

ICARE - localising Conditional AutoRegressive Expectiles

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Motivation

- Risk Exposure
 - ▶ Measure tail events
 - ▶ Conditional autoregressive expectile (CARE) model

- Time-varying parameters
 - ▶ Time-varying parameters in CARE ▶ Parameter Dynamics
 - ▶ Interval length reflects the structural changes in economy



Objectives

(i) Localising CARE Models

- ▶ Local parametric approach (LPA)
- ▶ Balance between modelling bias and parameter variability

(ii) Tail Risk Dynamics

- ▶ Estimation windows with varying lengths
- ▶ Time-varying expectile parameters



Econometrics and Risk Management

Econometrics

- Modelling bias vs. parameter variability
- Interval length and economic variables

Risk Management

- Parameter dynamics and structural changes
- Measuring tail risk



Risk Exposure

An investor observes daily DAX returns from 20050103 to 20141231 and estimates the underlying risk exposure via expectiles (UBS, e.g., 1% and 5%) over a one-year time horizon.

Modelling strategies

- (a) Data windows fixed on an ad hoc basis
- (b) Adaptively selected data intervals: time-varying parameters



Portfolio Protection

An investor decides about the daily allocation into a stock portfolio (DAX). Goal: the initial portfolio value (100) is preserved at the end of a horizon, i.e., the target floor equals 100.

Decision at day t : multiple of the difference between the portfolio value and the discounted floor up to t is invested into the stock portfolio (DAX), the rest into a riskless asset

Multiplier m selection: constant or time-varying (ICARE)



Research Questions

How to account for time-varying parameters in tail event risk measures estimation?

What are the typical data interval lengths assessing risk more accurately, i.e., striking a balance between bias and variability?

How well does the ICARE model perform in practice?



Outline

1. Motivation ✓
2. Conditional Autoregressive Expectile (CARE)
3. Local Parametric Approach (LPA)
4. Empirical Results
5. Applications
6. Conclusions



Conditional Autoregressive Expectile

- ▣ Taylor (2008), Kuan et al. (2009)
- ▣ Random variable Y (e.g. returns), i.i.d., y_t , $t = 1, \dots, n$
- ▣ CARE specification conditional on information set \mathcal{F}_{t-1}

$$y_t = e_{t,\tau} + \varepsilon_{t,\tau} \quad \varepsilon_{t,\tau} \sim \text{AND}(0, \sigma_{\varepsilon,\tau}^2, \tau)$$

$$e_{t,\tau} = \alpha_{0,\tau} + \alpha_{1,\tau} y_{t-1} + \alpha_{2,\tau} (y_{t-1}^+)^2 + \alpha_{3,\tau} (y_{t-1}^-)^2$$

- ▶ Expectile $e_{t,\tau}$ at $\tau \in (0, 1)$, $\theta_\tau = \{\alpha_{0,\tau}, \alpha_{1,\tau}, \alpha_{2,\tau}, \alpha_{3,\tau}, \sigma_{\varepsilon,\tau}^2\}^\top$
- ▶ Returns: $y_{t-1}^+ = \max\{y_{t-1}, 0\}$, $y_{t-1}^- = \min\{y_{t-1}, 0\}$



Parameter Estimation

- Data calibration with time-varying intervals
- Observed returns $\mathcal{Y} = \{y_1, \dots, y_n\}$
- Quasi maximum likelihood estimate (QMLE)

$$\tilde{\theta}_{I,\tau} = \arg \max_{\theta_\tau \in \Theta} \ell_I(\mathcal{Y}; \theta_\tau) \quad \triangleright \ell_I(\cdot)$$

- ▶ $I = [t_0 - \nu, t_0]$ - interval of $(\nu + 1)$ observations at t_0
- ▶ $\ell_I(\cdot)$ - quasi log likelihood



Estimation Quality

- Mercurio and Spokoiny (2004), Spokoiny (2009)
- Quality of estimating *true* parameter vector θ_τ^* by QMLE $\tilde{\theta}_{I,\tau}$ in terms of Kullback-Leibler divergence; $\mathcal{R}_r(\theta_\tau^*)$ - risk bound

$$E_{\theta_\tau^*} \left| \ell_I(\mathcal{Y}; \tilde{\theta}_{I,\tau}) - \ell_I(\mathcal{Y}; \theta_\tau^*) \right|^r \leq \mathcal{R}_r(\theta_\tau^*)$$

▶ $\mathcal{R}_r(\theta_\tau^*)$

▶ Gaussian Regression

- 'Modest' risk, $r = 0.5$ (shorter intervals of homogeneity)
- 'Conservative' risk, $r = 1$ (longer intervals of homogeneity)

Solomon Kullback and Richard A. Leibler on BBI:



Local Parametric Approach (LPA)

