(\varepsilon_i)]^2 / [\partial^2 \mathsf{E}\, \rho(\varepsilon_i)]^2$

$$\partial^2 \mathsf{E}\, \rho(\cdot) = \frac{\partial^2 \mathsf{E}\, \rho(\varepsilon_i - v)^2}{\partial v^2}\bigg|_{v=0} \tag{11}$$

Go to details

Theory

Theorem

Under A 1-5, $\mathcal{B}_n \stackrel{\text{def}}{=} \{\widehat{\beta} = \beta^*\} : \mathrm{P}(\mathcal{B}_n) \to 1$*. Let* $\mu_j \stackrel{\text{def}}{=} \int u^j K(u) du$*,* $\nu_j \stackrel{\text{def}}{=} \int u^j K^2(u) du$*,* $j = 0, 1, \ldots$*. If* $nh^3 \to \infty$ *and* $h \to 0$*, then*

$$\sqrt{nh}\sqrt{f_{Z_{(1)}}(z)/(\nu_0\sigma^2)}\left\{\widehat{g}(x^\top\widehat{\beta}) - g(x^\top\beta^*) - \frac{1}{2}h^2 g''(x^\top\beta^*)\mu_2\partial\,\mathrm{E}\,\psi(\varepsilon)\right\} \xrightarrow{\mathcal{L}} \mathrm{N}(0,\ 1),$$

and

$$\sqrt{nh^3}\sqrt{\{f_{Z_{(1)}}(z)\mu_2^2\}/(\nu_2\sigma^2)}\left\{\widehat{g}'(x^\top\widehat{\beta}) - g'(x^\top\beta^*)\right\} \xrightarrow{\mathcal{L}} \mathrm{N}(0,\ 1).$$

Go to details

Elements of Systemic Risk

- Spill-over-effects-based Network
- Single Institutions' Contribution to Systemic Risk
- Single Institutions' Exposure to System's Risk

Spill-over Effects

Step 1. Estimate linear quantile regressions and generate predicted values under assumption $F^{-1}_{\varepsilon_{j,t}}(\tau|M_{t-1}) = 0$ at $\tau \in (0,1)$:

$$
\begin{aligned}
X_{j,t} &= \alpha_j + \gamma_j^\top M_{t-1} + \varepsilon_{j,t}, \\
\widehat{VaR}^{\tau}_{j,t} &= \hat{\alpha}_j + \widehat{\gamma}_j^\top M_{t-1},
\end{aligned}
$$

where

- $X_{j,t}$ is the log-return of company j,
- M_{t-1} are macroprudential variables as in Adrian and Brunnermeier (2011).

Spill-over Effects

Step 2. Estimate single-index-model-based quantile regressions with variable selection and generate predicted values under assumption $F^{-1}_{\varepsilon_{j,t}}(\tau|X_{-j,t}) = 0$:

$$
\begin{aligned}
X_{j,t} &= g(\beta_{j|-j}^{\top} X_{-j,t}) + \varepsilon_{j,t}, \\
\widehat{\text{CoVaR}}^{\tau}_{j|-j,t} &= \sum_{l \in S_j^*} \widehat{\beta}_{j|-j,l} V_{-j,t,l},
\end{aligned}
$$

where

- $X_{-j,t} = \{M_{t-1}, X_{-j,t}\}$ and $V_{-j,t} = \{M_{t-1}, \text{VaR}_{-j,t}\}$,
- $\hat{\beta}$'s determine the spill-over effects from companies $-j$ to a company j, i. e. the network structure,
- S_j^* denotes the active set.

j's Contribution to Systemic Risk

Estimate single-index-model-based quantile regressions and generate predicted values under assumption $F^{-1}_{\varepsilon_{s,t}}(\tau|X_{-j,t}, X_{j,t}) = 0$:

$$\tau = \tau: \quad X_{s,t} = \alpha_{s|j} + \gamma^{\top}_{s|j} M^{*}_{t-1} + \beta_{s|-j^*} \widehat{VaR}^{\tau}_{-j*,t} + \beta_{s|j} \widehat{VaR}^{\tau}_{j,t} + \varepsilon_{s,t},$$

$$\tau = 0.5: \quad X_{s,t} = \alpha_{s|j} + \gamma^{\top}_{s|j} M^{**}_{t-1} + \beta_{s|-j^{**}} \widehat{VaR}^{\tau}_{-j^{**},t} + \beta_{s|j} \widehat{VaR}^{\tau}_{j,t} + \varepsilon_{s,t},$$

$$\triangle \widehat{CoVaR}^{\tau}_{s|j,t} = X_{s|j,t}(\tau = \tau) - X_{s|j,t}(\tau = 0.5)$$

- $X_{s,t}$ is log-return on financial index,
- $*$ denotes variables chosen at the quantile level $\tau = \tau$,
- $**$ denote variables chosen at the quantile level $\tau = 0.5$,
- $\triangle \widehat{CoVaR}^{\tau}_{s|j,t}$ measures the marginal contribution of company j to overall risk of the financial system.

"Exposure" CoVaR

Estimate OLS regressions to compute the sensitivity of company j's CoVaR to the shocks of the financial system:

$$
\begin{aligned}
\widehat{CoVaR}^{\tau}_{j|-j,t} &= f(\widehat{VaR}_{-j*}, M_{t-1}), \\
\widehat{CoVaR}^{\tau}_{j|-j,t} &= \beta_s X_{s,t} + \varepsilon_{s,t}, \\
\widehat{CoVaR}^{\tau}_{j|-j,t} &= \widehat{\beta}_s X_{s,t},
\end{aligned}
$$

Dataset

- Data: 200 financial companies and 7 macroprudential variables.
- Time period: January 6, 2006 - September 6, 2012, $T = 1669$.
- Frequency: daily.

