Weather Derivatives

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Weather

- Influences our daily lives and choices
- Impact on corporate revenues and earnings
- Metereological institutions: business activity is weather dependent
  - British Met Office: daily beer consumption increases by 10% if temperature decreases by 3°C
  - If temperature in Chicago is less than zero: consumption orange juice declines 10% in average

- Climate changes related to "El Niño"
- Global climate changes the volatility of weather
- Occurrence of extreme weather events increases
- International weather markets: Weather Derivatives (WD)

Examples

- Natural gas company suffers negative impact in mild winter
- Construction companies buy WD (rain period)
- Cloth retailers sell fewer clothes in hot summer
- Salmon fishery suffer losses by increase of sea temperatures
- Ice cream producers (cold summers)
- Disney World (rain period)

Outline

1. Motivation ✓
2. Weather Derivatives
3. Weather Indices
4. Valuation of Weather Derivatives
5. Application

What are WD?

- Emerged in the mid-1990s
- Hedge weather related risk exposures
- Payments based on weather-related measurements
- Formal exchanges: Chicago Mercantile Exchange (CME)
  - Monthly and seasonal temperature future contracts
  - European call and put options on these futures
- Underlying variables: temperature, rainfall, wind, snow, frost
  - No physical markets in weather: the underlying has no intrinsic financial value

Anatomy of a WD

A weather derivative is defined through:

- Reference Weather Station
- The underlying Index
- Term period
- Structure: puts, calls, swaps, collars, straddles, and strangles.
- Premium

Example

A beverage company hedges itself against a cold summer with the following option:

Weather Stations	Munich, Berlin and Frankfurt
Risk Period	01 Jul - 31 August
Weather Index	Average temp. for the 3 Weather Stations
Put Strike	19°C
Tick	EUR 30,000 per 0.01°C
Maximum Payout	3,000,000
Payout	Weather Index $<$ Put Strike

Weather indices: temperature

Heating degree day (HDD): over a period $[\tau_1, \tau_2]$

$$\int_{\tau_1}^{\tau_2} \max(T_0 - T_u, 0)du \tag{1}$$

Cooling degree day (CDD): over a period $[\tau_1, \tau_2]$

$$\int_{\tau_1}^{\tau_2} \max(T_u - T_0, 0)du \tag{2}$$

T_0 is the baseline temperature (typically 18°C or 65°F), T_u is the average temperature on day u.

Average of average temperature (PRIM): measure the "*excess*" or deficit of temperature. The average of average temperatures over $[\tau_1, \tau_2]$ days is:

$$\frac{1}{\tau_1 - \tau_2} \int_{\tau_1}^{\tau_2} T_u du \tag{3}$$

Cumulative averages (CAT): The accumulated average temperature over $[\tau_1, \tau_2]$ days is:

$$\int_{\tau_1}^{\tau_2} T_u du \tag{4}$$

Event indices: number of times a certain meteorological event occurs in the contract period

- Frost days: temperature at 7:00-10.00 local time is less than or equal to $-3.5^\circ C$

Weather indices: rain

Cumulative rainfall index I^c: sum of daily rainfall amount in a certain accumulation period

$$I^c = \sum_{t=1}^{x} y_t \tag{5}$$

x: length of the accumulation period

Rainfall deficit index I^d: measures the shortfall accumulated over z periods of the rainfall sum in an s-days period relative to a reference level $y^{\min}$

$$I^d = \sum_{\tau=1}^{z} \min\left(0, \sum_{t=(\tau-1)\cdot s+1}^{\tau \cdot s} y_t - y^{\min}\right) \tag{6}$$

Valuation of WD

The price of a contingent claim F can be calculated as:

$$F = e^{-rT}\,\mathsf{E}^{Q}\left[\psi(I)\right] \tag{7}$$

I: stochastic variable that expires at time T, weather index
$\psi(I)$: payoff of the derivative at expiration
r: risk free interest rate
E^{Q}: risk neutral probability measure

but...

