CDO Surfaces Dynamics

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iTraxx over Time

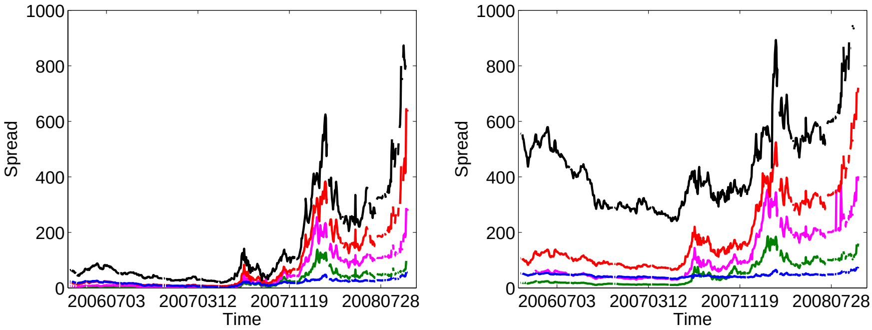


Figure 1: Spreads of iTraxx tranches, Series 5, maturity 5 (left) and 10 (right) years, data from 20060407-20081103. Tranches: 1, **2**, 3, 4, 5.

iTraxx Spread Surface

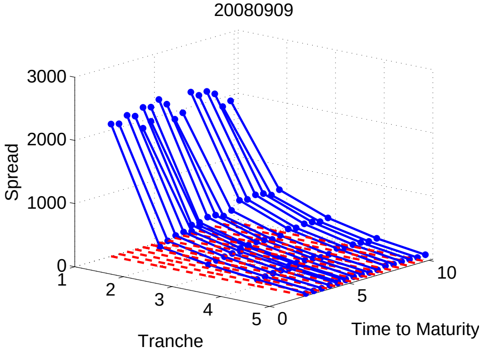


Figure 2: Spreads of tranches of all series observed on 20080909.

Research Goals

- Modelling the dynamics of CDO surfaces
- Applications in trading

Dynamic Semiparametric Factor Model

Applications:

1. Implied volatility surfaces in M. R. Fengler, W. Härdle and E. Mammen, *JFE* (2007) and B. Park, E. Mammen, W. Härdle, and S. Borak, *JASA* (2009)
2. Risk neutral densities in E. Giacomini, W. Härdle, and V. Krätschmer, *AStA* (2009)
3. Limit order book in W. Härdle, N., Hautsch, and A. Mihoci, *JEF* (2012)
4. Variance swaps in K. Detlefsen and W. Härdle, *QF* (2013)
5. fMRI images in A. Myšicková, S. Song, P. Majer, P. Mohr, H. Heekeren, W. Härdle, *Psychometrika* (2013)

Outline

1. Motivation ✓
2. CDOs
3. DSFM
4. Empirical Study
5. Applications
6. Outlook

CDOs

Use:

- Large Homogeneous Gaussian Copula Model
- Base correlations
- iTraxx Europe

Base Correlations over Time

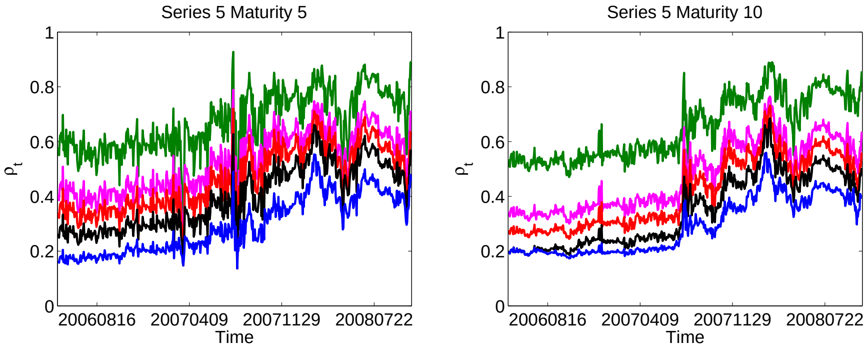


Figure 3: BC of iTraxx tranches, Series 5, maturity 5 (left) and 10 (right) years, data from 20060510-20081023. Tranches: 1, **2**, 3, 4, 5.