- LPA, Spokoiny (1998, 2009)
 - ▶ Time series parameters can be locally approximated
 - ▶ Finding the *interval of homogeneity* [▶ Details](#)
 - ▶ Balance between modelling bias and parameter variability

- Time series literature
 - ▶ GARCH(1, 1) models - Čížek et al. (2009)
 - ▶ Realized volatility - Chen et al. (2010)
 - ▶ Multiplicative Error Models - Härdle et al. (2014)



Interval Selection

- $(K + 1)$ nested intervals with length $n_k = |I_k|$

$$\begin{matrix} I_0 & \subset & I_1 & \subset & \dots & \subset & I_k & \subset & \dots & \subset & I_K \\ \tilde{\theta}_0 & & \tilde{\theta}_1 & & & & \tilde{\theta}_k & & & & \tilde{\theta}_K \end{matrix}$$

Example: Daily index returns

Fix i_0 , $I_k = [i_0 - n_k, i_0]$, $n_k = \lceil n_0 c^k \rceil$, $c > 1$

$\{n_k\}_{k=0}^{11} = \{20 \text{ days}, 25 \text{ days}, \dots, 250 \text{ days}\}$, $c = 1.25$



Local Change Point Detection

□ Fix t_0 , sequential test ($k = 1, \dots, K$)

H_0 : parameter homogeneity within I_k

H_1 : \exists change point within $J_k = I_k \setminus I_{k-1}$

$$T_{k,\tau} = \sup_{s \in J_k} \left\{ \ell_{A_{k,s}} \left(\mathcal{Y}, \tilde{\theta}_{A_{k,s},\tau} \right) + \ell_{B_{k,s}} \left(\mathcal{Y}, \tilde{\theta}_{B_{k,s},\tau} \right) - \ell_{I_{k+1}} \left(\mathcal{Y}, \tilde{\theta}_{I_{k+1},\tau} \right) \right\}$$

with $A_{k,s} = [t_0 - n_{k+1}, s]$ and $B_{k,s} = (s, t_0]$



Critical Values, $\mathfrak{z}_{k,\tau}$

- Simulate \mathfrak{z}_k - homogeneity of the interval sequence l_0, \dots, l_k
- 'Propagation' condition

$$E_{\theta_\tau^*} \left| \ell_{l_k} \left(\mathcal{Y}; \tilde{\theta}_{l_k, \tau} \right) - \ell_{l_k} \left(\mathcal{Y}; \hat{\theta}_\tau \right) \right|^r \leq \rho_k \mathcal{R}_r \left(\theta_\tau^* \right)$$

$\rho_k = \frac{\rho k}{K}$ for a given significance level ρ ▶ $\hat{\theta}_\tau$ - adaptive estimate

- Check $\mathfrak{z}_{k,\tau}$ for (six) different θ_τ^* ▶ Parameter Scenarios



Critical Values, $\mathfrak{z}_{k,\tau}$

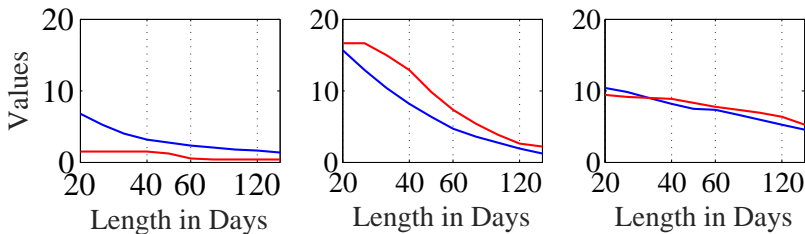


Figure 1: Simulated critical values across different parameter constellations

▶ Parameter Scenarios

for the modest case $r = 0.5$, $\tau = 0.05$ and $\tau = 0.01$



Critical Values, $\mathfrak{z}_{k,\tau}$

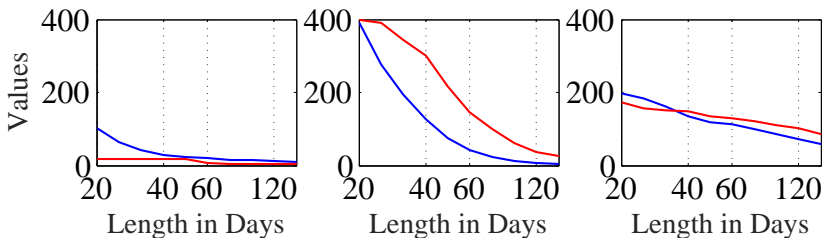


Figure 2: Simulated critical values across different parameter constellations

[Parameter Scenarios](#) for the conservative case $r = 1$, $\tau = 0.05$ and $\tau = 0.01$