- weather cannot be traded: no-arbitrage models to WD are impractical!
- Black and Scholes does not work: volatility changes
- Price of the derivative must account for the market price of weather risk (λ)
- Benth (2004): many arbitrage-free prices exist
- Hull (2006): Weather risk is not a systematic risk, $\lambda = 0$

Pricing Weather Derivatives

- Temperature derivatives: Campbell and Diebold (2005), Jewson and Brix (2005), Benth (2005)
- Rainfall: Cao, Li and Wei (2004)

Approaches:

- Burn analysis
- Index Value Simulation
- Daily simulation
- Stochastic Pricing Model

Burn analysis or Burning Cost Method

- Evaluates how a contract would have performed in previous years
- Uses the empirical distribution of the underlying index
- Implementation of the pricing formula

$$F = e^{-rT} \frac{1}{n} \sum_{t=1}^{n} \psi(I_t) \tag{8}$$

Steps:

1. Historical weather data are cleaned and detrended
2. Determine Index value - hypothetical payoffs for each year
3. Calculate payoff average and discount with r

Call option price:

$$F = e^{-rT}\frac{1}{n}\sum_{t=1}^{n}\max\left(I_t - K, 0\right)$$

Put option price:

$$F = e^{-rT}\frac{1}{n}\sum_{t=1}^{n}\max\left(K - I_t, 0\right)$$

r: risk free interest rate
K: strike
I_t: accumulated weather index for t-th year
n: number of years

Index Value simulation

- Utilizes a statistical model for the weather index or the derivative payoff
- Supported by goodness of fit tests

Steps:

1. Given an appropriate distribution, the values of the index are randomly drawn and the payoffs are determined
2. Calculate payoff average and discount with r

Daily simulation

- Describes the dynamics of the weather variable over time
- Yields smaller confidence intervals than Burn and index simulation: smaller for rainfall than for temperature

Steps:

1. Derive the weather index from the simulated sample path
   - summing up daily precipitation/daily average temperature
2. Determined pay offs
3. Calculate payoff average and discount with r

Stochastic Model for temperature

Define the Ornstein-Uhlenbleck process $X_t \in \mathbb{R}^p$:

$$dX_t = AX_t dt + e_{pt}\sigma_t dB_t$$

e_{tp}: t'th unit vector in $\mathbb{R}^p$
σ_t: temperature volatility
A: $p \times p$-matrix
B_t: Wiener Process

$$A = \begin{pmatrix} 0 & \dots & \mathtt{I} \\ -\alpha_p & \dots & -\alpha_1 \end{pmatrix}$$

Solution of X_t:

$$X_s = \exp(A_{s-t})x + \int_t^s \exp(A_{s-u})e_p\sigma_u dB_u$$

Temperature dynamics:

$$T_t = \Lambda_t + X_{1t}$$

- X_{1t}: continuous-time AR(p) (CAR(p)) model
- Λ_t: seasonality function
- Temperature will be normally distributed each time

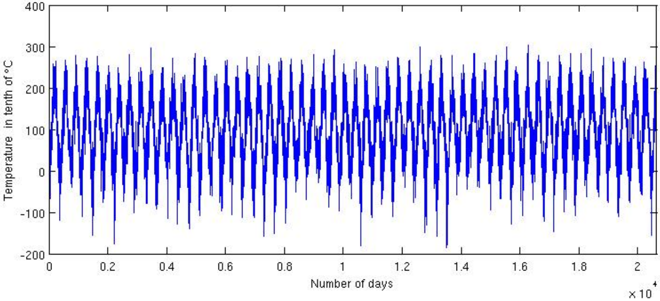
Let $p = 1$, implying $X_t = X_{1t}$:

$$dX_{1t} = -\alpha_1 X_{1t} dt + \sigma_t dB_t$$

Berlin temperature data

Daily average temperatures: 1950/1/1-2006/7/24

- Station: BERLIN-TEMP.(FLUGWEWA)
- 29 February removed
- 20645 recordings

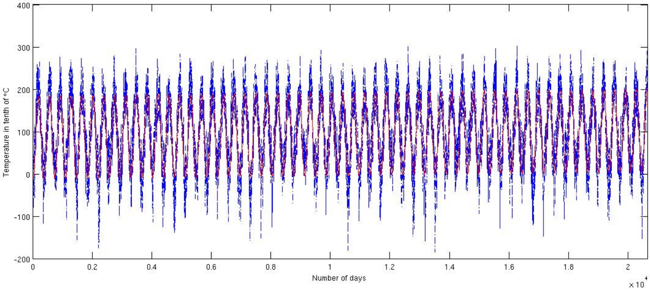


Seasonal function

Suppose seasonal function with trend:

$$\Lambda_t = a_0 + a_1 t + a_2 sin\left(\frac{2\pi t}{365} - a_3\right)$$

Estimates: $\hat{a}_0 = 86.05, \hat{a}_1 = 0.00, \hat{a}_2 = 98.05, \hat{a}_3 = 174.05$



Fitting an autoregressive model

Remove effect of Λ_t from the data: $Y_t = T_t - \Lambda_t$

ADF-Test statistic:

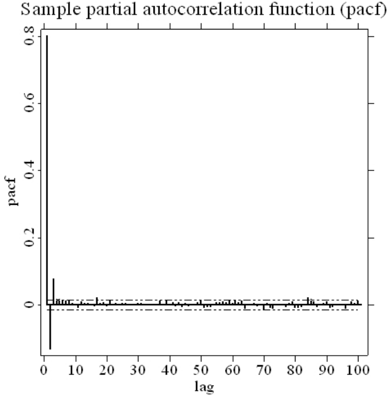
$$(1-L)y = c_1 + c_2\text{trend} + \tau Ly + \alpha_1(1-L)Ly + \ldots \alpha_p(1-L)L^p y + u$$

- $\tau = -51.53$, with 1% critical value equal to -2.5659
- Rejection of H_0 that $\tau = 0 \Rightarrow Y_i$ is a stationary process $I(0)$ $\Rightarrow$ no unit roots $d = 0$

Partial autocorrelation function suggest AR(3):

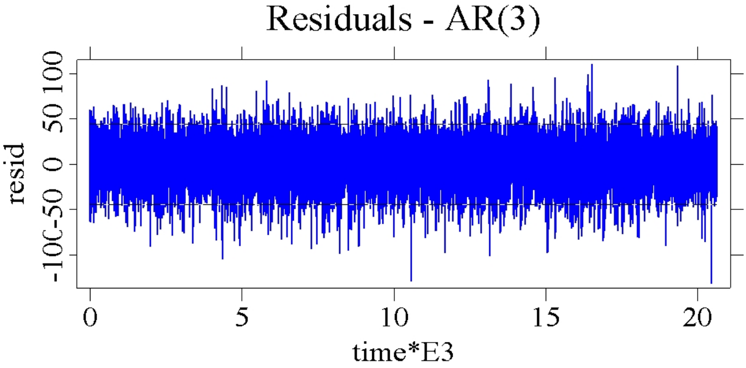
$$Y_{i+3} = \beta_1 Y_{i+2} + \beta_2 Y_{i+1} + \beta_3 Y_i + \sigma_i \varepsilon_i$$

Estimates: $\hat{\beta}_1 = 0.91, \hat{\beta}_2 = -0.20, \hat{\beta}_3 = 0.07, \hat{\sigma}_i^2 = 510.63$



Residuals

Reject H_0 for zero expected innovations: stat$= 2.1013$ (Xplore).