Base Correlation Surfaces

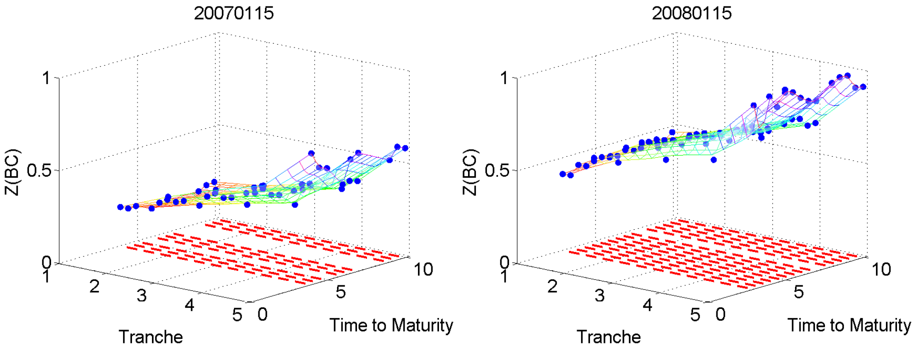


Figure 4: Z-transformed implied BC on day 20070115 (left) and 20080115 (right).

Dynamic Semiparametric Factor Model

$$Y_{t,j} = m_0(X_{t,j}) + \sum_{l=1}^{L} Z_{t,l} m_l(X_{t,j}) + \varepsilon_{t,j} = Z_t^\top A\psi(X_{t,j}) + \varepsilon_{t,j}$$

$Y_{t,j}$	log-spreads and Z-transformed BC on day t, $t = 1, \ldots, T$
j	intra-day numbering of BCs on day t, $j = 1, \ldots, J_t$
$X_{t,j}$	two-dimensional $(d = 2)$ vector of the tranche seniority and the time-to-maturity
m_l	factor functions, time invariant, nonparametric estimation
$Z_{t,l}$	time series, $l = 0, \ldots, L$, dynamic behavior
$\psi(X_{t,j})$	tensor B-spline basis
A	coefficient matrix

Estimation

$$(\widehat{Z}_t, \widehat{A}) = \arg \min_{Z_t, A} \sum_{t=1}^{T} \sum_{j=1}^{J_t} \left\{ Y_{t,j} - Z_t^\top A \psi(X_{t,j}) \right\}^2$$

Selection of L, K_1, K_2 and p_1, p_2 by explained variance criterion:

$$EV(L, K_2, p_2) = 1 - \frac{\sum_{t=1}^{T} \sum_{j=1}^{J_t} \left\{ Y_{t,j} - \sum_{l=1}^{L} Z_{t,l} m_l(X_{t,j}) \right\}^2}{\sum_{t=1}^{T} \sum_{j=1}^{J_t} \left\{ Y_{t,j} - \widetilde{m}_0(X_{t,j}) \right\}^2},$$

where $\widetilde{m}_0$ is an empirical mean surface.

DSFM without Mean

Extraction of the mean guarantees the surface monotonicity.

$$Y_{t,j} = \widetilde{m}_0(X_{t,j}) + \sum_{l=1}^{L} Z_{t,l} m_l(X_{t,j}) + \varepsilon_{t,j} = \widetilde{m}_0(X_{t,j}) + Z_t^\top A\psi(X_{t,j}) + \varepsilon_{t,j}$$

where

$\widetilde{m}_0$	empirical mean
m_l	factor functions, $l = 1, \ldots, L$

Data

- Series 1-10
- Maturities 5, 7, 10 years
- Time 20050921-20090202, 879 days.

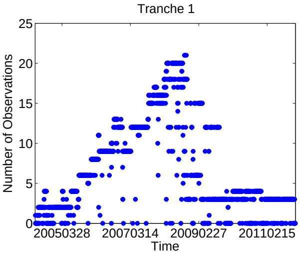
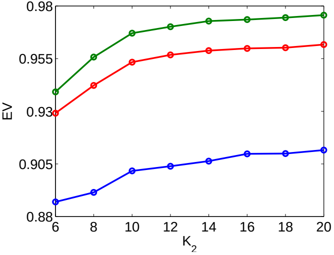
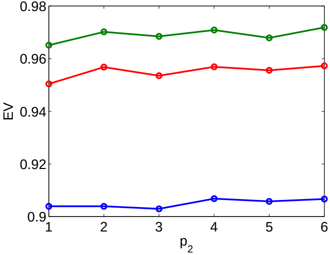


Figure 5: Number of observations of iTraxx tranche 1.