▸ Firms
▸ Macroprudential variables
▸ Summary statistics

Estimation

- VaR: linear quantile regression method
- CoVaR_L: linear quantile regression method with variable selection
- CoVaR_SIM: single-index-model-based quantile regression method with variable selection

$\mathrm{CoVaR_L}$

Li and Zhu (2008)

$$\widehat{\beta}_{j|-j} = \underset{\beta_{j|-j}}{\arg\min}\, n^{-1} \sum_{j=1}^{n} \rho(X_{j,t} - \beta_{j|-j}^{\top} X_{-j|t}) + \lambda \|\theta\|_1,$$

where

- $X_{-j,t} = \{M_{t-1}, X_{-j,t}\}$,
- λ is the penalty term.

$$\widehat{\mathrm{CoVaR}}^{\tau}_{L,j|-j,t} = \sum_{l \in S_j^*} \widehat{\beta}_{j|-j,l} X_{-j,t,l},$$

where S_j^* denotes the active set.

Lambda

- Schwarz Information Criteron (SIC) (Schwarz (1978), Koenker, Ng, and Portnoy (1994))

$$\mathrm{SIC}(\lambda) = \log[n^{-1} \sum_{i=1}^{n} \rho_\tau\{Y_i - f(X_i)\}] + \frac{\log n}{2n} \mathrm{df}$$

- Generalized Cross-Validation Criterion (GACV) (Yuan (2006))

$$\mathrm{GACV}(\lambda) = \frac{\sum_{i=1}^{n} \rho_\tau\{Y_i - f(X_i)\}}{n - \mathrm{df}},$$

where df is a measure of the effective dimensionality of the fitted model.

Effective dimension

Bandwidth

Symmetrized nearest neighbor estimation implies

$$\widehat{m}_h(X_0) = (nh)^{-1} \sum_{i=1}^{n} Y_i K_h\{F_n(X_i) - F_n(x_0)\}$$

where

- $\widehat{m}(x)$ denotes an estimator of the regression function,
- h is some bandwidth tending to zero.

Härdle and Carroll (1989)

VaR, CoVaR_L and CoVaR_SIM

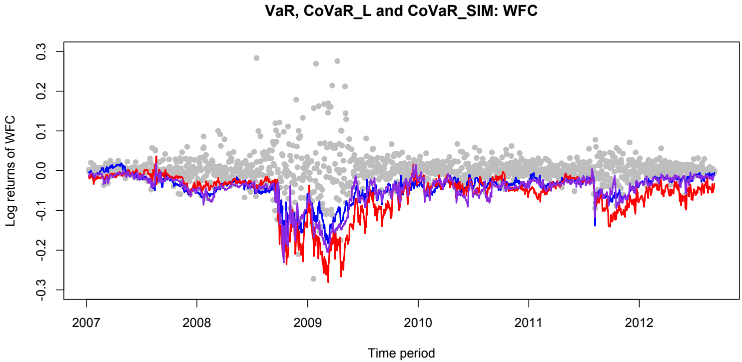


Figure 5: Log returns of WFC and the estimated CoVaR_SIM, CoVaR_L, VaR (violet), $\tau = 0.05$, $T = 1669$, window size $n = 250$.

$\mathrm{CoVaR_L}$: Lambda

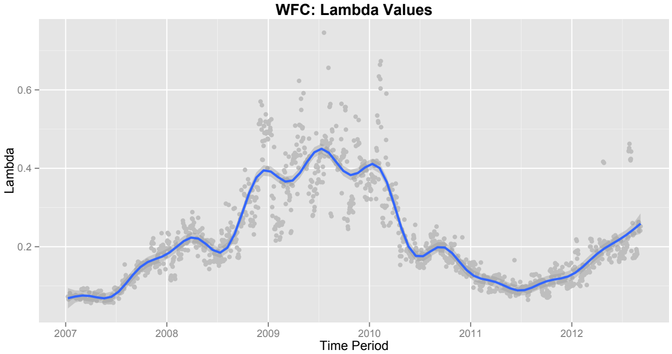


Figure 6: Lambda Values for WFC, $\tau = 0.05$, $T = 1417$, window size $n = 250$.

Smoothing

CoVaR_L: Number of selected variables

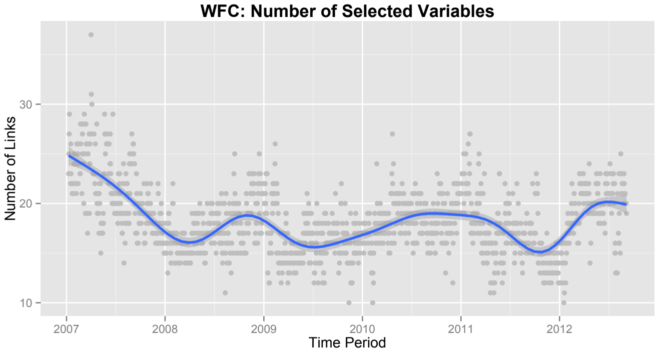


Figure 7: Number of Selected Variables for WFC, $\tau = 0.05$, $T = 1417$, window size $n = 250$.

