Calibration Risks for Exotic Options

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  [Cont (2005)]

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- 1999:
  $ 5 billion derivative **losses** in banking industry
  because of **model risk**
  [Williams (1999)]

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- The **choice** of the pricing model is important.
  $>$ Model risk (see e.g. [Schoutens et al. (2004)])
- But also the **implementation** of the model is vital for the pricing of exotic options.
  $>$ Calibration risk

Aims

We analyze calibration risk for the popular option pricing model of Heston.

To this end, we consider different ways to calibrate the model to a time series of DAX implied volatility surfaces and analyze the impact on the prices of exotic options.

Aims

In particular, we consider

- the special design of implied volatility surfaces
- different error functionals that are minimized in the calibration

Outline of the talk

1. motivation ✓
2. introduction
3. models and data
4. calibration
5. exotic options
6. conclusions
7. outlook

Option pricing under model uncertainty

[Schoutens et al. (2004)] calibrate several stochastic volatility and L'evy models to the plain vanilla market.
(This calibration is performed by minimization of RMSE of the prices.)
All models give good fits with comparable errors.
But the implied prices of exotics differ significantly.

Model uncertainty

Thus different parametric forms for the process of the underlying asset lead to different prices of exotic options although the plain vanilla prices coincide.

option	lookback	barrier	cliquet
price range	15 %	200 %	40 %

Table 1: Price differences of exotic options for different models.

(As [Schoutens et al. (2004)] consider only one day the price ranges in general are unclear.)

Risk measures

[Artzner et al. (1999)] have introduced risk measures as monotone, translation invariant, subadditive, positive homogene functions. Such a risk measure ρ represents the worst case expected payoff for a class $\mathcal{P}$ of probabilistic models:

$$\rho(X) = \sup_{P \in \mathcal{P}} \mathsf{E}_P(-X)$$

Risk measures

[Cont (2005)] proposes a quantitative framework for measuring model risk.
This framework takes into account special features of model risk like bid-ask spreads orthe existence of hedging instruments.
A simple risk measure that fits in Cont's set-up is given by

$$\mu_{\mathcal{P}}(X) \stackrel{\text{def}}{=} \sup_{P \in \mathcal{P}} \mathsf{E}_P(X) - \inf_{P \in \mathcal{P}} \mathsf{E}_P(X)$$

Risk measures

We consider this measure of model risk for a fixed option pricing model.
The class of probabilistic models $\mathcal{P}$ will be given by different calibration methods.
Thus we identify the price range for several exotic options using a time series of implied volatility surfaces.

Calibration methods

Calibration is done by minimization of an error functional.
This error functional can measure price of implied volatility differences in absolute or relative terms.
Moreover, the problem can be regularized in different ways.

options

We analyze the following options:

- up and out calls
- down and out puts
- cliquets

Model risk

We analyze the impact of the different calibration ways on the prices of these exotic options.
Moreover, we consider factors that influence these price differences.

In the end, we also look at the relation between calibration risk and model risk.
(Model risk is understood as the choice of a parametric option pricing model.)

Models

We focus on the Heston model that is quite popular because of its relative simplicity.
Moreover, we consider the Bates model, an extension of the Heston model.
The Bates model gives a fit to data and hence we can analyze the impact of goodness of fit for the same type of model.

The Heston model

The price process is given by

$$\frac{dS_t}{S_t} = \mu dt + \sqrt{V_t} dW_t^{(1)}$$

where the volatility process is modelled by a square-root process:

$$dV_t = \xi(\eta - V_t)dt + \theta\sqrt{V_t} dW_t^{(2)},$$

and W^1 and W^2 are Wiener processes with correlation ρ.

The Heston model

The mean-reversion speed ξ, the long vol η and the short vol V_0 control the term structure of the implied volatility surface (i.e. time to maturity direction).
The correlation ρ and the vol of vol θ control the smile/skew (i.e. moneyness direction).