Adaptive Estimation

▶ LPA

▶ $\mathfrak{z}_{k,\tau}$ - Critical Values

- Compare $T_{k,\tau}$ at every step k with $\mathfrak{z}_{k,\tau}$
- Data window index of the *interval of homogeneity* - \hat{k}
- Adaptive estimate

$$\hat{\theta}_\tau = \tilde{\theta}_{I_{\hat{k},\tau}}, \quad \hat{k} = \max_{k \leq K} \{k : T_{l,\tau} \leq \mathfrak{z}_{l,\tau}, l \leq k\}$$



Data

Series

- ▶ DAX, FTSE 100 and S&P 500 returns
20050103-20141231 (2608 days)
- ▶ Research Data Center (RDC) - Datastream

Setup

- ▶ Expectile levels: $\tau = 0.05$ and $\tau = 0.01$
- ▶ Modest ($r = 0.5$) and conservative ($r = 1$) risk cases
- ▶ $\{n_k\}_{k=0}^{11} = \{20 \text{ days}, 25 \text{ days}, \dots, 250 \text{ days}\}$



Adaptive Estimation

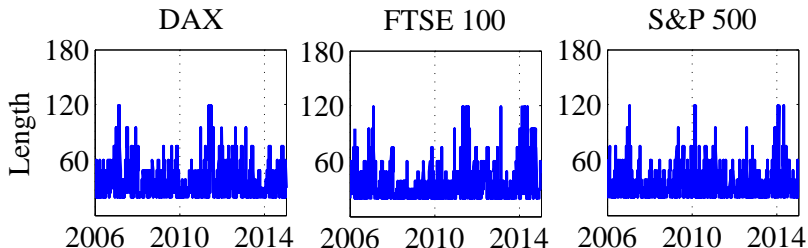


Figure 3: Estimated length $n_{\hat{k}}$ of intervals of homogeneity from 20060103-20141231 for the modest risk case $r = 0.5$, at expectile level $\tau = 0.05$

► Weekly



Adaptive Estimation

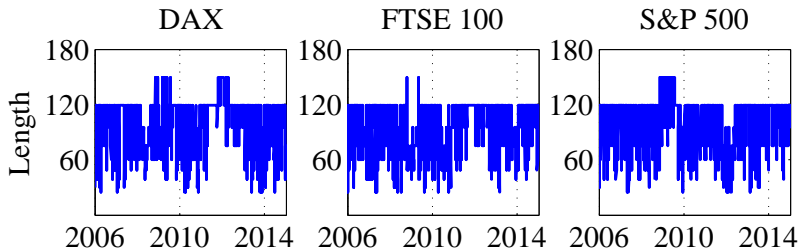


Figure 4: Estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the conservative risk case $r = 1$, at expectile level $\tau = 0.05$

▶ Weekly



Adaptive Estimation

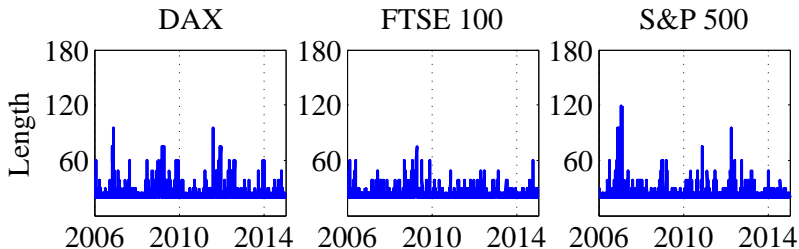


Figure 5: Estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the modest risk case $r = 0.5$, at expectile level $\tau = 0.01$

► Weekly



Adaptive Estimation

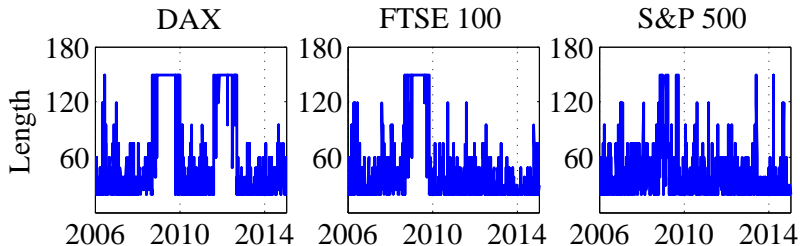


Figure 6: Estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the conservative risk case $r = 1$, at expectile level $\tau = 0.01$

► Weekly



Risk Exposure

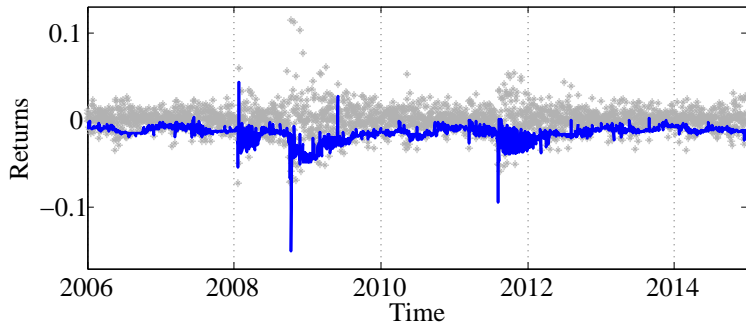


Figure 7: DAX index returns (*) and adaptively estimated expectile $e_{t,\tau}$ ($r = 1$ and $\tau = 0.05$) from 20060103-20141231