Seasonal volatility

Close to zero ACF for residuals of AR(3) and highly seasonal ACF for squared residuals of AR(3)

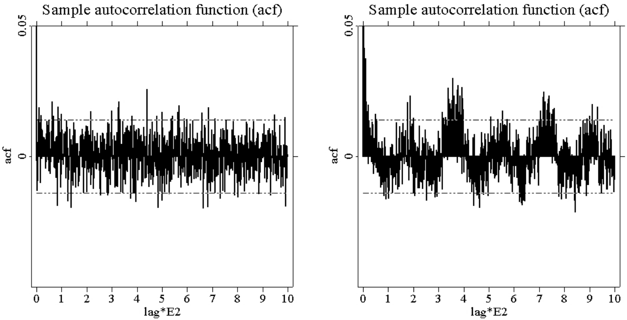


Figure 1: ACF for residuals (left), ACF for squared residuals (right)

Calculate the daily variances of residuals for 56 years and calibrate it to the next truncated Fourier series:

$$\sigma_t^2 = c + \sum_{i=1}^{4} c_i sin\left(\frac{2i\pi t}{365}\right) + \sum_{j=1}^{4} d_j cos\left(\frac{2j\pi t}{365}\right)$$

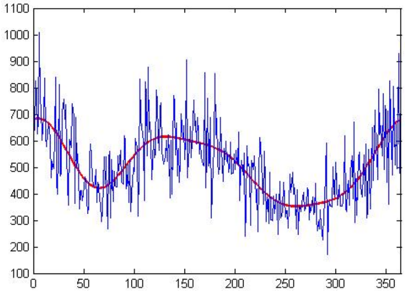


Figure 2: Seasonal variance

Dividing out the seasonal volatility from the regression residuals:

- ACF for residuals unchanged
- ACF for squared residuals non-seasonal

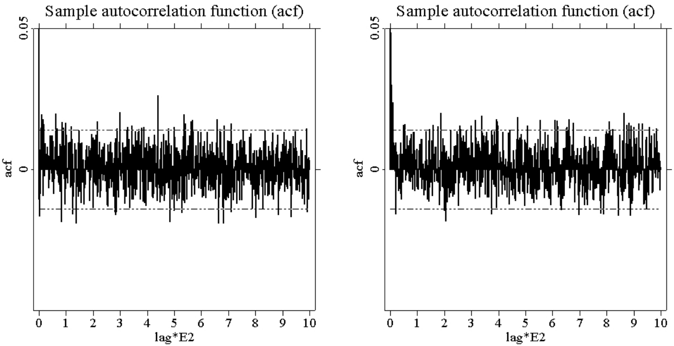


Figure 3: ACF for residuals (left), ACF for squared residuals (right)

Residuals become normally distributed

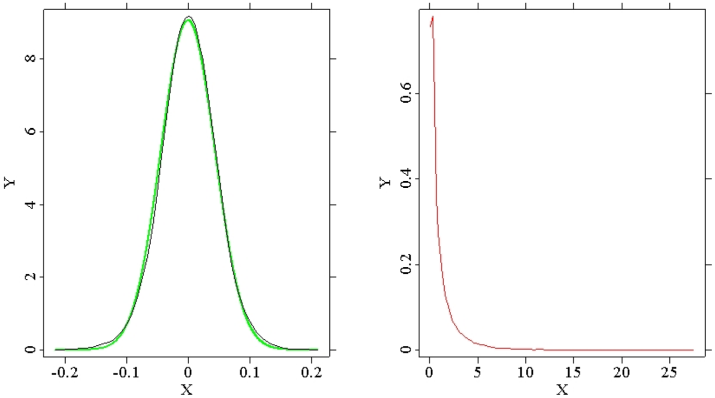


Figure 4: Left: pdf for residuals (black line)and a normal pdf (green line). Right: pdf for squared residuals

Temperature futures

HDD-futures price $F_{HDD(t,\tau_1,\tau_2)}$ at time $t \leq \tau_1$:

$$F_{HDD(t,\tau_1,\tau_2)} = \mathsf{E}^Q \left[\int_{\tau_1}^{\tau_2} \max(18^\circ C - T_u, 0) du | \mathcal{F}_t \right]$$

No trade in settlement period
Constant interest rate r
Q is a risk neutral probability

- Not unique since market is incomplete
- Temperature is not tradable

Analogously: CDD and CAT future prices

Risk neutral probabilities

Defined by the Girsanov transformation of B_t: $dB_t^\theta = dB_t - \theta_t dt$
θ_t: time-dependent market price of risk.