DSFM for BC





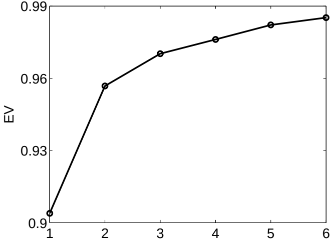


Figure 6: Proportion of the explained variance as a function of K_2 (up left) with $p_2 = 2$, as a function of p_2 (up right) with $K_2 = 12$, as a function of L (down) for $L = 1$, $L = 2$, $L = 3$, $p_1 = 1$ and $K_1 = 5$.

Models Considered

1. DSFM for Log-Spreads
2. DSFM without Mean for Log-Spreads
3. DSFM for Z-transformed-BC
4. DSFM without Mean for Z-transformed-B

DSFM for Log-Spreads

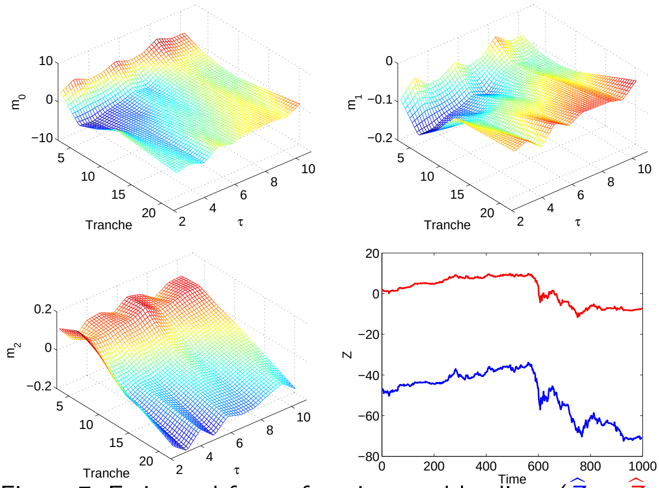


Figure 7: Estimated factor functions and loadings ($\widehat{Z}_{t,1}$, $\widehat{Z}_{t,2}$).

DSFM without Mean for Log-Spreads

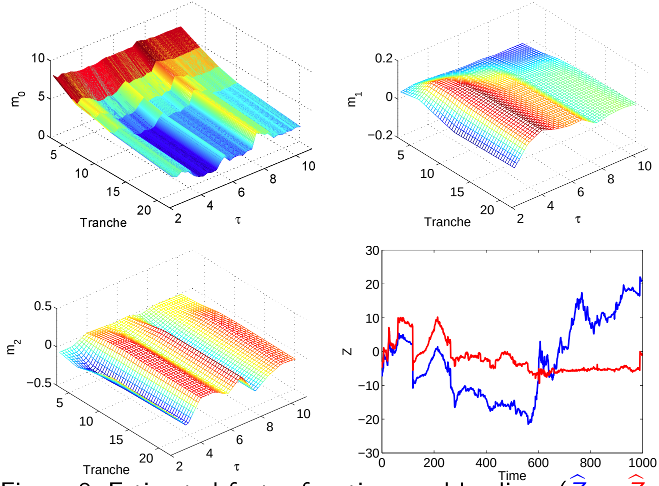


Figure 8: Estimated factor functions and loadings ($\widehat{Z}_{t,1}$, $\widehat{Z}_{t,2}$).

DSFM for Z-transformed-BC

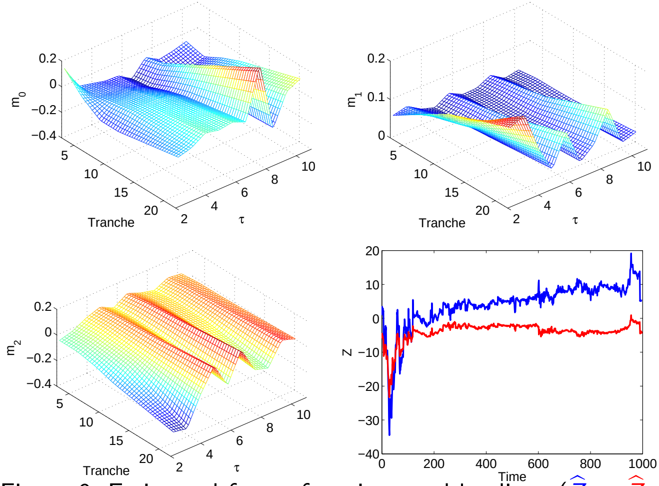


Figure 9: Estimated factor functions and loadings ($\widehat{Z}_{t,1}$, $\widehat{Z}_{t,2}$).