Risk Exposure

▶ Expected Shortfall $ES_{e_t, \tau}$

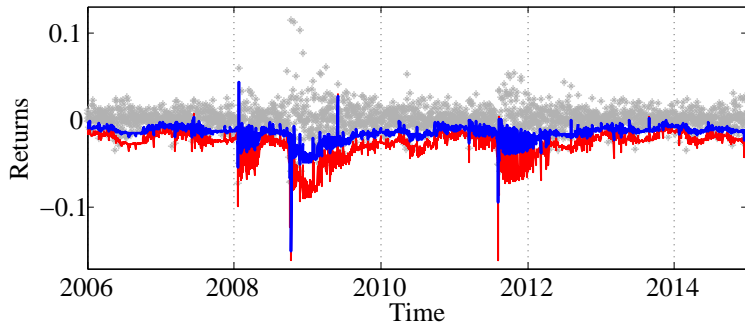


Figure 8: DAX index returns (*), adaptively estimated **expectile** $e_{t, \tau}$ and **expected shortfall** $ES_{e_t, \tau}$ ($r = 1$ and $\tau = 0.05$) from 20060103-20141231



Portfolio Protection

- Multiplier selection - Hamidi et al. (2014), ICARE

$$m_{t,\tau} = |ES_{e_{t,\tau}}|^{-1} \quad \text{Details}$$

- ▶ Practice: threshold range for $m_{t,\tau}$, [1, 12]

Example

Decision at day t : multiple of the difference between the portfolio value and the discounted floor up to t is invested into the stock portfolio (DAX), the rest into a riskless asset



Multiplier Dynamics

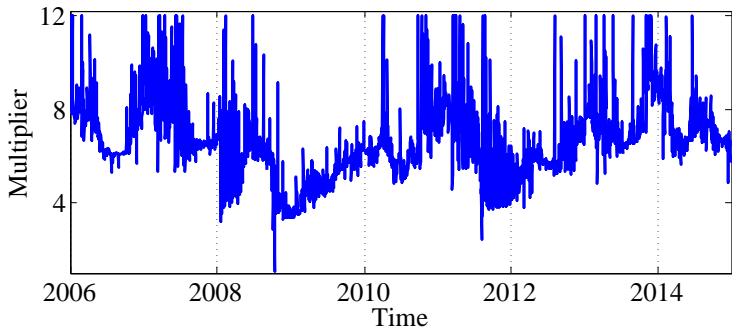


Figure 9: Time-varying multiplier $m_{t,\tau}$ for DAX index returns based on ICARE ($r = 1$ and $\tau = 0.05$) from 20060103-20141231 [▶ Multiplier Density](#)



Performance [▸ Details](#)

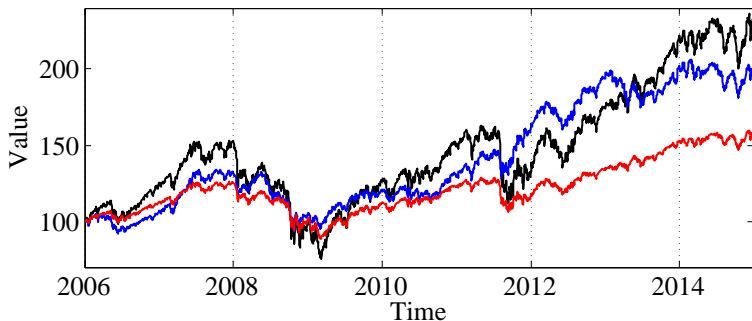


Figure 10: Portfolio value: (a) DAX index, (b) $m = 5$, (c) $m_{t,\tau} - \text{ICARE}$ ($r = 1$ and $\tau = 0.05$) from 20060103-20141231. Target floor equals 100.



Conclusions

(i) Localising CARE Models

- ▶ Balance between modelling bias and parameter variability
- ▶ Parameter dynamics

(ii) Tail Risk Dynamics

- ▶ Varying distributional characteristics
- ▶ Expectile levels $\tau = 0.05$ and $\tau = 0.01$



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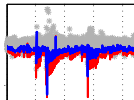
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References



Chen, Y. and Härdle, W. and Pigorsch, U.

Localized Realized Volatility

Journal of the American Statistical Association **105**(492):
1376–1393, 2010



Čížek, P., Härdle, W. and Spokoiny, V.

