ICARE - localising Conditional AutoRegressive Expectiles

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Motivation

- Risk Exposure
    - Measure tail events
    - Conditional autoregressive expectile (CARE) model

- Time-varying parameters
    - Time-varying parameters in CARE Parameter Dynamics
    - Interval length reflects the structural changes in economy

Objectives

(i) Localising CARE Models
- Local parametric approach (LPA)
- Balance between modelling bias and parameter variability

(ii) Tail Risk Dynamics
- Estimation windows with varying lengths
- Time-varying expectile parameters

Econometrics and Risk Management

Econometrics

- Modelling bias vs. parameter variability
- Interval length and economic variables

Risk Management

- Parameter dynamics and structural changes
- Measuring tail risk

Risk Exposure

An investor observes daily DAX returns from 20050103 to 20141231 and estimates the underlying risk exposure via expectiles (UBS, e.g., 1% and 5%) over a one-year time horizon.

Modelling strategies

(a) Data windows fixed on an ad hoc basis

(b) Adaptively selected data intervals: time-varying parameters

Portfolio Protection

An investor decides about the daily allocation into a stock portfolio (DAX). Goal: the initial portfolio value (100) is preserved at the end of a horizon, i.e., the target floor equals 100.

Decision at day t: multiple of the difference between the portfolio value and the discounted floor up to t is invested into the stock portfolio (DAX), the rest into a riskless asset

Multiplier m selection: constant or time-varying (ICARE) ▸ Constant m

Research Questions

How to account for time-varying parameters in tail event risk measures estimation?

What are the typical data interval lengths assessing risk more accurately, i.e., striking a balance between bias and variability?

How well does the lCARE model perform in practice?

Outline

1. Motivation ✓
2. Conditional Autoregressive Expectile (CARE)
3. Local Parametric Approach (LPA)
4. Empirical Results
5. Applications
6. Conclusions

Conditional Autoregressive Expectile

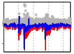
- Taylor (2008), Kuan et al. (2009)
- Random variable Y (e.g. returns), i.i.d., y_t, $t = 1, ..., n$
- CARE specification conditional on information set $\mathcal{F}_{t-1}$

$$y_t = e_{t,\tau} + \varepsilon_{t,\tau}$$

▸ $\varepsilon_\tau \sim \text{AND}\left(0, \sigma^2_{\varepsilon,\tau}, \tau\right)$

$$e_{t,\tau} = \alpha_{0,\tau} + \alpha_{1,\tau} y_{t-1} + \alpha_{2,\tau}\left(y^+_{t-1}\right)^2 + \alpha_{3,\tau}\left(y^-_{t-1}\right)^2$$

- Expectile $e_{t,\tau}$ at $\tau \in (0,1)$, $\theta_\tau = \left\{\alpha_{0,\tau}, \alpha_{1,\tau}, \alpha_{2,\tau}, \alpha_{3,\tau}, \sigma^2_{\varepsilon,\tau}\right\}^\top$
- Returns: $y^+_{t-1} = \max\{y_{t-1}, 0\}$, $y^-_{t-1} = \min\{y_{t-1}, 0\}$



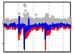
Parameter Estimation

- Data calibration with time-varying intervals
- Observed returns $\mathcal{Y} = \{y_1, \dots, y_n\}$
- Quasi maximum likelihood estimate (QMLE)

$$\widetilde{\theta}_{I,\tau} = \arg \max_{\theta_\tau \in \Theta} \ell_I \left(\mathcal{Y}; \theta_\tau\right)$$

▸ $\ell_I(\cdot)$

- ▶ $I = [t_0 - v, t_0]$ - interval of $(v+1)$ observations at t_0
- ▶ $\ell_I(\cdot)$ - quasi log likelihood



Estimation Quality

- Mercurio and Spokoiny (2004), Spokoiny (2009)
- Quality of estimating *true* parameter vector θ^*_τ by QMLE $\widetilde{\theta}_{I,\tau}$ in terms of Kullback-Leibler divergence; $\mathcal{R}_r(\theta^*_\tau)$ - risk bound

$$\mathsf{E}_{\theta^*_\tau}\left|\ell_I(\mathcal{Y};\widetilde{\theta}_{I,\tau}) - \ell_I(\mathcal{Y};\theta^*_\tau)\right|^r \leq \mathcal{R}_r(\theta^*_\tau)$$

▸ $\mathcal{R}_r(\theta^*_\tau)$ ▸ Gaussian Regression

- 'Modest' risk, $r = 0.5$ (shorter intervals of homogeneity)
- 'Conservative' risk, $r = 1$ (longer intervals of homogeneity)

Solomon Kullback and Richard A. Leibler on BBI:

Local Parametric Approach (LPA)

- LPA, Spokoiny (1998, 2009)
  - Time series parameters can be locally approximated
  - Finding the *interval of homogeneity* ▸ Details
  - Balance between modelling bias and parameter variability

- Time series literature
  - GARCH$(1,1)$ models - Čížek et al. (2009)
  - Realized volatility - Chen et al. (2010)
  - Multiplicative Error Models - Härdle et al. (2014)