The Bates model

In this model, the price process is given by

$$\begin{aligned} \frac{dS_t}{S_t} &= \mu dt + \sqrt{V_t} dW_t^{(1)} + dZ_t \\ dV_t &= \xi(\eta - V_t)dt + \theta \sqrt{V_t} dW_t^{(2)} \end{aligned}$$

where Z is a compound Poisson process.

This model extends the Heston model and has three more parameters.

The Bates model

The meaning of the parameters ξ, η, θ, ρ and V_0 is the same as in the Heston model.

The parameters of the compound Poisson process control the smile/skew especially for short times to maturity.

Data

- Eurex settlement volatilities of European options
- underlying : dax
- time period: March 2003 - April 2004
- risk free interest rate: Euribor
- no dividends because dax is performance index

Because of computation time we consider only one day for aec week.
Hence, we consider 51 implied volatility surfaces.

Data

Arbitrage:
The implied volatility surfaces have been preprocessed in order to eliminate arbitrage.

Illiquidity:
Only options with moneyness $m \in [0.75, 1.35]$ for small times to maturity $T \leq 1$ have been considered because of illiquidity.

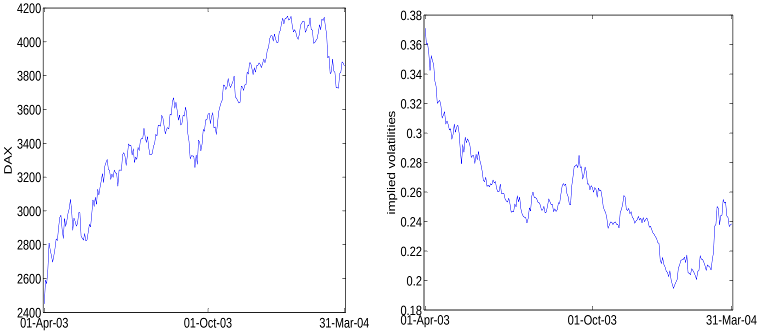

Figure 1: DAX and ATM implied volatility with 1 year to maturity on the trading days from 01 April 2003 to 31 March 2004.

	mean number of maturities	mean number of obervations	mean money-ness range
short maturities ($0.25 \leq T < 1.0$)	3.06	64	0.553
long maturities ($1.0 \leq T$)	5.98	76	0.699
total	9.04	140	0.649

Table 2: Description of the implied volatility surfaces.

Error functionals I

For the minimization we consider the four objective functions based on the root weighted square error:

$$ap \stackrel{\text{def}}{=} \sqrt{\sum_{i=1}^{n} w_i (P_i^{mod} - P_i^{mar})^2}$$

$$rp \stackrel{\text{def}}{=} \sqrt{\sum_{i=1}^{n} w_i (\frac{P_i^{mod} - P_i^{mar}}{P_i^{mar}})^2}$$

Error functionals II

$$ai \stackrel{\text{def}}{=} \sqrt{\sum_{i=1}^{n} w_i (IV_i^{mod} - IV_i^{mar})^2}$$

$$ri \stackrel{\text{def}}{=} \sqrt{\sum_{i=1}^{n} w_i \left(\frac{IV_i^{mod} - IV_i^{mar}}{IV_i^{mar}}\right)^2}$$

where *mod* refers to a model quantity and *mar* to a quantity observed on the market, P to a price and IV to an implied volatility.

Data design

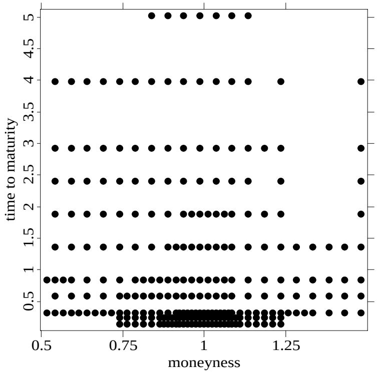

Figure 2: Grid of the DAX implied volatility surface on March 1st, 2004. (only for moneyness between 0.5 and 1.5)

Data design

The observations per day have a special design:

- data come in strings
- strings are not uniformly distributed in time-to-maturity
- strings have different moneyness ranges
- strings are moving in time
- strings disappear
- new strings appear

Data design

We use the RMSE to measure the difference between market and model. In order to take the special data design into account we

use the weights such that

- each string get the same weight
- all observations in a string have the same weight

These weights imply an average time to maturity of 2.02. Hence, it is a good weighting to analyze options with 1,2 or 3 years to expiration.