DSFM without Mean for Z-transformed-BC

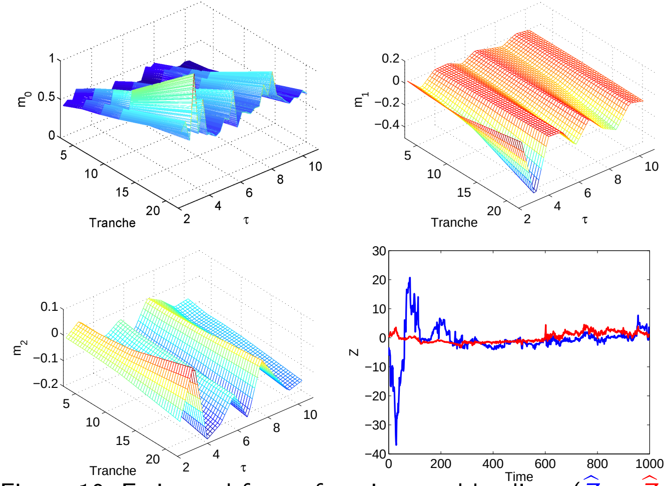


Figure 10: Estimated factor functions and loadings ($\widehat{Z}_{t,1}$, $\widehat{Z}_{t,2}$).

DSFM for Z-transformed-BC Fit Over Time

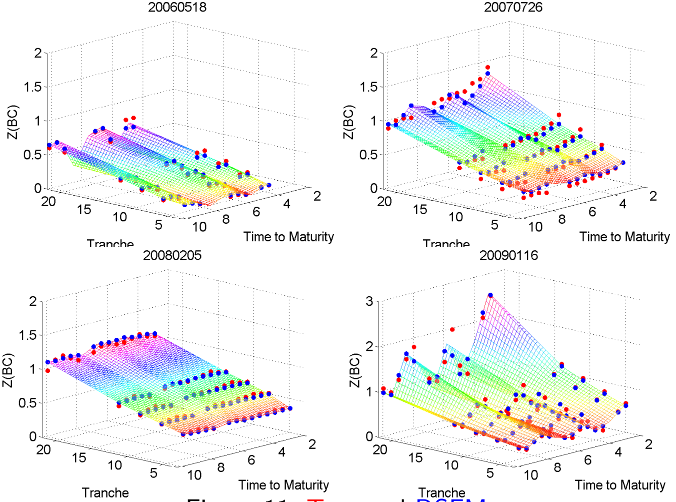


Figure 11: True and DSFM.

Forecasting with Rolling Window

- Consider a static rolling window of $h = 250$.
- Estimate DSFM using $\{Y_t\}_{t=t_0-h}^{t_0-1}$ for every $h < t_0 < T$.
- Forecast factor loadings using VAR, VECM or DCC-GARCH.
- Check what are the possible tomorrow's times-to-maturity for the tranches, i.e. J_{t+1} and $X_{t+1,j}$, $j = 1, \ldots, J_{t+1}$.
- Calculate the forecast of $\hat{Y}_{t+1}$.
- Calculate the expected correlation and spread for tomorrow.

Forecasting Spreads

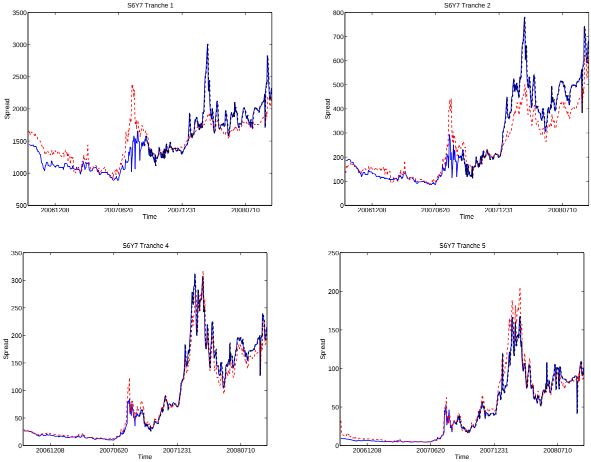


Figure 12: **Market spreads**, **GARCH** and **DSFM**, S6 Y7.