Interval Selection

- $(K+1)$ nested intervals with length $n_k = |I_k|$

$$\begin{array}{ccccccccc} I_0 & \subset & I_1 & \subset \cdots \subset & I_k & \subset \cdots \subset & I_K \\ \widetilde{\boldsymbol{\theta}}_0 & & \widetilde{\boldsymbol{\theta}}_1 & & \widetilde{\boldsymbol{\theta}}_k & & \widetilde{\boldsymbol{\theta}}_K \end{array}$$

Example: Daily index returns

Fix i_0, $I_k = [i_0 - n_k, i_0]$, $n_k = \left[n_0 c^k\right]$, $c > 1$

$\{n_k\}_{k=0}^{11} = \{20 \text{ days}, 25 \text{ days}, \ldots, 250 \text{ days}\}$, $c = 1.25$

Local Change Point Detection

- Fix t_0, sequential test $(k = 1, \ldots, K)$

H_0 : parameter homogeneity within I_k
H_1 : $\exists$ change point within $J_k = I_k \setminus I_{k-1}$

$$T_{k,\tau} = \sup_{s \in J_k} \left\{ \ell_{A_{k,s}} \left(\mathcal{Y}, \widetilde{\theta}_{A_{k,s},\tau} \right) + \ell_{B_{k,s}} \left(\mathcal{Y}, \widetilde{\theta}_{B_{k,s},\tau} \right) - \ell_{I_{k+1}} \left(\mathcal{Y}, \widetilde{\theta}_{I_{k+1},\tau} \right) \right\}$$

with $A_{k,s} = [t_0 - n_{k+1}, s]$ and $B_{k,s} = (s, t_0]$

Critical Values, $\mathfrak{z}_{k,\tau}$

- Simulate $\mathfrak{z}_k$ - homogeneity of the interval sequence $I_0, \dots, I_k$
- 'Propagation' condition

$$\mathsf{E}_{\theta_\tau^*} \left| \ell_{I_k} \left(\mathcal{Y}; \widetilde{\theta}_{I_k,\tau} \right) - \ell_{I_k} \left(\mathcal{Y}; \widehat{\theta}_\tau \right) \right|^r \leq \rho_k \mathcal{R}_r \left(\theta_\tau^* \right)$$

$\rho_k = \frac{\rho k}{K}$ for a given significance level ρ ▸ $\widehat{\theta}_\tau$ - adaptive estimate

- Check $\mathfrak{z}_{k,\tau}$ for (six) different θ_τ^* ▸ Parameter Scenarios

Critical Values, $\mathfrak{z}_{k,\tau}$

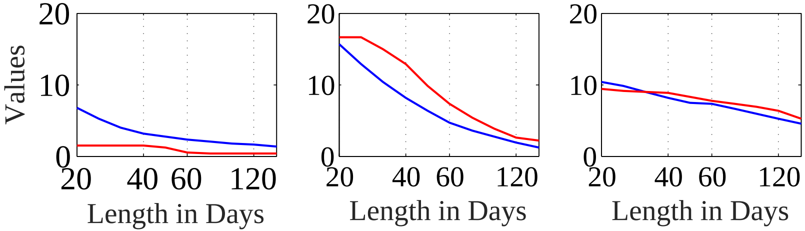


Figure 1: Simulated critical values across different parameter constellations ▸ Parameter Scenarios for the modest case $r = 0.5$, $\tau = 0.05$ and $\tau = 0.01$

Critical Values, $\mathfrak{z}_{k,\tau}$

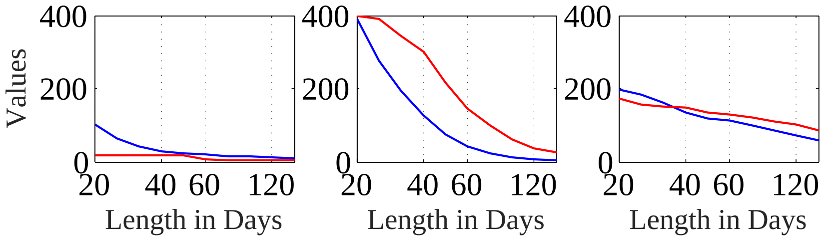


Figure 2: Simulated critical values across different parameter constellations ▸ Parameter Scenarios for the conservative case $r = 1$, $\tau = 0.05$ and $\tau = 0.01$

Adaptive Estimation

▸ LPA ▸ $\mathfrak{z}_{k,\tau}$ - Critical Values

- Compare $T_{k,\tau}$ at every step k with $\mathfrak{z}_{k,\tau}$
- Data window index of the *interval of homogeneity* - $\widehat{k}$

- Adaptive estimate

$$\widehat{\theta}_{\tau} = \widetilde{\theta}_{I_{\hat{k}},\tau}, \quad \widehat{k} = \max_{k \leq K} \{k : T_{\ell,\tau} \leq \mathfrak{z}_{\ell,\tau}, \ell \leq k\}$$

Data

- Series
  - DAX, FTSE 100 and S&P 500 returns 20050103-20141231 (2608 days)
  - Research Data Center (RDC) - Datastream

- Setup
  - Expectile levels: $\tau = 0.05$ and $\tau = 0.01$
  - Modest ($r = 0.5$) and conservative ($r = 1$) risk cases
  - $\{n_k\}_{k=0}^{11} = \{20 \text{ days}, 25 \text{ days}, \ldots, 250 \text{ days}\}$