CoVaR$_L$: Influential variables

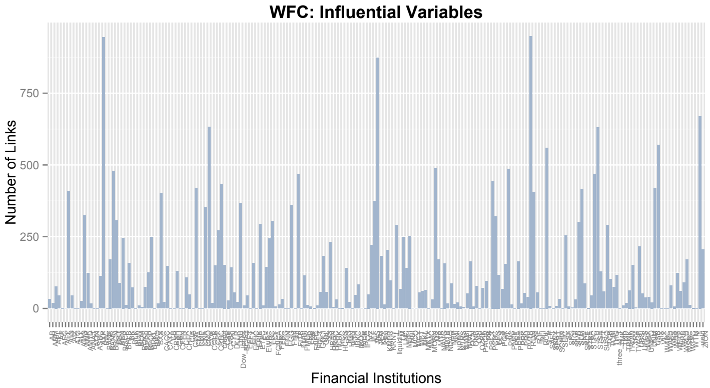


Figure 8: Influential Variables for WFC, $\tau = 0.05$, $T = 1417$, window size $n = 250$.

$\text{CoVaR}_{\text{SIM}}$: Lambda

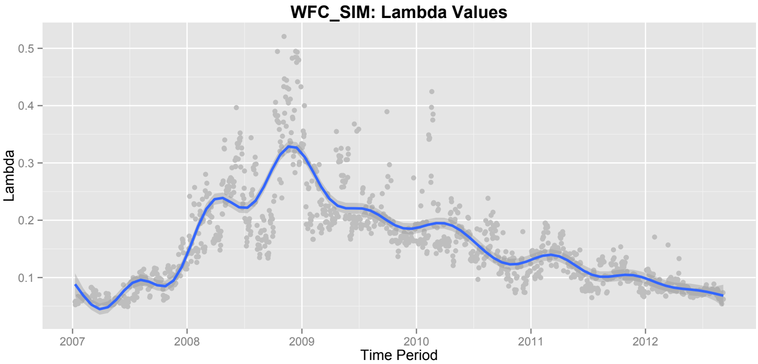


Figure 9: Lambda Values for WFC_{SIM}, $\tau = 0.05$, $T = 1417$, window size $n = 250$.

$\text{CoVaR}_{\text{SIM}}$: Number of selected variables

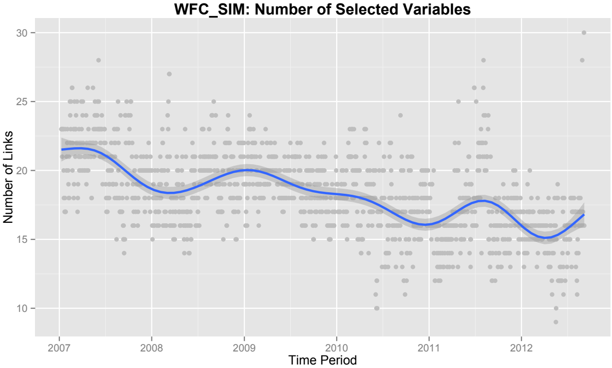


Figure 10: Number of Selected Variables for WFC_{SIM}, $\tau = 0.05$, $T = 1417$, window size $n = 250$.

$\text{CoVaR}_{\text{SIM}}$: Influential variables

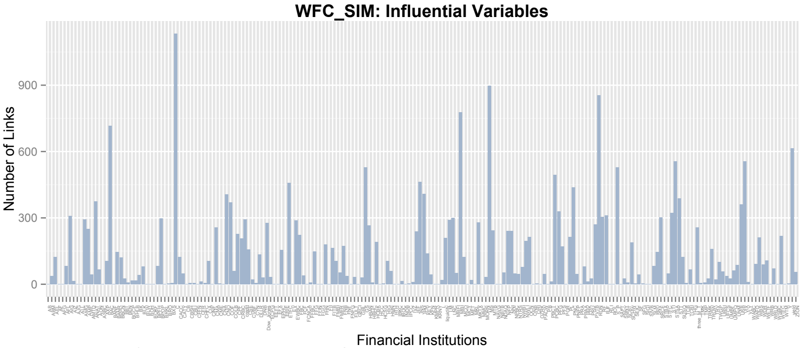


Figure 11: Influential Variables for WFC_{SIM}, $\tau = 0.05$, $T = 1417$, window size $n = 250$.

$\mathrm{CoVaR}_{\mathrm{SIM}}$: Link Function

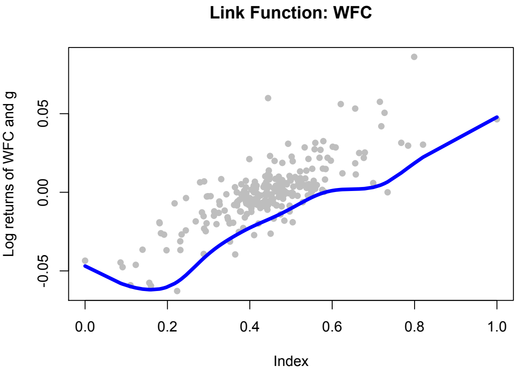


Figure 12: Example of a Link Function for $\mathrm{WFC}_{\mathrm{SIM}}$, $\tau = 0.05$, $T = 1417$, window size $n = 250$.

CoVaR_L – Network dynamics

Figure 13: Network Dynamics, positive and negative Links for 2007.01.11-2009.01.09, $\tau = 0.05$.

Network Analysis

Let $D_t = [d_{ij,t}]$ be a $p \times p$ matrix, where d_{ij} represents a link from i to j at time t.