Calibration method

The error functionals are minimized with respect to the model parameters by a global stochastic minimization routine.

The plain vanilla prices are calculated by a method of Carr and Madan:

$$C(K,T) = \frac{\exp\{-\alpha \ln(K)\}}{\pi} \int_0^{+\infty} \exp\{-\mathbf{i} v \ln(K)\} \psi_T(v) dv$$

for a damping factor $\alpha > 0$. The function ψ_T is given by

$$\psi_T(v) = \frac{\exp(-rT)\phi_T\{v - (\alpha+1)\mathbf{i}\}}{\alpha^2 + \alpha - v^2 + \mathbf{i}(2\alpha+1)v}$$

where ϕ_T is the characteristic function of $\log(S_T)$.

Calibration method

mean objective fct.	AP	RP $[E^{-2}]$	AI $[E^{-2}]$	RI $[E^{-2}]$
AP	7.3	9.7	0.81	3.1
RP	11.	6.1	0.74	2.9
AI	9.4	7.3	0.68	2.6
RI	8.8	7.0	0.70	2.5

Table 3: Calibration errors in the Heston model for 51 days.

Calibration method

mean objective fct.	AP	RP $[E^{-2}]$	AI $[E^{-2}]$	RI $[E^{-2}]$
AP	7.0	13.	0.76	2.8
RP	12.	5.1	0.67	2.6
AI	8.9	6.4	0.60	2.3
RI	8.7	6.2	0.62	2.2

Table 4: Calibration errors in the Bates model for 51 days.

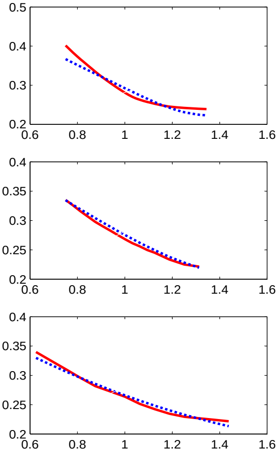

Figure 3: IVS on 25/06/03 for AI in Heston (maturities: 0.26, 0.52, 0.78, 1.04, 1.56, 2.08). (market: blue, model: red; X: moneyness, Y:iv)

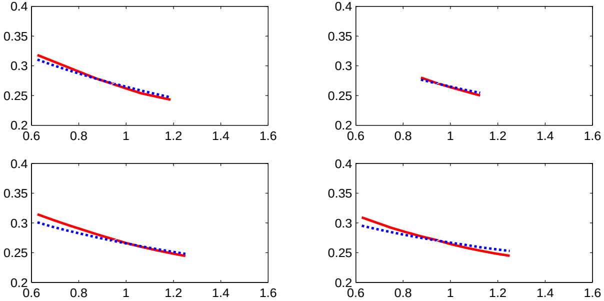

Figure 4: IVS on 25/06/03 for AI in Heston (maturities: 2.60, 3.12, 3.64, 4.70). (market: blue, model: red; X: moneyness, Y:iv)

	ξ	η	θ	ρ	V_0
AP	0.87	0.07	0.34	-0.82	0.07
	(0.48)	(0.02)	(0.08)	(0.08)	(0.02)
RP	1.38	0.07	0.44	-0.74	0.08
	(0.35)	(0.02)	(0.06)	(0.03)	(0.02)
AI	1.32	0.07	0.43	-0.77	0.08
	(0.40)	(0.02)	(0.06)	(0.04)	(0.02)
RI	1.20	0.07	0.41	-0.75	0.08
	(0.35)	(0.02)	(0.06)	(0.05)	(0.02)

Table 5: Mean parameters (std.) in the Heston model for 51 days.