Adaptive Estimation

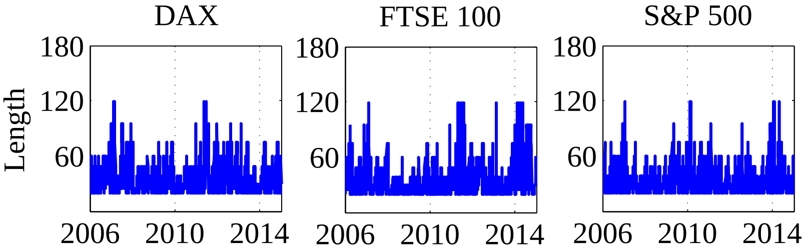


Figure 3: Estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the modest risk case $r = 0.5$, at expectile level $\tau = 0.05$

▸ Weekly

Adaptive Estimation

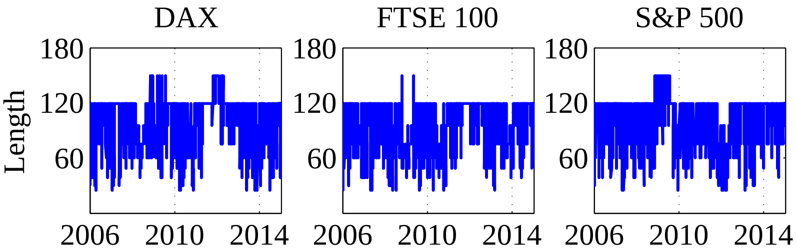


Figure 4: Estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the conservative risk case $r = 1$, at expectile level $\tau = 0.05$

Weekly

Adaptive Estimation

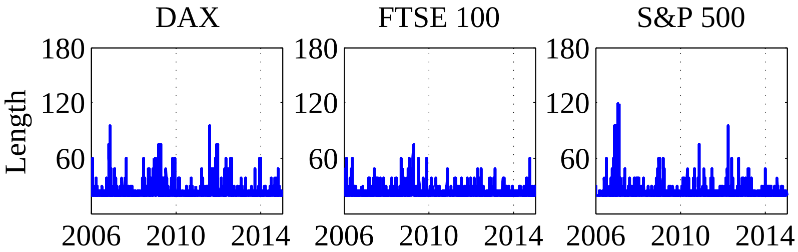


Figure 5: Estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the modest risk case $r = 0.5$, at expectile level $\tau = 0.01$

▸ Weekly

Adaptive Estimation

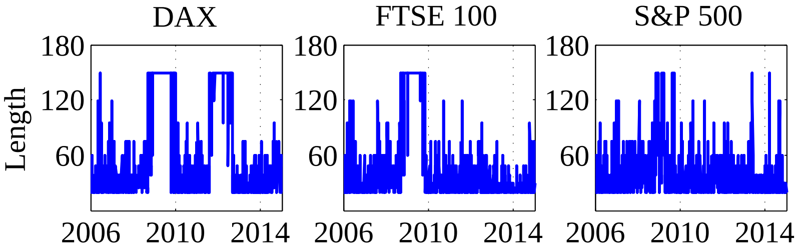


Figure 6: Estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the conservative risk case $r = 1$, at expectile level $\tau = 0.01$

▸ Weekly

Risk Exposure

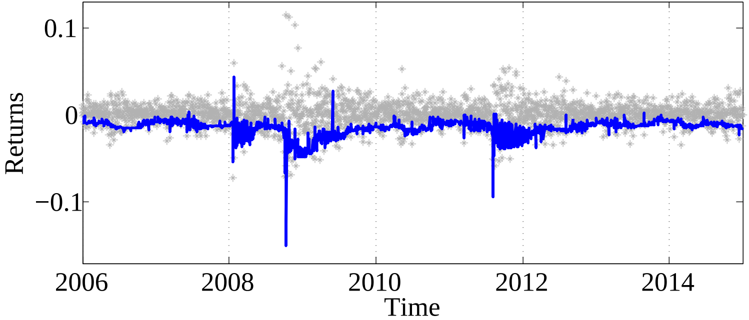
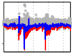


Figure 7: DAX index returns $(*)$ and adaptively estimated expectile $e_{t,\tau}$ ($r = 1$ and $\tau = 0.05$) from 20060103-20141231



Risk Exposure

▸ Expected Shortfall $ES_{e_{t,\tau}}$

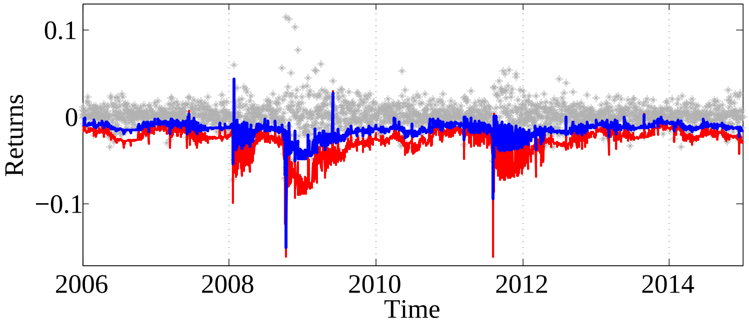


Figure 8: DAX index returns $(*)$, adaptively estimated expectile $e_{t,\tau}$ and expected shortfall $ES_{e_{t,\tau}}$ $(r = 1$ and $\tau = 0.05)$ from 20060103-20141231