*Adaptive Pointwise Estimation in Time-Inhomogeneous
Conditional Heteroscedasticity Models*

Econometrics Journal **12**: 248–271, 2009



References



Estep, T. and Kritzman, M.

Time-invariant portfolio protection: insurance without complexity

Journal of Portfolio Management 14(4): 38–42, 1988



Föllmer, H. and Leukert, P.

Quantile hedging

Finance and Stochastics 3: 251–273, 1999



References



Gerlach, R.H., Chen, C.W.S. and Lin, L.Y.

*Bayesian GARCH Semi-parametric Expected Shortfall
Forecasting in Financial Markets*

Business Analytics Working Paper No. 01/2012, 2012





Hamidi, B., Maillet, B. and Prigent, J.L.

*A dynamic autoregressive expectile for time-invariant portfolio
protection strategies*

Journal of Economic Dynamics & Control **46**: 1–29, 2014



References

-  Hädle, W. K., Hautsch, N. and Mihoci, A.
Local Adaptive Multiplicative Error Models for High-Frequency Forecasts
Journal of Applied Econometrics, 2014
-  Kuan, C.M., Yeh, J.H. and Hsu, Y.C.
Assessing value at risk with CARE, the Conditional Autoregressive Expectile models
Journal of Econometrics **150**(2): 261–270, 2009



References



Mercurio, D. and Spokoiny, V.

Statistical inference for time-inhomogeneous volatility models

The Annals of Statistics **32**(2): 577–602, 2004



Spokoiny, V.

Estimation of a function with discontinuities via local polynomial fit with an adaptive window choice

The Annals of Statistics **26**(4): 1356–1378, 1998



References



Spokoiny, V.

Multiscale Local Change Point Detection with Applications to Value-at-Risk

The Annals of Statistics **37**(3): 1405–1436, 2009



Taylor, J.W.

Estimating Value at Risk and Expected Shortfall Using Expectiles

Journal of Financial Econometrics **6**(2): 231–252, 2008



Yao, Q. and Tong, H.

Asymmetric least squares regression estimation: a nonparametric approach

Journal of Nonparametric Statistics **6**(2): 273–292, 1996



Asymmetric Normal Distribution (AND) ▶ ICARE

□ If $\varepsilon_\tau \sim \text{AND}(\mu, \sigma_{\varepsilon_\tau}^2, \tau)$ with pdf

$$f_\varepsilon(w) = \frac{2}{\sigma_{\varepsilon_\tau}} \left(\sqrt{\frac{\pi}{|\tau-1|}} + \sqrt{\frac{\pi}{\tau}} \right)^{-1} \exp \left\{ -\rho_\tau \left(\frac{w-\mu}{\sigma_{\varepsilon_\tau}} \right) \right\}$$

- ▶ Check function: $\rho_\tau(u) = |\tau - \mathbf{1}\{u \leq 0\}| u^2$
- ▶ Specification following Gerlach et al. (2012)



PDF

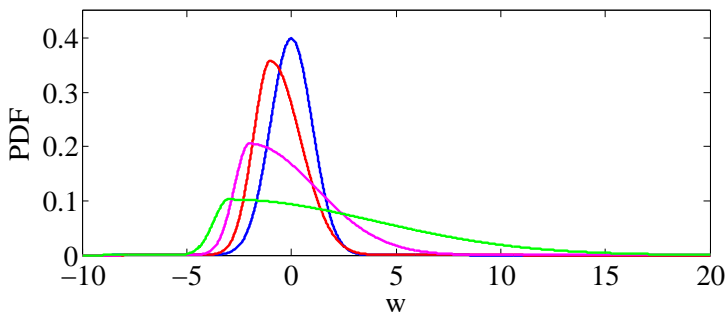


Figure 11: Density function for selected ANDs: (a) $\mu = 0, \tau = 0.5$
(b) $\mu = -1, \tau = 0.25$ (c) $\mu = -2, \tau = 0.05$ (d) $\mu = -3, \tau = 0.01$,
with $\sigma_{\varepsilon_\tau}^2 = 1$



Quasi Log Likelihood Function

▶ Parameter Estimation

- If $\varepsilon_\tau \sim \text{AND}(\mu, \sigma_\varepsilon^2, \tau)$ with pdf $f_\varepsilon(\cdot)$
then $Y \sim \text{AND}(e_\tau + \mu, \sigma_\varepsilon^2, \tau)$
- Quasi log likelihood function for observed data
 $\mathcal{Y} = \{y_1, \dots, y_n\}$ over a fixed interval I

$$\ell_I(\mathcal{Y}; \theta_\tau) = \sum_{t \in I} \log f_\varepsilon(y_t - e_{t,\tau})$$