- Total Connectedness (at time t)

$$TC_t = \sum_{i=1}^{p} |\beta_{ij,t}|$$

- Total IN Links

$$C_{i \leftarrow \bullet} = \sum_{i=1, i \neq j}^{p} d_{ij,t}$$

- Total OUT Links

$$C_{\bullet \leftarrow j} = \sum_{j=1, i \neq j}^{p} d_{ij,t}$$

Network Analysis

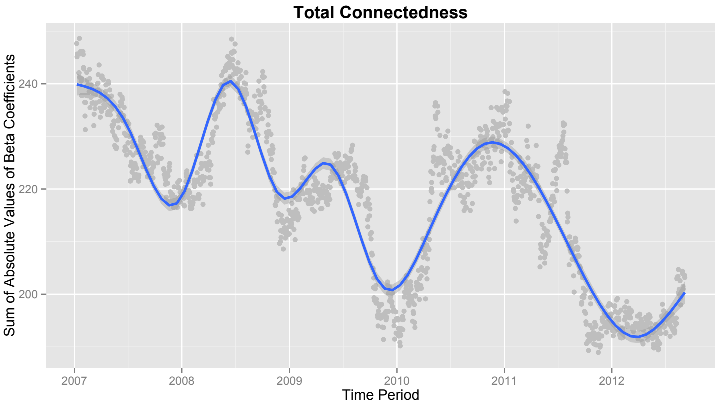


Figure 14: Total Connectedness (in absolute values), T = 1417, $\tau = 0.05$, window size n=250, a smoothed curve and original data.

Average Lambda

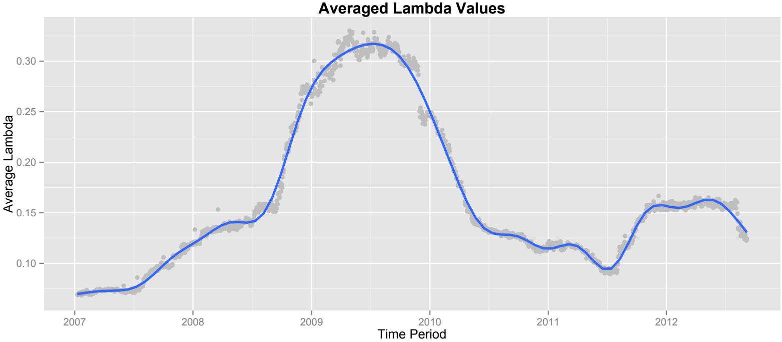


Figure 15: Averaged Lambda, T = 1417, $\tau = 0.05$, window size n=250, a smoothed curve and original data.

Network Analysis

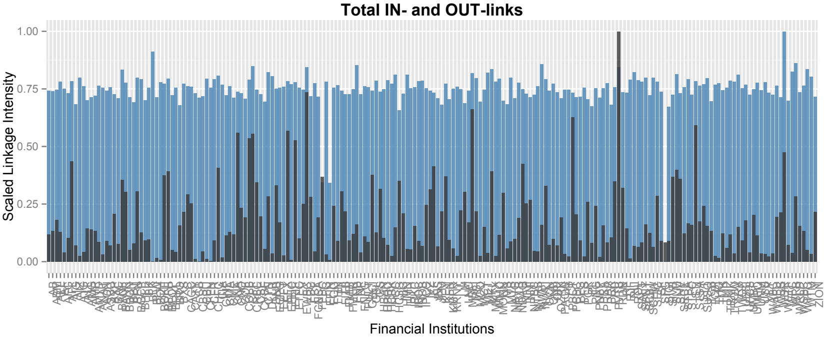


Figure 16: Total IN and OUT Links, T = 1417, $\tau = 0.05$, window size n=250.

▸ IN and OUT Links in Absolute Values

Network Analysis

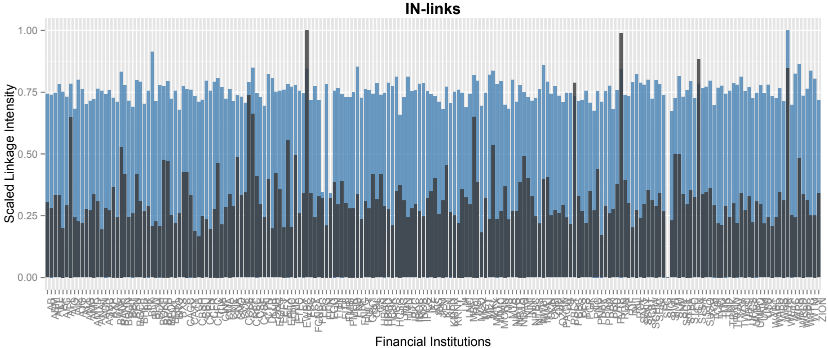


Figure 17: Total Number of IN Links scaled on $[0, 1]$, T $= 1417$, $\tau = 0.05$, window size n=250, number of links and sum of absolute values of beta coefficients.

Network Analysis

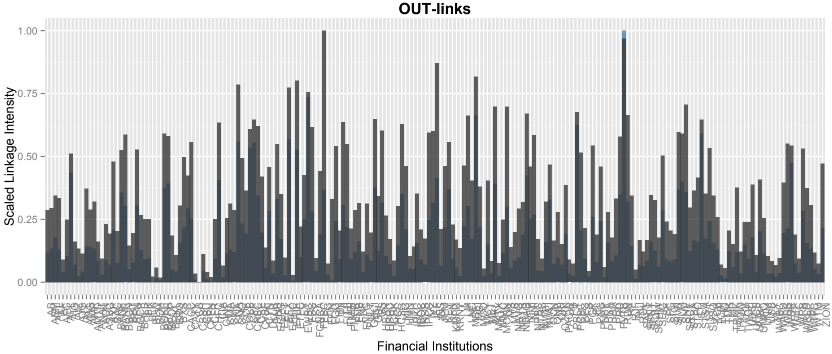


Figure 18: Total Number of OUT Links scaled on $[0,1]$, T = 1417, $\tau = 0.05$, window size n=250, number of links and sum of absolute values of beta coefficients.