Portfolio Protection

Constant m

- Multiplier selection - Hamidi et al. (2014), ICARE

$$m_{t,\tau} = \left| ES_{e_{t,\tau}} \right|^{-1}$$

Details

  - Practice: threshold range for $m_{t,\tau}$, $[1, 12]$

Example

Decision at day t: multiple of the difference between the portfolio value and the discounted floor up to t is invested into the stock portfolio (DAX), the rest into a riskless asset

Multiplier Dynamics

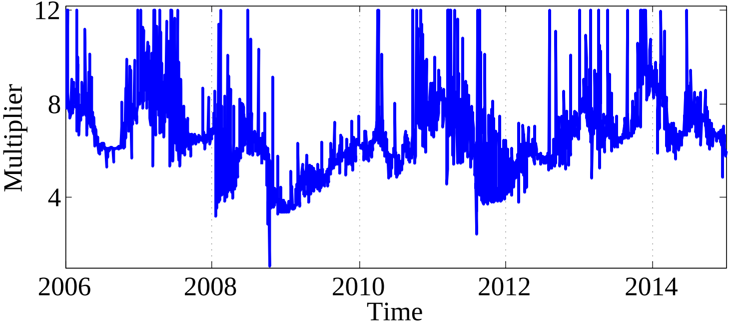


Figure 9: Time-varying multiplier $m_{t,\tau}$ for DAX index returns based on lCARE ($r = 1$ and $\tau = 0.05$) from 20060103-20141231 ▸ Multiplier Density

Performance

Details

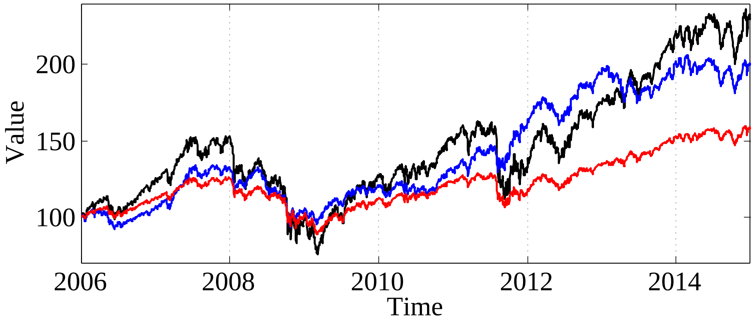
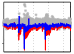


Figure 10: Portfolio value: (a) DAX index, (b) $m = 5$, (c) $m_{t,\tau}$ - ICARE ($r = 1$ and $\tau = 0.05$) from 20060103-20141231. Target floor equals 100.



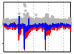
Conclusions

(i) Localising CARE Models

- Balance between modelling bias and parameter variability
- Parameter dynamics

(ii) Tail Risk Dynamics

- Varying distributional characteristics
- Expectile levels $\tau = 0.05$ and $\tau = 0.01$



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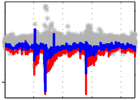

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References

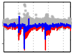
Chen, Y. and Härdle, W. and Pigorsch, U.
Localized Realized Volatility
Journal of the American Statistical Association **105**(492): 1376–1393, 2010

Čížek, P., Härdle, W. and Spokoiny, V.
Adaptive Pointwise Estimation in Time-Inhomogeneous Conditional Heteroscedasticity Models
Econometrics Journal **12**: 248–271, 2009

References

Estep, T. and Kritzman, M.
Time-invariant portfolio protection:insurance without complexity
Journal of Portfolio Management **14**(4): 38–42, 1988

Föllmer, H. and Leukert, P.
Quantile hedging
Finance and Stochastics **3**: 251–273, 1999



References

Gerlach, R.H., Chen, C.W.S. and Lin, L.Y.
Bayesian GARCH Semi-parametric Expected Shortfall Forecasting in Financial Markets
Business Analytics Working Paper No. 01/2012, 2012

Hamidi, B., Maillet, B. and Prigent, J.L.
A dynamic autoregressive expectile for time-invariant portfolio protection strategies
Journal of Economic Dynamics & Control **46**: 1–29, 2014

References

Härdle, W. K., Hautsch, N. and Mihoci, A.
Local Adaptive Multiplicative Error Models for High-Frequency Forecasts
Journal of Applied Econometrics, 2014

Kuan, C.M., Yeh, J.H. and Hsu, Y.C.
Assessing value at risk with CARE, the Conditional Autoregressive Expectile models
Journal of Econometrics **150**(2): 261–270, 2009

References

Mercurio, D. and Spokoiny, V.
Statistical inference for time-inhomogeneous volatility models
The Annals of Statistics **32**(2): 577–602, 2004

Spokoiny, V.
Estimation of a function with discontinuities via local polynomial fit with an adaptive window choice
The Annals of Statistics **26**(4): 1356–1378, 1998

References

Spokoiny, V.
Multiscale Local Change Point Detection with Applications to Value-at-Risk
The Annals of Statistics **37**(3): 1405–1436, 2009

Taylor, J.W.
Estimating Value at Risk and Expected Shortfall Using Expectiles
Journal of Financial Econometrics **6**(2): 231–252, 2008

Yao, Q. and Tong, H.
Asymmetric least squares regression estimation: a nonparametric approach
Journal of Nonparametric Statistics **6**(2): 273–292, 1996