	ξ	η	θ	ρ	V_0	λ	$\overline{k}$	δ
AP	0.92	0.07	0.33	-0.94	0.07	0.33	0.07	0.08
	(0.50)	(0.02)	(0.08)	(0.07)	(0.02)	(0.21)	(0.03)	(0.06
RP	1.56	0.07	0.45	-0.89	0.08	0.54	0.05	0.08
	(0.47)	(0.02)	(0.07)	(0.07)	(0.02)	(0.23)	(0.03)	(0.06
AI	1.43	0.07	0.43	-0.95	0.07	0.50	0.06	0.09
	(0.44)	(0.02)	(0.06)	(0.06)	(0.02)	(0.22)	(0.03)	(0.04
RI	1.36	0.07	0.41	-0.93	0.07	0.52	0.05	0.08
	(0.44)	(0.02)	(0.07)	(0.09)	(0.02)	(0.26)	(0.04)	(0.08

Table 6: Mean parameters (std.) in the Bates model for 51 days.

Monte Carlo simulation

We compue the prices of the options by Euler discretization using 1000000 paths.

	Heston			Bates		
	$T = 1$	$T = 2$	$T = 3$	$T = 1$	$T = 2$	$T =$
up and out calls	0.17	0.10	0.08	0.17	0.11	0.09
down and out puts	0.18	0.11	0.08	0.19	0.12	0.10
cliquet options	0.06	0.05	0.05	0.07	0.06	0.05

Table 7: Maximal relative standard error of Monte Carlo simulations. $[E^{-2}]$

Monte Carlo simulation

The payoffs of the barrier options is based on a maximum/minimum in continuous time.
This is replaced in the simulations by maximum/minimum over 250 days a year.
Although there is still some bias it can regarded as payoff of an approximate instrument.

Barrier options

The prices of up and out calls are given by

$$\exp(-rT)\,\mathsf{E}[(S_T - K)^+ \mathbf{1}_{\{M_T < B\}}]$$

where

$$M_T \stackrel{\text{def}}{=} \max_{0 \le t \le T} S_t.$$

Barrier options II

We consider for the barrier B and the strike K

$$B = 1 + T * 0.2$$
$$K = 1 - T * 0.1$$

where $T = 1, 2, 3$ denotes time to maturity.

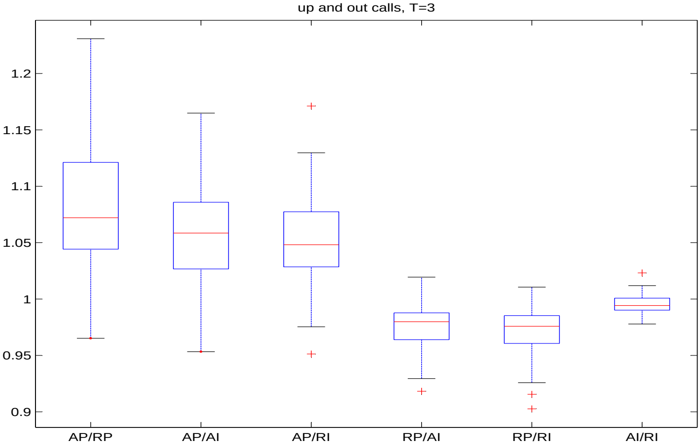

Figure 5: Relative prices of the up and out calls in the Heston model for 3 years to maturity.