Network Analysis

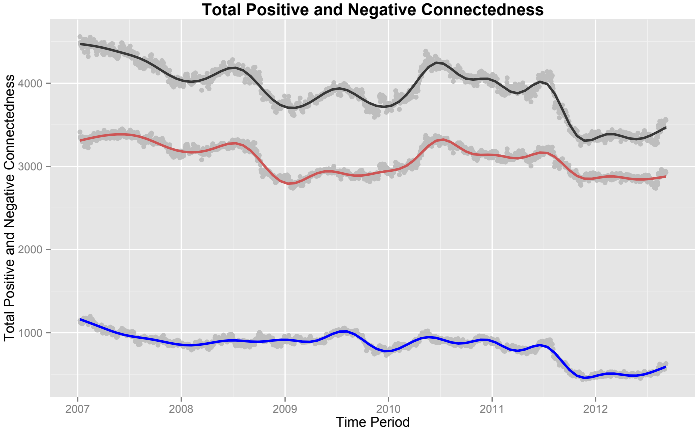


Figure 19: Total Positive and Negative Connectedness, $T = 1417$, $\tau = 0.05$, window size n=250, smoothed curves and original data.

Conclusions and Outlook

- Performance of linear vs. non-linear networks?
- What are underlying network drivers?
- Non-stationarity?
- Interdependencies between financial sector and other sectors in the economy?

TENET: Tail Event driven NETwork risk

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Adrian and Brunnermeier (2011)

Step 1. Estimate linear quantile regressions

$$X_{i,t} = \alpha_i + \gamma_i M_{t-1} + \varepsilon_{i,t},$$
$$X_{system,t} = \alpha_{system|i} + \beta_{system|i} X_{i,t} + \gamma_{system|i} M_{t-1} + \varepsilon_{system|i,t},$$

where

- $X_{i,t}$ is the log return of institution i,
- $X_{system,t}$ is the log return of the financial system, e.g. market index,
- M_{t-1} are lagged macroprudential variables.

Adrian and Brunnermeier (2011)

Step 2. Generate predicted values under assumption $F^{-1}_{\varepsilon_{i,t}}(\tau|M_{t-1}) = 0$ and $F^{-1}_{\varepsilon_{system|i,t}}(\tau|M_{t-1}, X_{i,t}) = 0$

$$\widehat{\text{VaR}}^{\tau}_{i,t} = \hat{\alpha}_i + \hat{\gamma}_{j|i} M_{t-1},$$
$$\widehat{\text{CoVaR}}^{\tau}_{j|i,t} = \hat{\alpha}_{j|i} + \hat{\beta}_{j|i} \widehat{\text{VaR}}^{\tau}_{i,t} + \hat{\gamma}_{j|i} M_{t-1},$$
$$\triangle\, \widehat{\text{CoVaR}}^{\tau}_{j|i,t} = \widehat{\text{CoVaR}}^{\tau}_{j|X_i=\widehat{\text{VaR}}^{\tau}_{i,t}} - \widehat{\text{CoVaR}}^{\tau}_{j|X_i=\widehat{\text{VaR}}^{0.5}_{i,t}} .$$

Return

The macroprudential variables

1. VIX
2. Short term liquidity spread (liquidity)
3. Daily change in the 3-month Treasury maturities (3MT)
4. Change in the slope of the yield curve (yield)
5. Change in the credit spread (credit)
6. Daily Dow Jones U.S. Real Estate index returns (D_J)
7. S&P500 returns (S&P)

Source: AB (2011), Datastream.

Return

The penalty term

- Lasso, Tibshirani (1996): $\gamma_\lambda(x) = \lambda$
- SCAD, Fan and Li (2001):

$$\gamma_\lambda(x) = \lambda\{\mathbf{I}(x \le \lambda) + \frac{(a\lambda - x)_+}{(a-1)\lambda}\mathbf{I}(x > \lambda)\},$$

- The adaptive Lasso, Zou (2006): $\gamma_\lambda(x) = \lambda|x|^{-a}$ for some $a > 0$.

Return

Assumptions

A1 K a cts symmetric pdf, $g(\cdot) \in C^2$.

A2 $\rho(x)$ convex. Suppose $\psi(x)$, subgradient of $\rho(x)$:
i) Lipschitz continuous; ii) $\mathsf{E}\,\psi(\varepsilon_i) = 0$ and
$\inf_{|v| \le c} \partial\, \mathsf{E}\,\psi(\varepsilon_i - v) = C_1$.

A3 ε_i is independent of X_i. Let $Z_i = X_i^\top \beta^*$ and $Z_{ij} = Z_i - Z_j$.
$C_{0(1)} \stackrel{\text{def}}{=} \mathsf{E}\{g'(Z_i)^2(\mathsf{E}(X_{i(1)}|Z_i) - X_{i(1)})(\mathsf{E}(X_{i(1)}|Z_i - X_{i(1)}))\}^\top\}$,
and the matrix $C_{0(1)}$ satisfies
$0 < L_1 \le \lambda_{\min}(C_{0(1)}) \le \lambda_{\max}(C_{0(1)}) \le L_2$ for positive constants L_1 and L_2. There exists a constant $c_0 > 0$ such that $\sum_{i=1}^n \{\|X_{i(1)}\|/\sqrt{n}\}^{2+c_0} \xrightarrow{p} 0$, with $0 < c_0 < 1$. Also
$\|\sum_i \sum_j X_{(0)ij}\omega_{ij}X_{(1)ij}^\top \partial\, \mathsf{E}\,\psi(v_{ij})\|_{2,\infty} = \mathcal{O}_p(n^{1-\alpha_1})$.