Asymmetric Normal Distribution (AND) ▸ CARE

- If $\varepsilon_\tau \sim \text{AND}\left(\mu, \sigma^2_{\varepsilon_\tau}, \tau\right)$ with pdf

$$f_\varepsilon(w) = \frac{2}{\sigma_{\varepsilon_\tau}}\left(\sqrt{\frac{\pi}{|\tau-1|}} + \sqrt{\frac{\pi}{\tau}}\right)^{-1} \exp\left\{-\rho_\tau\left(\frac{w-\mu}{\sigma_{\varepsilon_\tau}}\right)\right\}$$

  - Check function: $\rho_\tau(u) = |\tau - \mathbf{I}\{u \leq 0\}|\, u^2$
  - Specification following Gerlach et al. (2012)

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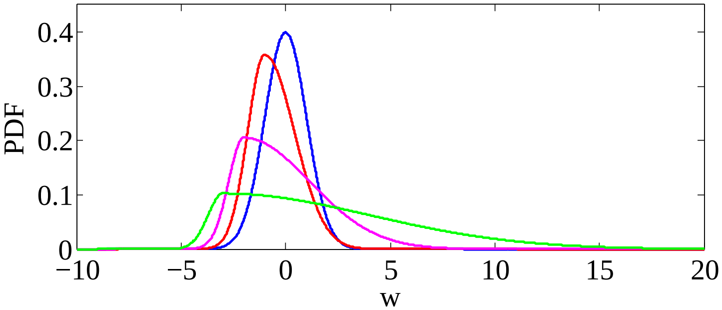


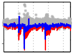
Figure 11: Density function for selected ANDs: (a) $\mu = 0, \tau = 0.5$ (b) $\mu = -1, \tau = 0.25$ (c) $\mu = -2, \tau = 0.05$ (d) $\mu = -3, \tau = 0.01$, with $\sigma^2_{\varepsilon_\tau} = 1$

Quasi Log Likelihood Function

▸ Parameter Estimation

- If $\varepsilon_\tau \sim \text{AND}\left(\mu, \sigma_\varepsilon^2, \tau\right)$ with pdf $f_\varepsilon(\cdot)$ then $Y \sim \text{AND}\left(e_\tau + \mu, \sigma_\varepsilon^2, \tau\right)$
- Quasi log likelihood function for observed data $\mathcal{Y} = \{y_1, \ldots, y_n\}$ over a fixed interval I

$$\ell_I(\mathcal{Y}; \theta_\tau) = \sum_{t \in I} \log f_\varepsilon(y_t - e_{t,\tau})$$



Gaussian Regression

▸ Estimation Quality

$$Y_i = f(X_i) + \varepsilon_i, i = 1, \ldots, n, \text{ weights } W = \{w_i\}_{i=1}^{n}$$

$$L(W, \theta) = \sum_{i=1}^{n} \ell\{Y_i, f_\theta(X_i)\} w_i, \text{ log-density } \ell(\cdot), \ \widetilde{\theta} = \arg\max_{\theta \in \Theta} L(W, \theta)$$

1. Local constant, $f(X_i) \approx \theta^*$, $\varepsilon_i \sim \mathrm{N}(0, \sigma^2)$

$$\mathrm{E}_{\theta^*} \left| L(W, \widetilde{\theta}) - L(W, \theta^*) \right|^r \leq \mathrm{E}\, |\xi|^{2r}, \quad \xi \sim \mathrm{N}(0, 1)$$

2. Local linear, $f(X_i) \approx \theta^{*\top} \Psi_i$, $\varepsilon_i \sim \mathrm{N}(0, \sigma^2)$, basis functions $\Psi = \{\psi_1(X_1), \ldots, \psi_p(X_p)\}$, multivariate ξ

$$\mathrm{E}_{\theta^*} \left| L(W, \widetilde{\theta}) - L(W, \theta^*) \right|^r \leq \mathrm{E}\, |\xi|^{2r}, \quad \xi \sim \mathrm{N}(0, \mathcal{I}_p)$$

Risk Bound ▸ Estimation Quality

	$\tau = 0.05$			$\tau = 0.01$		
	Low	Mid	High	Low	Mid	High
$r = 0.5$	0.24	0.33	0.25	0.38	0.38	0.15
$r = 1.0$	2.40	4.62	2.75	5.90	5.81	1.15

Table 1: Simulated $\mathcal{R}_r(\theta_\tau^*)$, with expectile levels $\tau = 0.05$ and $\tau = 0.01$, for six selected parameter constellation groups ▸ Parameter Scenarios

Parameter Scenarios Risk Bound Critical Values

	$\tau = 0.05$			$\tau = 0.01$		
	Low	Mid	High	Low	Mid	High
$\widetilde{\alpha}_{0,\tau}$	-0.0003	0.0003	0.0007	-0.0003	0.0003	0.0007
$\widetilde{\alpha}_{1,\tau}$	-0.1058	-0.0306	0.0524	-0.1035	-0.0312	0.0547
$\widetilde{\alpha}_{2,\tau}$	-0.5800	-0.5288	0.2438	-0.5808	-0.5266	0.2089
$\widetilde{\alpha}_{3,\tau}$	0.5050	0.5852	2.1213	0.5134	0.5871	2.2066
$\widetilde{\sigma}^2_{\varepsilon,\tau}$	0.0001	0.0001	0.0002	0.0001	0.0001	0.0002