Gaussian Regression ▶ Estimation Quality

$Y_i = f(X_i) + \varepsilon_i, i = 1, \dots, n$, weights $W = \{w_i\}_{i=1}^n$

$$L(W, \theta) = \sum_{i=1}^n \ell\{Y_i, f_\theta(X_i)\} w_i, \text{ log-density } \ell(\cdot), \tilde{\theta} = \arg \max_{\theta \in \Theta} L(W, \theta)$$

1. Local constant, $f(X_i) \approx \theta^*$, $\varepsilon_i \sim N(0, \sigma^2)$

$$E_{\theta^*} \left| L(W, \tilde{\theta}) - L(W, \theta^*) \right|^r \leq E |\xi|^{2r}, \quad \xi \sim N(0, 1)$$

2. Local linear, $f(X_i) \approx \theta^{*\top} \Psi_i$, $\varepsilon_i \sim N(0, \sigma^2)$, basis functions $\Psi = \{\psi_1(X_1), \dots, \psi_p(X_p)\}$, multivariate ξ

$$E_{\theta^*} \left| L(W, \tilde{\theta}) - L(W, \theta^*) \right|^r \leq E |\xi|^{2r}, \quad \xi \sim N(0, \mathcal{I}_p)$$



Risk Bound ▶ Estimation Quality

	$\tau = 0.05$			$\tau = 0.01$		
	Low	Mid	High	Low	Mid	High
$r = 0.5$	0.24	0.33	0.25	0.38	0.38	0.15
$r = 1.0$	2.40	4.62	2.75	5.90	5.81	1.15

Table 1: Simulated $\mathcal{R}_r(\theta_\tau^*)$, with expectile levels $\tau = 0.05$ and $\tau = 0.01$, for six selected parameter constellation groups ▶ Parameter Scenarios



Parameter Scenarios

[▶ Risk Bound](#)
[▶ Critical Values](#)

	$\tau = 0.05$			$\tau = 0.01$		
	Low	Mid	High	Low	Mid	High
$\tilde{\alpha}_{0,\tau}$	-0.0003	0.0003	0.0007	-0.0003	0.0003	0.0007
$\tilde{\alpha}_{1,\tau}$	-0.1058	-0.0306	0.0524	-0.1035	-0.0312	0.0547
$\tilde{\alpha}_{2,\tau}$	-0.5800	-0.5288	0.2438	-0.5808	-0.5266	0.2089
$\tilde{\alpha}_{3,\tau}$	0.5050	0.5852	2.1213	0.5134	0.5871	2.2066
$\tilde{\sigma}_{\varepsilon,\tau}^2$	0.0001	0.0001	0.0002	0.0001	0.0001	0.0002

Table 2: Quartiles of estimated CARE parameters based on one-year estimation window, i.e., 250 observations, for the three stock market returns - DAX, FTSE 100, S&P 500 - from 20050103-20141231 (2608 trading days)



Expected Shortfall

► Risk Exposure

- Expectile level τ_α such that $e_{t,\tau_\alpha} = q_\alpha$ (α -quantile), Yao and Tong (1996)

$$\tau_\alpha = \frac{\alpha \cdot q_\alpha - \int_{-\infty}^{q_\alpha} y dF(y)}{E[Y] - 2 \int_{-\infty}^{q_\alpha} y dF(y) - (1 - 2\alpha) q_\alpha}$$

- Expected Shortfall (ES), Kuan et al. (2009)

$$ES_{e_{t,\tau_\alpha}} = \left| 1 + \tau_\alpha (1 - 2\tau_\alpha)^{-1} \alpha^{-1} \right| e_{t,\tau_\alpha}$$



Strategy Setup ▶ Performance

- V_t the value of the portfolio at time t , $t \in (0, T]$
- Predetermine a protection value (floor), F_t the discount floor up to time t , Estep and Kritzman (1988)

$$V_t \geq \max \left\{ F_t, k \times \sup_{p \leq t} V_p \right\} = F_t^s$$

- ▶ k exogenous parameter $(0, 1)$
- ▶ Cushion value $C_t = V_t - F_t^s \geq 0$
- Allocate $G_t = m \cdot C_t$ proportion into stock portfolio (DAX), and the remaining $V_t - G_t$ into riskless asset
- The multiplier m is non negative ▶ Solution