Forecasting Error

DSFM	BC	Spread
Classic with VECM	127.78	219.77
Classic with VAR	126.16	218.54
Without mean with VECM	126.34	186.56
Without mean with VAR	**126.13**	**186.43**
Without mean with MGARCH	135.51	191.25

Table 1: Sum of relative errors $\times 1000$ over 9124 days (8 series, 3 maturities) for 5 tranches.

Curve Trades

So, how can I make money with this?

Combine tranches of different time to maturity:

- Flattener – sell a short-term tranche, buy a long-term tranche
  Example: sell 10Y 3-6% and buy 5Y 6-9%
  Outlook: bullish long-term, bearish short-term
- Steepener – opposite trade

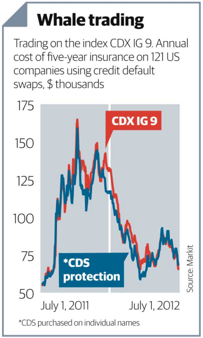
References:

Felsenheimer et al. (2004). DJ iTraxx: Credit at its best!, Credit derivatives special, HVB Corporates & Markets.

Kakodkar et al. (2006). Credit derivatives handbook, A guide to the exotics credit derivatives market, Technical report, Merrill Lynch.

JP Morgan Trading Loss, May 2012

J.P. Morgan's flattener – bought 5Y CDX IG 9 index, sold 10Y CDX IG 9 index in a 3:1 ratio. The final loss reached $6.2 billion.



Flattener

Sell protection at $s_1(t_0)$ for the period $[t_0, T_1]$ and buy protection at $s_2(t_0)$ for $[t_0, T_2]$, $T_1 > T_2$. At t_0 for $j = 1, 2$:

$$\mathrm{MTM}_j(t_0) = \sum_{t=t_1}^{T_j} \beta(t_0, t)\left[s_j(t_0)\Delta t \mathrm{E}\{F_j(t)\} - \mathrm{E}\{L_j(t) - L_j(t-\Delta t)\}\right] = 0.$$

At $\tilde{t} > t_0$, the market quotes $s_j(\tilde{t})$ and

$$\mathrm{MTM}_j(\tilde{t}) = \{s_j(t_0) - s_j(\tilde{t})\} \sum_{t=t_1}^{T_j} \beta(\tilde{t}, t)\Delta t \mathrm{E}\{F_j(t)\}.$$

Curve Trade

- A positive MTM means a positive value to the protection seller.
- If the protection seller closes the position at time $\tilde{t}$, then receives from the protection buyer $\mathrm{MTM}_j(\tilde{t})$.
- Flattener-trader aims to maximize the total MTM value

$$\max\left\{\mathrm{MTM}_1(\tilde{t}) - \mathrm{MTM}_2(\tilde{t})\right\}.$$

Empirical Study

- Consider long and short term tranches of equal notional amounts. Remark: by adjusting the notionals a trade can be structured so that it is risky duration neutral, carry neutral, correlation neutral, or theta (sensitivity to implied correlation changes) neutral.
- Consider trades that generate no or a positive carry, i.e. the spread of the long tranche does not exceed the spread of the short tranche.

Empirical Study

- If one buys 5Y 6-9% and sells 10Y 6-9%, then the trade is hedged for default until the maturity of the 5Y tranche. Defaults that emerage from 10Y 6-9% are covered by 5Y 6-9% till it expires.
- Series differ in the composition of the collateral.
- If one buys 5Y 6-9% and sells 10Y 3-6%, then these tranches provide protection of different portion of portfolio risk. If there is any default in 10Y 3-6%, then we must deliver a payment obligation and incur a loss.

In the study: do not account for default payments (no data of historical defaults in iTraxx), do not account for the positive carry.

Trading Strategies

Consider flatteners and steepeners

1. for fixed tranche and fixed maturities
2. for fixed tranche and all maturities
3. for all tranches and fixed maturities
4. for all tranches and all maturities
5. combined flatteners and steepeners for all tranches and all maturities

Time horizons: 1, 5, 10, 20 days.