Table 2: Quartiles of estimated CARE parameters based on one-year estimation window, i.e., 250 observations, for the three stock market returns - DAX, FTSE 100, S&P 500 - from 20050103-20141231 (2608 trading days)

Expected Shortfall

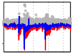
▸ Risk Exposure

- Expectile level τ_α such that $e_{t,\tau_\alpha} = q_\alpha$ (α-quantile), Yao and Tong (1996)

$$\tau_\alpha = \frac{\alpha \cdot q_\alpha - \int_{-\infty}^{q_\alpha} y dF(y)}{\mathsf{E}[Y] - 2\int_{-\infty}^{q_\alpha} y dF(y) - (1 - 2\alpha)\, q_\alpha}$$

- Expected Shortfall (ES), Kuan et al. (2009)

$$ES_{e_{t,\tau_\alpha}} = \left| 1 + \tau_\alpha (1 - 2\tau_\alpha)^{-1} \alpha^{-1} \right| e_{t,\tau_\alpha}$$



Strategy Setup ▸ Performance

- V_t the value of the portfolio at time t, $t \in (0, T]$
- Predetermine a protection value (floor), F_t the discount floor up to time t, Estep and Kritzman (1988)

$$V_t \geq \max\left\{F_t,\ k \times \sup_{p \leq t} V_p\right\} = F_t^s$$

  - k exogenous parameter $(0, 1)$
  - Cushion value $C_t = V_t - F_t^s \geq 0$
- Allocate $G_t = m \cdot C_t$ proportion into stock portfolio (DAX), and the remaining $V_t - G_t$ into riskless asset
- The multiplier m is non negative ▸ Solution

Multiplier

Mutiplier

- Portfolio value V_t Setup

$$V_{t+1} = V_t + G_t r_{t+1} + (V_t - G_t)\, rf_{t+1}$$

with r_t stock index return and rf_t riskless rate

- Cushion value $C_t = V_t - F_t^s \geq 0$

$$C_{t+1} = C_t\,[1 + m \cdot r_{t+1} + (1-m)\, rf_{t+1}]$$

- $\forall t \leq T$, since the value $C_t \geq 0$

$$m \cdot r_{t+1} + (1-m)\, rf_{t+1} \geq -1$$

- rf_t is relatively small, yield the upper bound on the multiple

$$m \leq (-r_{t+1})^{-1}, \; \forall t \leq T$$

Multiplier

▸ Mutiplier

- Given a confidence leve α, the protection portfolio condition

$$\mathrm{P}\left(C_t \geq 0,\ \forall t \leq T\right) \ \geq \ 1 - \alpha$$

- The above equation is equivalent to (set time-varying multiplier)

$$\mathrm{P}\left(m_t \leq (-r_{t+1})^{-1},\ \forall t \leq T\right) \ \geq \ 1 - \alpha$$

- Multiplier m_t with quantile - Föllmer and Leukert (1999)

$$m_t = \left|VaR_{1-\alpha}(r_{t+1})\right|^{-1}$$

- Multiple m_t with expected shortfall - Hamidi et al. (2014)

$$m_t = \left|ES_{e_{t,\tau}}\right|^{-1}$$

Multiplier Density

▸ Multiplier Dynamics

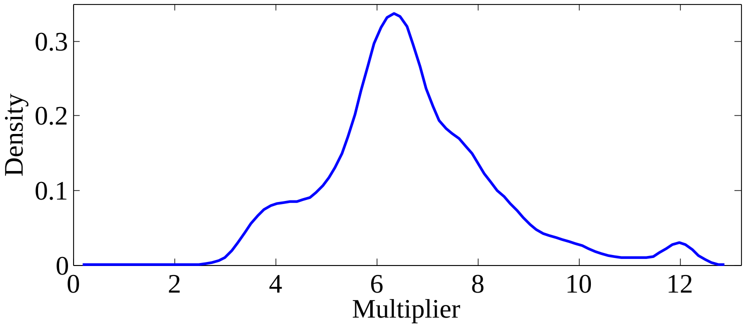


Figure 12: Kernel density estimate of the multiplier $m_{t,\tau}$ for DAX index returns based on lCARE ($r = 1$ and $\tau = 0.05$) from 20060103-20141231

Portfolio Protection ▸ Motivation ▸ Mutiplier

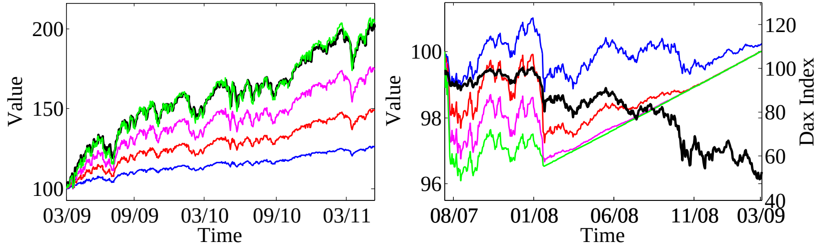


Figure 13: Portfolio value: (a) DAX index, (b) $m = 3$, (c) $m = 6$, (d) $m = 9$, (e) $m = 12$ on DAX index in a bull market from 20090309-20110510 (left panel, 567 observations) and in a bear market from 20070716-20090306 (right panel, 431 observations). Target floor equals 100.