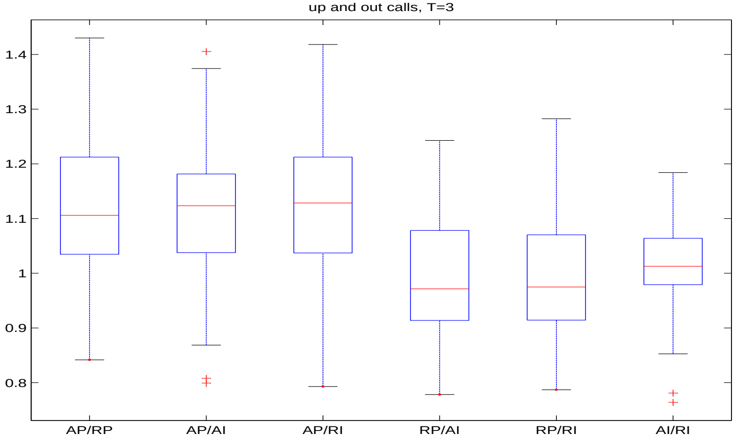

Figure 6: Relative prices of the up and out calls in the Bates model for 3 years to maturity.

		AP/RP	AP/AI	AP/RI	RP/AI	RP/RI	AI/RI
Heston	$T=1$	0.986	0.968	0.967	0.984	0.984	0.999
	$T=2$	1.051	1.024	1.022	0.979	0.978	0.998
	$T=3$	1.072	1.059	1.048	0.980	0.976	0.994
Bates	$T=1$	0.988	0.985	1.002	1.002	1.006	1.012
	$T=2$	1.070	1.083	1.104	0.970	0.986	1.018
	$T=3$	1.106	1.123	1.129	0.972	0.975	1.013

Table 8: Median of price quotients of up and out calls.

down and out puts

The situation is similar for down and out puts but the order of the prices is the way round.

Cliquet options

We consider cliquet options with prices

$$\exp(-rT)\,\mathrm{E}[H]$$

where the payoff H is given by

$$H \stackrel{\mathrm{def}}{=} \min(c_g, \max(f_g, \sum_{i=1}^{N} \min(c_l^i, \max(f_l^i, \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}})))).$$

Here c_g (f_g) is a global cap (floor) and c_g^i (f_g^i) is a local cap (floor) for the period $[t_{i-1}, t_i]$.

Here we consider three periods with $t_i = i\frac{T}{3}$ $(i = 0, \ldots, 3)$ and the caps and floors are given by

$$\begin{aligned} c_g &= \infty \\ f_g &= 0 \\ c_l^i &= 0.08,\ \ i = 1, 2, 3 \\ f_l^i &= -0.08,\ \ i = 1, 2, 3 \end{aligned}$$

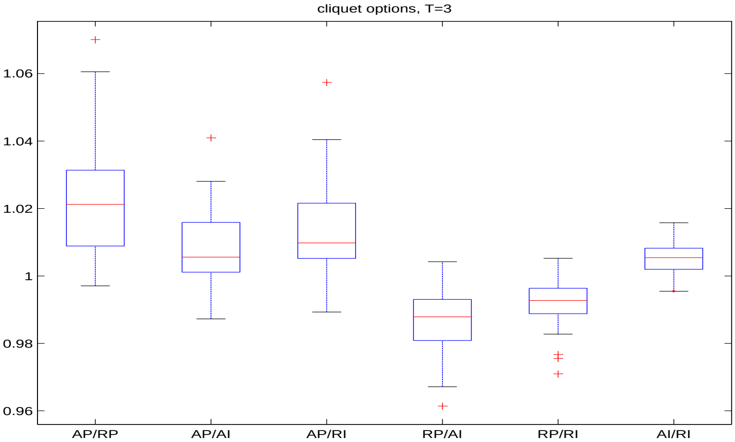

Figure 7: Relative prices of the cliquet options in the Heston model for 3 years to maturity.