Assumptions

A4 The penalty parameter λ is chosen such that $\lambda D_n = \mathcal{O}\{n^{-1/2}\}$, with $D_n \stackrel{\text{def}}{=} \max\{d_l : l \in \mathcal{M}_*\} = \mathcal{O}(n^{\alpha_1 - \alpha_2/2})$, $d_l \stackrel{\text{def}}{=} \gamma_\lambda(|\beta_l^*|)$, $\mathcal{M}_* = \{l : \beta_l^* \neq 0\}$ be the true model. Furthermore assume $qh \to 0$ as $n \to \infty$, $q = \mathcal{O}(n^{\alpha_2})$, $p = \mathcal{O}(\exp\{n^\delta\})$, $nh^3 \to \infty$ and $h \to 0$. Also, $0 < \delta < \alpha < \alpha_2/2 < 1/2$, $\alpha_2/2 < \alpha_1 < 1$. For example, $\delta = 1/5$, $\alpha = 1/4$, $\alpha_2 = 3/5$, $\alpha_1 = 3/5$.

A5 The error term ε_i satisfies $\mathsf{E}\,\varepsilon_i = 0$ and $\mathrm{Var}(\varepsilon_i) < \infty$. Assume that $\mathsf{E}\left|\psi^m(\varepsilon_i)/m!\right| \leq s_0 c^m$ where s_0 and c are constants.

Return

Subgradient

If $f : U \to \mathbb{R}$ is a real-valued convex function defined on a convex open set in the Euclidean space $\mathbb{R}^n$, a vector v in that space is called a subgradient at a point x_0 in U if for any x in U one has

$$f(x) - f(x_0) \geq v \cdot (x - x_0)$$

where the dot denotes the dot product.

Return

Matrix norm

Assume A is a $m \times n$ matrix

$$\|A\|_{\alpha,\beta} = \max_{x \neq 0} \frac{\|Ax\|_\beta}{\|x\|_\alpha}$$

Return

Sparsistency

The result of (8) is stronger than the oracle property defined in Fan and Li (2001) once the properties of $\widehat{\beta}^0$ are established. It was formulated by Kim et al. (2008) for the SCAD estimator with polynomial dimensionality p. It implies not only the model selection consistency and but also sign consistency (Zhao and Yu, 2006; Bickel et al., 2008, 2009):

$$P\{sgn(\widehat{\beta}) = sgn(\beta^*)\} = P\{sgn(\widehat{\beta}^0) = sgn(\beta^*)\} \to 1$$

Return

The confidence interval

The $100(1-\alpha)\%$ confidence interval:

$$\Big[\widehat{g}(z) - \frac{1}{\sqrt{nh}} \cdot \frac{\sigma\sqrt{\nu_0}}{\sqrt{\widehat{f}_{Z_{(1)}}(z)}} \cdot \mathfrak{z}_\alpha + \frac{1}{2}h^2\widehat{g}''(z)\mu_2\partial\widehat{\mathsf{E}}\psi(\varepsilon);$$

$$\widehat{g}(z) + \frac{1}{\sqrt{nh}} \cdot \frac{\sigma\sqrt{\nu_0}}{\sqrt{\widehat{f}_{Z_{(1)}}(z)}} \cdot \mathfrak{z}_\alpha + \frac{1}{2}h^2\widehat{g}''(z)\mu_2\partial\widehat{\mathsf{E}}\psi(\varepsilon)\Big]$$

where $\mathfrak{z}_\alpha$ is the α-Quantile of the standard normal distribution, and

$\widehat{f}_{Z_{(1)}}(z) = n^{-1}\sum_{i=1}^{n} K_h(z - Z_{i(1)})$, where $Z_{i(1)} = X_{i(1)}^\top \widehat{\beta}_{(1)}$.

Return

Expectile-Quantile Correspondence

Let $v(x)$ represents expectile regression, $l(x)$ represents quantile regression.
Fixed x, define $w(\tau)$ such that $v_{w(\tau)}(x) = l(x)$ then $w(\tau)$ is related to $l(x)$ via

$$w(\tau) = \frac{\tau l(x) - \int_{-\infty}^{l(x)} ydF(y|x)}{2\,\mathrm{E}(Y|x) - 2\int_{-\infty}^{l(x)} ydF(y|x) - (1-2\tau)l(x)}$$

For example, $Y \sim U(-1,1)$, then $w(\tau) = \tau^2/(2\tau^2 - 2\tau + 1)$
Expectile corresponds to quantile with transformation w.

▸ Return to 2-1 ▸ Return to 3-11

The financial firms and macroprudential variables

The financial frims:	
1. Wells Fargo & Co (WFC)	15. Franklin Resources Inc. (BEN)
2. JP Morgan Chase & Co (JPM)	16. The Travelers Companies, Inc. (TRV)
3. Bank of America Corp (BAC)	17. AFLAC Inc. (AFL)
4. Citigroup Inc (C)	18. Prudential Financial, Inc. (PRU)
5. American Express Company (AXP)	19. State Street Corporation (STT)
6. U.S. Bancorp (USB)	20. The Chubb Corporation (CB)
7. The Goldman Sachs Group, Inc. (GS)	21. BB&T Corporation (BBT)
8. American International Group, Inc. (AIG)	22. Marsh & McLennan Companies, Inc. (MMC)
9. MetLife, Inc. (MET)	23. The Allstate Corporation (ALL)
10. Capital One Financial Corp. (COF)	24. Aon plc (AON)
11. BlackRock, Inc. (BLK)	25. CME Group Inc. (CME)
12. Morgan Stanley (MS)	26. The Charles Schwab Corporation (SCHW)
13. PNC Financial Services Group Inc. (PNC)	27. T. Rowe Price Group, Inc. (TROW)
14. The Bank of New York Mellon Corporation (BK)	28. Loews Corporation (L)