Multiplier ▶ Multiplier

- Portfolio value V_t ▶ Setup

$$V_{t+1} = V_t + G_t r_{t+1} + (V_t - G_t) r_{t+1}^f$$

with r_t stock index return and r_t^f riskless rate

- Cushion value $C_t = V_t - F_t^s \geq 0$

$$C_{t+1} = C_t [1 + m \cdot r_{t+1} + (1 - m) r_{t+1}^f]$$

- $\forall t \leq T$, since the value $C_t \geq 0$

$$m \cdot r_{t+1} + (1 - m) r_{t+1}^f \geq -1$$

- r_t^f is relatively small, yield the upper bound on the multiple

$$m \leq (-r_{t+1})^{-1}, \forall t \leq T$$



Multiplier ▶ Multiplier

- Given a confidence level α , the protection portfolio condition

$$P(C_t \geq 0, \forall t \leq T) \geq 1 - \alpha$$

- The above equation is equivalent to
(set time-varying multiplier)

$$P(m_t \leq (-r_{t+1})^{-1}, \forall t \leq T) \geq 1 - \alpha$$

- Multiplier m_t with quantile - Föllmer and Leukert (1999)

$$m_t = |VaR_{1-\alpha}(r_{t+1})|^{-1}$$

- Multiple m_t with expected shortfall - Hamidi et al. (2014)

$$m_t = |ES_{e_t, \tau}|^{-1}$$



Multiplier Density ▶ Multiplier Dynamics

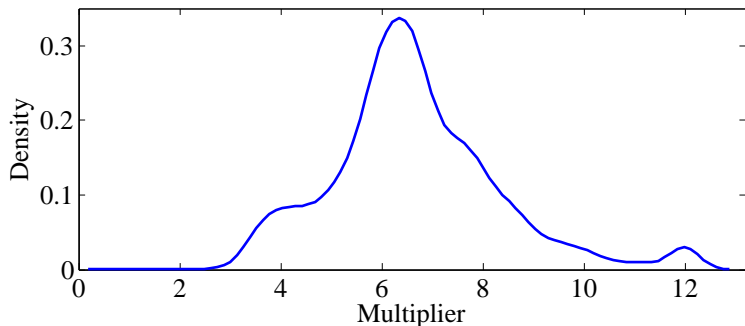


Figure 12: Kernel density estimate of the multiplier $m_{t,\tau}$ for DAX index returns based on ICARE ($r = 1$ and $\tau = 0.05$) from 20060103-20141231



Parameter Dynamics [▶ Motivation](#)

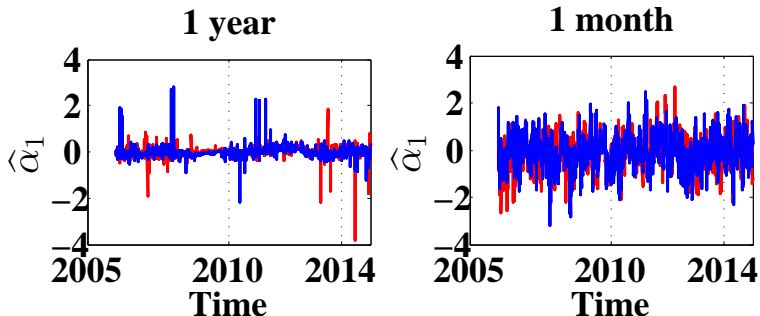


Figure 13: Estimated $\alpha_{1,0.05}$ for **DAX** and **FTSE100** using 20 (1 month) or 250 (1 year) observations [▶ more parameters](#)



Parameter Dynamics [▶ Motivation](#)

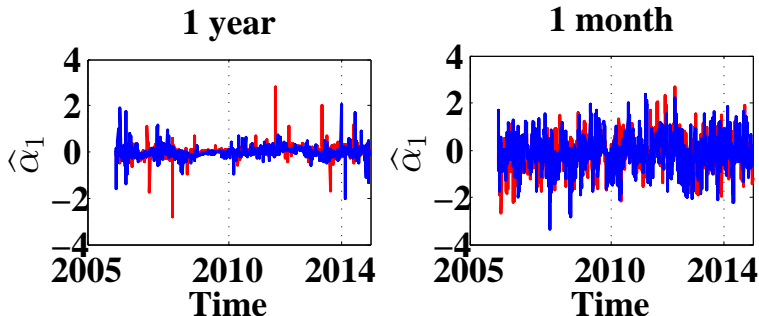


Figure 14: Estimated $\alpha_{1,0.01}$ for **DAX** and **FTSE100** using 20 (1 month) or 250 (1 year) observations [▶ more parameters](#)



Parameter Distributions ▶ Motivation

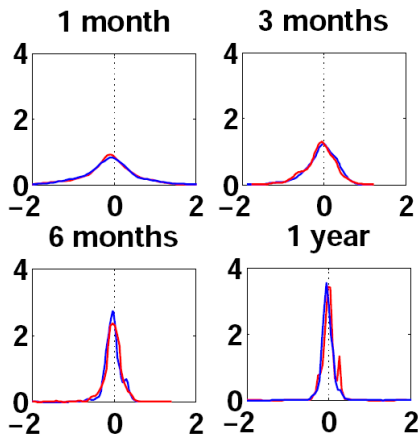


Figure 15: Kernel density estimates of $\alpha_{1,0.05}$ for **DAX** and **FTSE100** using 20, 60, 125 or 250 observations