Parameter Dynamics

Motivation

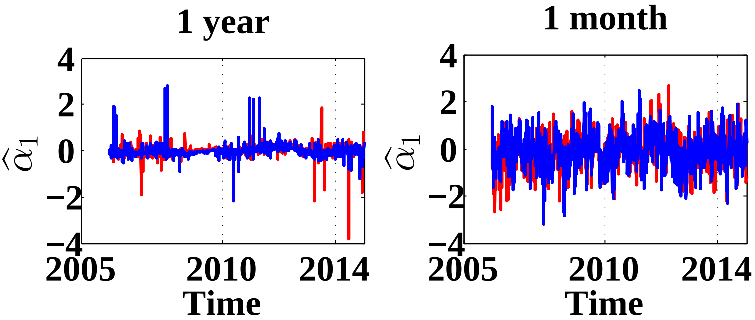


Figure 14: Estimated $\alpha_{1,0.05}$ for DAX and FTSE100 using 20 (1 month) or 250 (1 year) observations more parameters

Parameter Dynamics ▸ Motivation

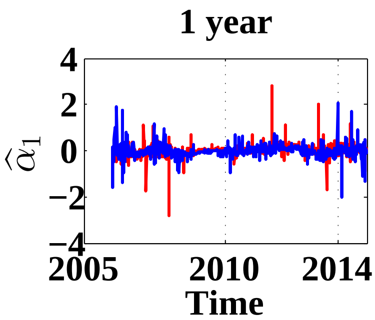
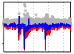


Figure 15: Estimated $\alpha_{1,0.01}$ for DAX and FTSE100 using 20 (1 month) or 250 (1 year) observations ▸ more parameters



Parameter Distributions

Motivation

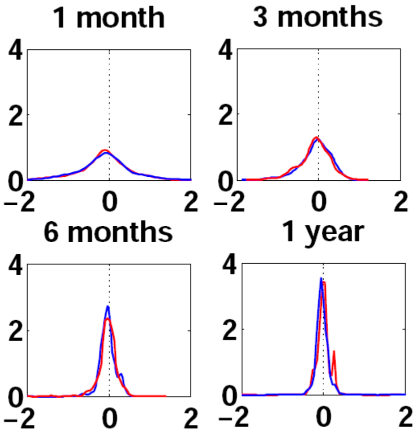


Figure 16: Kernel density estimates of $\alpha_{1,0.05}$ for DAX and FTSE100 using 20, 60, 125 or 250 observations

Parameter Distributions ▸ Motivation

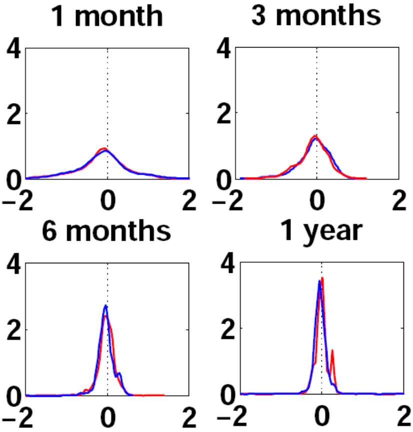


Figure 17: Kernel density estimates of $\alpha_{1,0.01}$ for DAX and FTSE100 using 20, 60, 125 or 250 observations

Parameter Dynamics

Parameter Dynamics

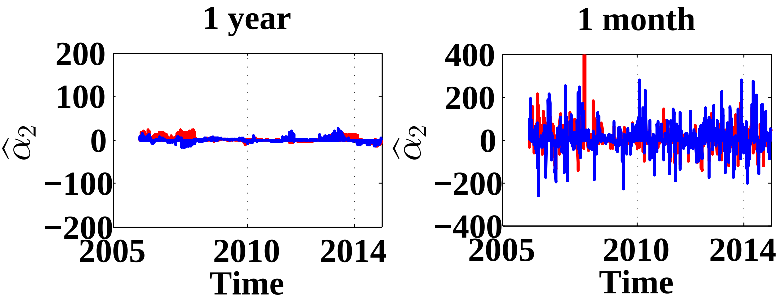


Figure 18: Estimated $\alpha_{2,0.05}$ for DAX and FTSE100 using 20 (1 month) or 250 (1 year) observations

Parameter Dynamics

Parameter Dynamics

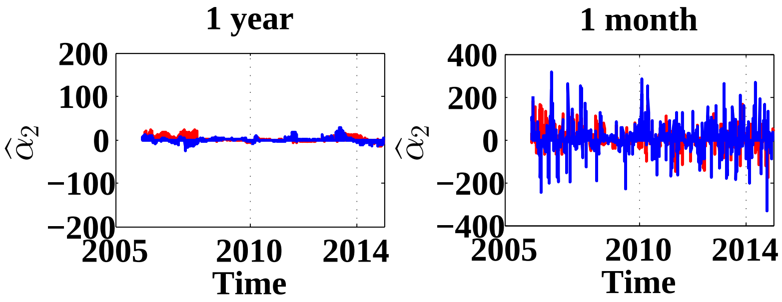


Figure 19: Estimated $\alpha_{2,0.01}$ for DAX and FTSE100 using 20 (1 month) or 250 (1 year) observations