The financial firms and macroprudential variables

29. SunTrust Banks, Inc. (STI)	44. Lincoln National Corporation (LNC)
30. Fifth Third Bancorp (FITB)	45. Affiliated Managers Group Inc. (AMG)
31. Progressive Corp. (PGR)	46. Cincinnati Financial Corp. (CINF)
32. M&T Bank Corporation (MTB)	47. Equifax Inc. (EFX)
33. Ameriprise Financial Inc. (AMP)	48. Alleghany Corp. (Y)
34. Northern Trust Corporation (NTRS)	49. Unum Group (UNM)
35. Invesco Ltd. (IVZ)	50. Comerica Incorporated (CMA)
36. Moody's Corp. (MCO)	51. W.R. Berkley Corporation (WRB)
37. Regions Financial Corp. (RF)	52. Fidelity National Financial, Inc. (FNF)
38. The Hartford Financial Services Group, Inc. (HIG)	53. Huntington Bancshares Incorporated (HBAN)
39. TD Ameritrade Holding Corporation (AMTD)	54. Raymond James Financial Inc. (RJF)
40. Principal Financial Group Inc. (PFG)	55. Torchmark Corp. (TMK)
41. SLM Corporation (SLM)	56. Markel Corp. (MKL)
42. KeyCorp (KEY)	57. Ocwen Financial Corp. (OCN)
43. CNA Financial Corporation (CNA)	58. Arthur J Gallagher & Co. (AJG)

The financial firms and macroprudential variables

59. Hudson City Bancorp, Inc. (HCBK)	74. Commerce Bancshares, Inc. (CBSH)
60. People's United Financial Inc. (PBCT)	75. Signature Bank (SBNY)
61. SEI Investments Co. (SEIC)	76. Jefferies Group, Inc. (JEF)
62. Nasdaq OMX Group Inc. (NDAQ)	77. Rollins Inc. (ROL)
63. Brown & Brown Inc. (BRO)	78. Morningstar Inc. (MORN)
64. BOK Financial Corporation (BOKF)	79. East West Bancorp, Inc. (EWBC)
65. Zions Bancorp. (ZION)	80. Waddell & Reed Financial Inc. (WDR)
66. HCC Insurance Holdings Inc. (HCC)	81. Old Republic International Corporation (ORI)
67. Eaton Vance Corp. (EV)	82. ProAssurance Corporation (PRA)
68. Erie Indemnity Company (ERIE)	83. Assurant Inc. (AIZ)
69. American Financial Group Inc. (AFG)	84. Hancock Holding Company (HBHC)
70. Dun & Bradstreet Corp. (DNB)	85. First Niagara Financial Group Inc. (FNFG)
71. White Mountains Insurance Group, Ltd. (WTM)	86. SVB Financial Group (SIVB)
72. Cullen-Frost Bankers, Inc. (CFR)	87. First Horizon National Corporation (FHN)
73. Legg Mason Inc. (LM)	88. E-TRADE Financial Corporation (ETFC)

The financial firms and macroprudential variables

89. SunTrust Banks, Inc. (STI)	104. Valley National Bancorp (VLY)
90. Mercury General Corporation (MCY)	105. KKR Financial Holdings LLC (KFN)
91. Associated Banc-Corp (ASBC)	106. Synovus Financial Corporation (SNV)
92. Credit Acceptance Corp. (CACC)	107. Texas Capital BancShares Inc. (TCBI)
93. Protective Life Corporation (PL)	108. American National Insurance Co. (ANAT)
94. Federated Investors, Inc. (FII)	109. Washington Federal Inc. (WAFD)
95. CNO Financial Group, Inc. (CNO)	110. First Citizens Bancshares Inc. (FCNCA)
96. Popular, Inc. (BPOP)	111. Kemper Corporation (KMPR)
97. Bank of Hawaii Corporation (BOH)	112. UMB Financial Corporation (UMBF)
98. Fulton Financial Corporation (FULT)	113. Stifel Financial Corp. (SF)
99. AllianceBernstein Holding L.P. (AB)	114. CapitalSource Inc. (CSE)
100. TCF Financial Corporation (TCB)	115. Portfolio Recovery Associates Inc. (PRAA)
101. Susquehanna Bancshares, Inc. (SUSQ)	116. Janus Capital Group, Inc. (JNS)
102. Capitol Federal Financial, Inc. (CFFN)	117. MBIA Inc. (MBI)
103. Webster Financial Corp. (WBS)	118. Healthcare Services Group Inc. (HCSG)

The financial firms and macroprudential variables

119. The Hanover Insurance Group Inc. (THG)	134. BancorpSouth, Inc. (BXS)
120. F.N.B. Corporation (FNB)	135. Privatebancorp Inc. (PVTB)
121. FirstMerit Corporation (FMER)	136. United Bankshares Inc. (UBSI)
122. FirstMerit Corporation (FMER)	137. Old National Bancorp. (ONB)
123. RLI Corp. (RLI)	138. International Bancshares Corporation (IBOC)
124. StanCorp Financial Group Inc. (SFG)	139. First Financial Bankshares Inc. (FFIN)
125. Trustmark Corporation (TRMK)	140. Westamerica Bancorp. (WABC)
126. IberiaBank Corp. (IBKC)	141. Northwest Bancshares, Inc. (NWBI)
127. Cathay General Bancorp (CATY)	142. Bank of the Ozarks, Inc. (OZRK)
128. National Penn Bancshares Inc. (NPBC)	143. Huntington Bancshares Incorporated (HBAN)
129. Nelnet, Inc. (NNI)	144. Euronet Worldwide Inc. (EEFT)
130. Wintrust Financial Corporation (WTFC)	145. Community Bank System Inc. (CBU)
131. Umpqua Holdings Corporation (UMPQ)	146. CVB Financial Corp. (CVBF)
132. GAMCO Investors, Inc. (GBL)	147. MB Financial Inc. (MBFI)
133. Sterling Financial Corp. (STSA)	148. ABM Industries Incorporated (ABM)