Results

Strategy	1 day	5 days	10 days	20 days
F-S-AllTM	0.30	0.12	0.08	0.05
F-S-T2-AllM	0.27	0.12	0.08	0.05
F-S-T3-AllM	0.22	0.07	0.04	0.03
F-S-T4-AllM	0.14	0.05	0.03	0.02
F-S-T5-AllM	0.09	0.03	0.02	0.02
F-T2-AllM	0.38	0.13	0.08	0.06
F-T3-AllM	0.27	0.08	0.05	0.03
F-T4-AllM	0.20	0.06	0.04	0.03
F-T5-AllM	0.12	0.04	0.03	0.02
S-T2-AllM	0.38	0.13	0.08	0.06
S-T3-AllM	0.27	0.08	0.05	0.03
S-T4-AllM	0.20	0.06	0.04	0.03
S-T5-AllM	0.12	0.04	0.03	0.02

Table 2: Mean of daily gain in %. Flatteners and steepeners calculated from predictions of spreads and BC.

Strategy over time

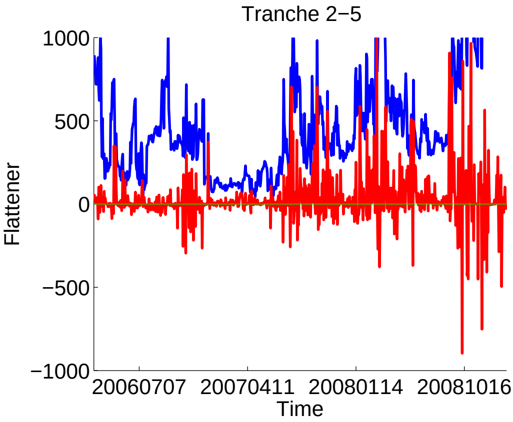


Figure 13: Combined flatteners and steepeners for all tranches and all maturities, **prediction**, **realized**, one day horizon.

Investor's Strategy

At t_0 enter an optimal flattener/steepener for h-day horizon. At $t_0 + h$

1. keep the current position for the next h-days
2. close the current position and enter a new one

Assume a margin of 10% of your notional. Every time the position is close add to the margin the realized P&L. If margin ≤ 0, quit the trade. Otherwise, trade up to 250 days.

Assume *no trading costs, no difference between bid and ask spreads*.

Investor's Strategy

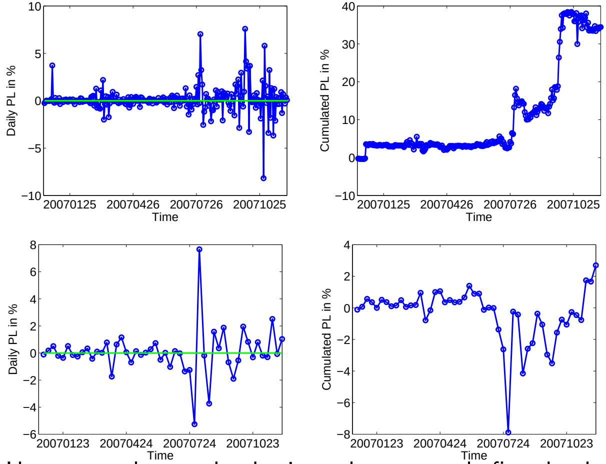


Figure 14: Upper panel: one-day horizon, lower panel: five-day horizon. Starting date 20061219.

Investor's Strategy

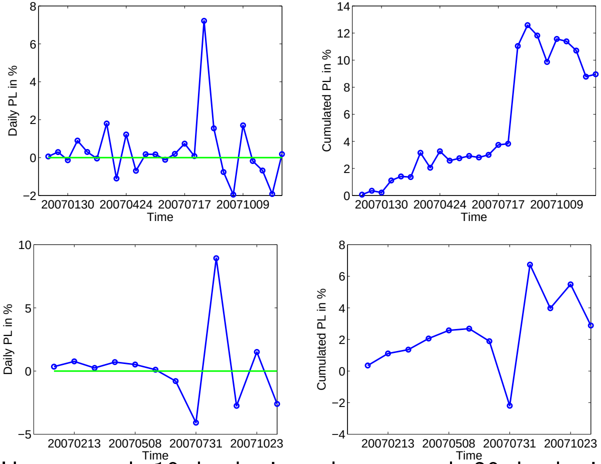


Figure 15: Upper panel: 10-day horizon, lower panel: 20-day horizon. Starting date 20061219.

Outlook

- DSFM provides a flexible way of handling CDO data.
- DSFM gives worse prediction results than GARCH models (but CDO is illiquid). However, it is able to forecast spreads of new coming series that do not have many observations.
- Forecasting the evolution of the surfaces is successfully applied in curve trading.

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Chapman & Hall/Crc Financial Mathematics Series, 2006

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