Parameter Dynamics

▸ Parameter Dynamics

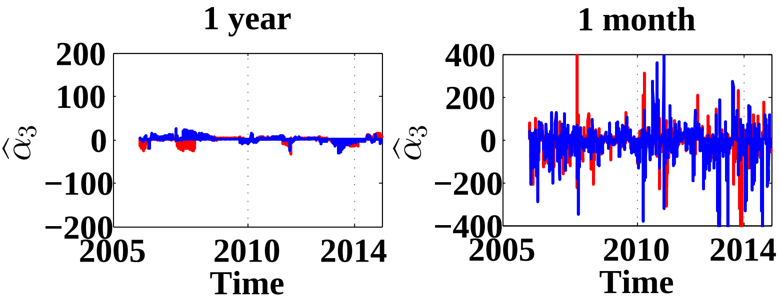


Figure 20: Estimated $\alpha_{3,0.05}$ for DAX and FTSE100 using 20 (1 month) or 250 (1 year) observations

Parameter Dynamics

Parameter Dynamics

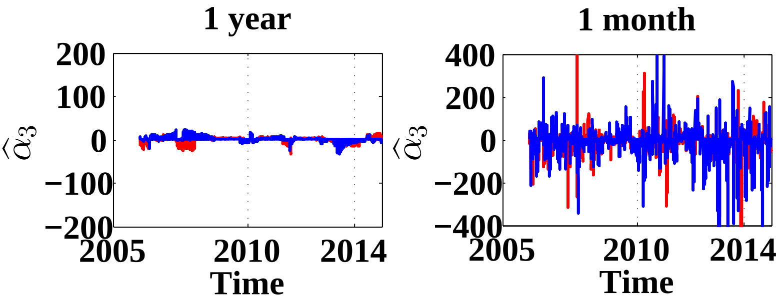


Figure 21: Estimated $\alpha_{3,0.01}$ for DAX and FTSE100 using 20 (1 month) or 250 (1 year) observations

Adaptive Estimation - Weekly

▸ Adaptive Estimation

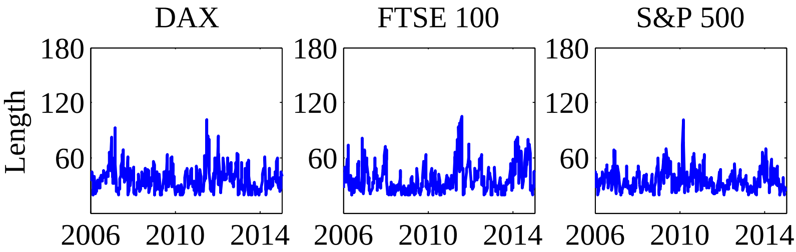


Figure 22: Weekly estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the modest risk case $r = 0.5$, at expectile level $\tau = 0.05$

Adaptive Estimation - Weekly

Adaptive Estimation

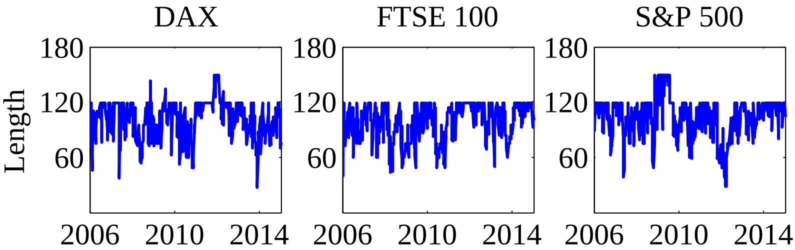


Figure 23: Weekly estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the conservative risk case $r = 1$, at expectile level $\tau = 0.05$

Adaptive Estimation - Weekly

▸ Adaptive Estimation

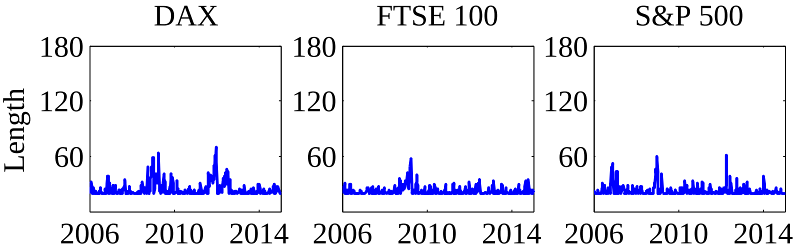
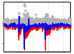


Figure 24: Weekly estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the modest risk case $r = 0.5$, at expectile level $\tau = 0.01$



Adaptive Estimation - Weekly

Adaptive Estimation

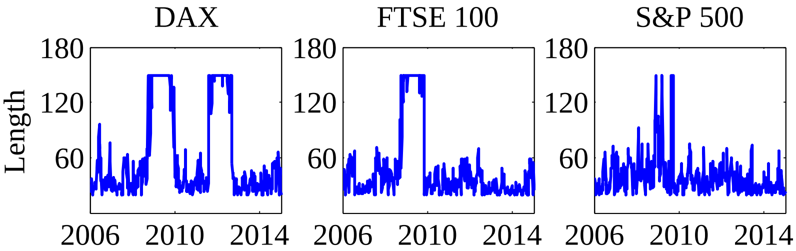


Figure 25: Weekly estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the conservative risk case $r = 1$, at expectile level $\tau = 0.01$