The financial firms and macroprudential variables

149. Glacier Bancorp Inc. (GBCI)	164. Citizens Republic Bancorp, Inc (CRBC)
150. Selective Insurance Group Inc. (SIGI)	165. Horace Mann Educators Corp. (HMN)
151. Park National Corp. (PRK)	166. DFC Global Corp. (DLLR)
152. Flagstar Bancorp Inc. (FBC)	167. Navigators Group Inc. (NAVG)
153. FBL Financial Group Inc. (FFG)	168. Boston Private Financial Holdings, Inc. (BPFH)
154. Astoria Financial Corporation (AF)	169. American Equity Investment Life Holding Co. (AEL)
155. World Acceptance Corp. (WRLD)	170. BlackRock Limited Duration Income Trust (BLW)
156. First Midwest Bancorp Inc. (FMBI)	171. Columbia Banking System Inc. (COLB)
157. PacWest Bancorp (PACW))	172. Safety Insurance Group Inc. (SAFT)
158. First Financial Bancorp. (FFBC)	173. National Financial Partners Corp. (NFP)
159. BBCN Bancorp, Inc. (BBCN)	174. NBT Bancorp, Inc. (NBTB)
160. Provident Financial Services, Inc. (PFS)	175. Tower Group Inc. (TWGP)
161. FBL Financial Group Inc. (FFG)	176. Encore Capital Group, Inc. (ECPG)
162. WisdomTree Investments, Inc. (WETF)	177. Pinnacle Financial Partners Inc. (PNFP)
163. Hilltop Holdings Inc. (HTH)	178. First Commonwealth Financial Corp. (FCF)

The financial firms and macroprudential variables

179. BancFirst Corporation (BANF)	190. Berkshire Hills Bancorp Inc. (BHLB)
180. Independent Bank Corp. (INDB)	191. Brookline Bancorp, Inc. (BRKL)
181. Infinity Property and Casualty Corp. (IPCC)	192. National Western Life Insurance Company (NWLI)
182. Central Pacific Financial Corp. (CPF)	193. Tompkins Financial Corporation (TMP)
183. Kearny Financial Corp. (KRNY)	194. BGC Partners, Inc. (BGCP)
184. Chemical Financial Corporation (CHFC)	195. Epoch Investment Partners, Inc. (EPHC)
185. Banner Corporation (BANR)	196. United Fire Group, Inc (UFCS)
186. State Auto Financial Corp. (STFC)	197. 1st Source Corporation (SRCE)
187. Radian Group Inc. (RDN)	198. Citizens Inc. (CIA)
188. SCBT Financial Corporation (SCBT)	199. S&T Bancorp Inc. (STBA)
189. WesBanco Inc. (WSBC)	

The financial firms and macroprudential variables

The macroprudential variables:
200. VIX
201. Short term liquidity spread (liquidity)
202. Daily change in the 3-month Treasury maturities (3MT)
203. Change in the slope of the yield curve (yield)
204. Change in the credit spread (credit)
205. Daily Dow Jones U.S. Real Estate index returns (D_J)
206. S&P500 returns (S&P)

Return

The macroprudential variables

1. VIX
2. Short term liquidity spread (liquidity)
3. Daily change in the 3-month Treasury maturities (3MT)
4. Change in the slope of the yield curve (yield)
5. Change in the credit spread (credit)
6. Daily Dow Jones U.S. Real Estate index returns (D_J)
7. S&P500 returns (S&P)

Source: Adrian and Brunnermeier (2011), Datastream.

Return

Effective dimension

Let $\{X_i, Y_i\}_{i=1}^n$ be independent r. v.
Given X, let $Y_i \sim (\mu(X), \sigma^2)$, where $\mu(X)$ is the true mean and σ^2 is the common variance.

$$\mathrm{df}(\hat{f}) = \sum_{i=1}^{n} \frac{\mathrm{Cov}\{\hat{f}(X_i), Y_i\}}{\sigma^2}.$$

Under certain mild conditions an unbiased estimator of df is

$$\mathrm{df}(\hat{f}) = \sum_{i=1}^{n} \frac{\partial \hat{f}(X_i)}{\partial Y_i}$$

Stein (1981)

Return

Smoothing

Let $\{X_i, Y_i\}_{i=1}^{n}$ be independent r. v., $Y_i = f(X_i)$.

$$\hat{f} = \min \sum_{i=1}^{n} \{Y_i - \hat{f}(X_i)\}^2 + \lambda \int_{x_1}^{x_n} \{\hat{f}''(x)\}^2 \partial x$$

Example of a cubic spline

$$f(X) = a_i(X - X_i)^3 + b_i(X - X_i)^2 + c_i(X - X_i) + d_i$$

Given $m_1 < x_1 < x_2 < ... < x_n < m_2$

- on each interval $(m_1, x_1), (x_1, x_2), \ldots, (x_n, m_2)$, f is a cubic polynomial,
- the polynomial pieces connect at points $x_1, \ldots, x_n$ called knots so that f' and f'' are continuous on $[m_1, m_2]$.

Return

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