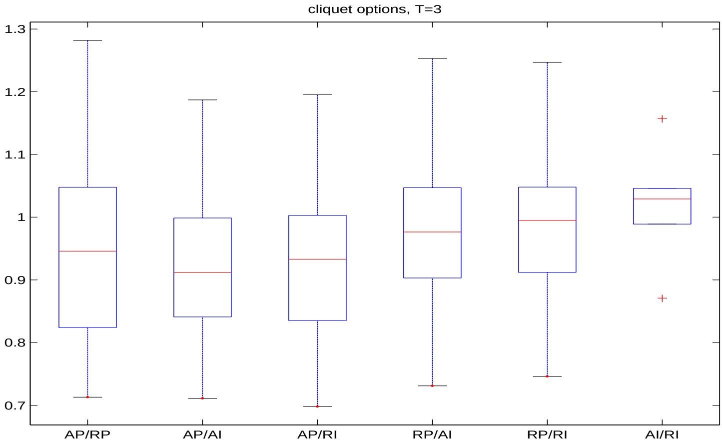

Figure 8: Relative prices of the cliquet options in the Bates model for 3 years to maturity.

		AP/RP	AP/AI	AP/RI	RP/AI	RP/RI	AI/RI
Heston	$T=1$	0.983	0.976	0.989	0.993	1.006	1.013
	$T=2$	1.002	0.991	1.000	0.989	0.998	1.010
	$T=3$	1.022	1.008	1.014	0.987	0.992	1.005
Bates	$T=1$	0.917	0.899	0.917	0.987	1.005	1.024
	$T=2$	0.931	0.903	0.923	0.980	0.999	1.029
	$T=3$	0.946	0.912	0.933	0.976	0.995	1.029

Table 9: Median of price quotients of cliquet options.

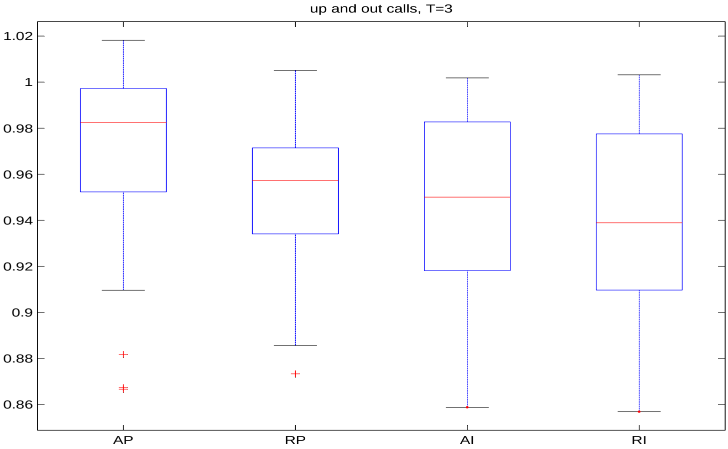

Figure 9: Bates prices over Heston prices for up and out calls with 3 years to maturity on 51 days.

		AP	RP	AI	RI
up and out calls	$T = 1$	0.973	0.953	0.944	0.941
	$T = 2$	0.980	0.954	0.953	0.940
	$T = 3$	0.983	0.957	0.950	0.939
down and out puts	$T = 1$	0.933	0.892	0.877	0.878
	$T = 2$	0.918	0.883	0.872	0.860
	$T = 3$	0.916	0.881	0.873	0.860
cliquets	$T = 1$	1.057	1.100	1.109	1.119
	$T = 2$	1.076	1.128	1.130	1.144
	$T = 3$	1.086	1.138	1.140	1.162

Table 10: Median of Bates prices over Heston prices.

calibration risk

- exotic prices from AP differ from exotic prices from RP,AI,RI
- price differences grow for longer times to maturity
- price differences bigger for barrier options than for cliquets
- model risk (between Heston and Bates) smallest for AP and biggest for RI

calibration risk

If the choice of the parametric model is unclear for the considered option then calibration w.r.t AP reduces model risk. For a given

model calibration w.r.t AI gives (relatively) stable parameters, good fits and exotics prices with small variance that lie between the prices from AP and RP calibrations.

Stability

Traders in banks are not only interested in good calibrations.
It is vital to stable results.
But the stability does not concern the model parameter.
Instead they interested in stable prices and greeks.

Stability

Possible regularization:

- parameters
- (finite dimensional) distribution of the stock process
- other liquid markets (e.g. variance swap market)

For Further Reading

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