Density of Q^θ:

$$Z_t^\theta = \exp\left(\int_0^t \theta_s dB_s - \frac{1}{2}\int_0^t \theta_s^2 ds\right)$$

Dynamics X_t under Q^θ:

$$dX_t = (AX_t + e_p\sigma_t\theta_t)dt + e_p\sigma_t dB_t^\theta$$

$$\begin{aligned} X_s &= \exp(A_{s-t})x + \int_t^s \exp(A_{s-u})e_p\sigma_u\theta_u du \\ &+ \int_t^s \exp(A_{s-u})e_p\sigma_u dB_u^\theta \end{aligned} \tag{9}$$

CDD futures

The future price is a function of the lagged temperatures $T_t, T_{t-1}, ..., T_{t-p}$:

$$F_{CDD(t,\tau_1,\tau_2)} = \int_{\tau_1}^{\tau_2} v_{t,s}\psi \left[\frac{m\left\{t, s, e_1^\top \exp(A_{s-t})X_t\right\}}{v_{t,s}} \right] ds \qquad (10)$$

where $m\{t,s,x\} = \Lambda_s - c + \int_{\tau_1}^{\tau_2} \sigma_u \theta_u e_1^\top \exp\left(A_{s-t}\right) e_p du + x$
$v_{t,s}^2 = \int_t^s \sigma_u^2 \left\{e_1^\top \exp\left(A_{s-t}\right) e_p\right\}^2 du$
$\psi[x] = x\Phi(x) + \Phi^{'}(x)$
Φ is the standard normal cdf

From the martingale property and $It\hat{o}'s$ formula, the time dynamics of the CDD-futures prices:

$$
\begin{aligned}
dF_{CDD(t,\tau_1,\tau_2)} &= \sigma_t \int_{\tau_1}^{\tau_2} \left\{ \mathrm{e}_1^\top \exp(A_{s-t}) e_p \right\} \\
&\quad \times \Phi \left[\frac{m\left\{t, s, e_1^\top \exp\left(A_{s-t}\right) X_t\right\}}{v_{t,s}} \right] ds dB_t^\theta
\end{aligned}
$$

Benth (2005): No analytical solution, do Monte Carlo simulation

CAT futures price

$$
\begin{aligned}
F_{CAT(t,\tau_1,\tau_2)} &= \int_{\tau_1}^{\tau_2} \Lambda_u du + a_{t,\tau_1,\tau_2} X_t \\
&\quad + \int_{\tau_1}^{\tau_2} \theta_u \sigma_u a_{t,\tau_1,\tau_2} e_p du \\
&\quad + \int_{\tau_1}^{\tau_2} \theta_u \sigma_u e_1^\top A^{-1} \left\{ \exp\left(A_{\tau_2 - u}\right) - I_p \right\} e_p du
\end{aligned}
$$

where I_p : $p \times p$ identity matrix and
$a_{t,\tau_1,\tau_2} = e_1^\top A^{-1} \left\{ \exp(A_{\tau_2 - t}) - \exp(A_{\tau_1 - t}) \right\}$

Time dynamics of F_{CAT}:

$$
dF_{CAT(t,\tau_1,\tau_2)} = \sigma_t a_{t,\tau_1,\tau_2} e_p dB_t^\theta
$$

Data adjustments

Station changes

- Instrumentation
- Location

Trends: to capture weather surprise

- Global Climate Cycles
- Urban heat

Forecasting

Questions

- Memory in AR(3): volatility with monthly/weekly measurement period
- Role of the strike value
- Longterm (interannual) variability of parameters - capture volatility due to climate changes
- Relationship between weather variables and production
- Formula for options on CAT/CDD/HDD futures
- WD related to catastrophes

Conclusion

- CAR(p) model for the temperature dynamics
  - AR with seasonal mean and seasonal volatility
- Analytical future prices for the traded contract on CME
  - HDD/CDD/CAT/ futures with delivery over months or seasons

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