Statistics II in English
Exercises from *Übungsaufgaben und Lösungen zu Statistik I und II*
Part 1

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Exercise 5-13:

A Baker has in his store marzipan pigs. The random variable X represents the weight of **one** marzipan pig and is normally distributed with mean $E(X) = 150$g and variance $Var(X) = 16$g. Baker assumes that weights of marzipan pigs are independent.

a) What is the distribution of the weight of 4 marzipan pigs.

b) What is the probability that the weight of 4 marzipan pigs
   - is exactly 600g?
   - differs not more then 1% from the expected value?

B Shop assistant Mona knows that during lunch time $(13.00 - 15.00)$ on average one customer comes per 15 minutes.

a) U_1 is the waiting time until the next customer comes. What is the distribution of U_1? What are the parameters of this distribution?

b) What is the probability that Mona will have to wait more than 30 minutes until the first customer comes?

c) Mona has already waited 30 minutes. What is the probability that within the next 15 minutes nobody will come?

C Mona is receiving fresh food evry 3 hours. Today she forgot her watch and asked her friend Leonardo for statistical advice.

a) U_1 is the waiting time for the arrival of fresh food. What is the distribution of U_2? What are the parameters of this distribution?

b) Mona has already waited 1 hour and nothing has come. What is the probability that during the next 30 minutes the food will arrive?

Exercise 6-1:
From a population of $N = 3$ people aged 20, 22 and 24 years random samples of size $n = 2$ are drawn with repetition.

a) Determine the arithmetic mean μ and the variance σ^2 of this population.

b) How many 2-tuples are in this sample space?

c) List all possible 2-tuples resulting from random sampling.

d) Determine the probability distribution of the following sample functions:

   (i) $Y = \sum_{i=1}^{2} X_i$.
   Calculate $E(Y)$ and $Var(Y)$.

   (ii) $\bar{X} = \frac{1}{n}\sum_{i=1}^{2} X_i$.
   Calculate $E(\bar{X})$ and $Var(\bar{X})$.

   (iii) $S'^2 = \frac{1}{n}\sum_{i=1}^{2}(X_i - \bar{X})^2$.
   Calculate $E(S'^2)$.

   (iv) $S^2 = \frac{1}{n-1}\sum_{i=1}^{2}(X_i - \bar{X})^2$.
   Calculate $E(S^2)$.

e) Check, with the help of the results obtained in d), the following formulas:

   (i) $E(Y) = n\mu$ and $Var(Y) = n\sigma^2$,

   (ii) $E(\bar{X}) = \mu$ and $Var(\bar{X}) = \sigma^2/n$,

   (iii) $E(S'^2) = \sigma^2(n-1)/n$,

   (iv) $E(S^2) = \sigma^2$.

Exercise 6-2:
The random variable Y can be expressed as sum of random variables X_i, $(i = 1, \ldots, n)$. The distribution of the X_i is unknown.

1. Under which conditions on the X_i and n Y can be regarded as approximatively normally distributed?

2. Which fundamental statistical theorem is linked to this?

3. If the conditions stated in (1) are fulfilled, what is the approximative distribution of Y if:

   (i) X_i are not identically distributed?

   (ii) X_i are identically distributed?

Exercise 6-3:
The amount of active substance in pills against headache is normally distributed. Since a too low dosis does not yield desired effects and a too high dosis provokes side effects, the production has to be supervised constantly. With the help of simple random sampling the average amount of active substance μ (in mg) shall be estimated. What is the probability that the function $\bar{X}$ takes values deviating more than $+0.5$mg of the true μ for:

1. $\sigma = 1$mg and $n = 16$,
2. $\sigma = 1$mg and $n = 64$,
3. $\sigma = 2$mg and $n = 64$?

Exercise 6-4:
For the following distributions of the X_i, what is the distribution, the expectation and the variance of the sample function $\sum X_i$. Assume the X_i are independent.

1. $X_i \sim B(n, p)$
2. $X_i \sim N(\mu, \sigma)$
3. $X_i \sim$ discrete uniform single point distribution(μ)

Exercise 6-9:
A tennis teacher gives every month (30 days) 8 lessons (1 hour) per day. Throughout his career he remarked that per lesson students throw two or seven balls over the court's fence with probability 0.1 and one or six balls with probability 0.3. However none of his students have ever done this for three, four, less than one or more than seven balls.

1. What is the approximate probability that in one month between 900 and 1000 balls get lost over the fence?
2. What is the approximate probability that in one month more than 1050 balls get lost over the fence?
3. Provide symmetric borders for the expectation so that the probability that the loss of balls lies within these borders is 0.99.
4. Check if the assumptions of the used statistical theorem are verified in this exercies.

Exercise 6-12:
From an urn with N balls, of which a proportion of π is red, samples of size n are drawn without repetition. Determine the probability of receiving proportions of red balls between p_1 and p_2 in the sample if:

1. $N = 5$, $n = 3$, $\pi = 0.4$, $p_1 = 1/3$, $p_2 = 2/3$;
2. $N = 1000$, $n = 4$, $\pi = 0.3$, $p_1 = 0.25$, $p_2 = 0.75$;
3. $N = 250$, $n = 100$, $\pi = 0.2$, $p_1 = 0.1$, $p_2 = 0.3$;
4. $N = 2500$, $n = 100$, $\pi = 0.2$, $p_1 = 0.14$, $p_2 = 0.3$.