Parameter Distributions ▶ Motivation

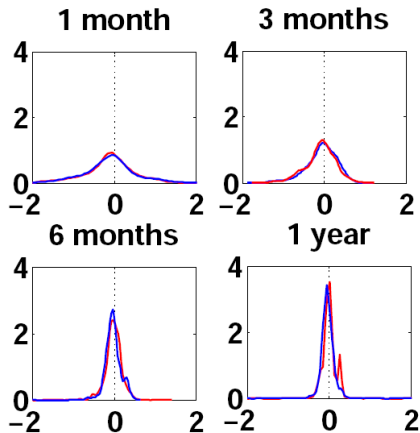


Figure 16: Kernel density estimates of $\alpha_{1,0.01}$ for **DAX** and **FTSE100** using 20, 60, 125 or 250 observations



Parameter Dynamics

▶ Parameter Dynamics

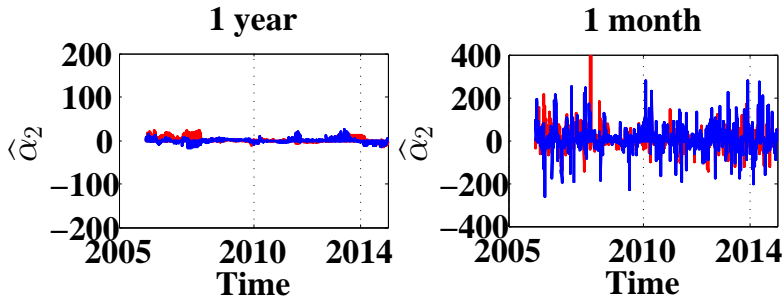


Figure 17: Estimated $\alpha_{2,0.05}$ for **DAX** and **FTSE100** using 20 (1 month) or 250 (1 year) observations



Parameter Dynamics

▶ Parameter Dynamics

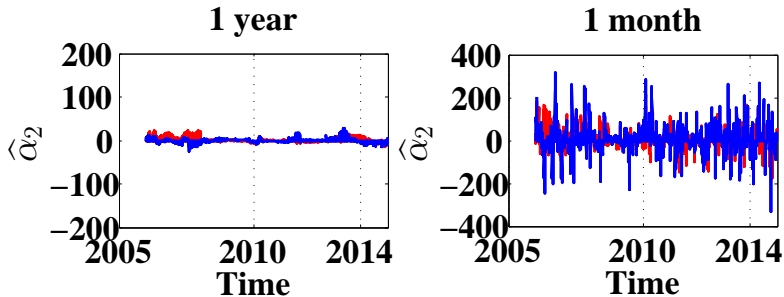


Figure 18: Estimated $\alpha_{2,0.01}$ for **DAX** and **FTSE100** using 20 (1 month) or 250 (1 year) observations



Parameter Dynamics

▶ Parameter Dynamics

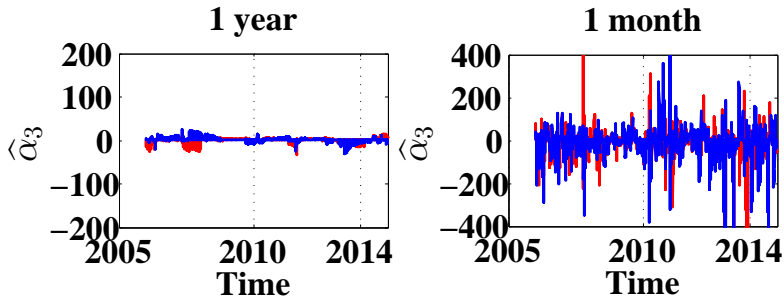


Figure 19: Estimated $\alpha_{3,0.05}$ for **DAX** and **FTSE100** using 20 (1 month) or 250 (1 year) observations



Parameter Dynamics

▶ Parameter Dynamics

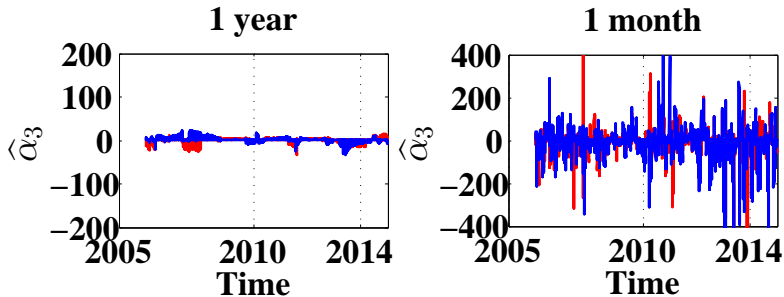


Figure 20: Estimated $\alpha_{3,0.01}$ for **DAX** and **FTSE100** using 20 (1 month) or 250 (1 year) observations



Adaptive Estimation - Weekly

▶ Adaptive Estimation

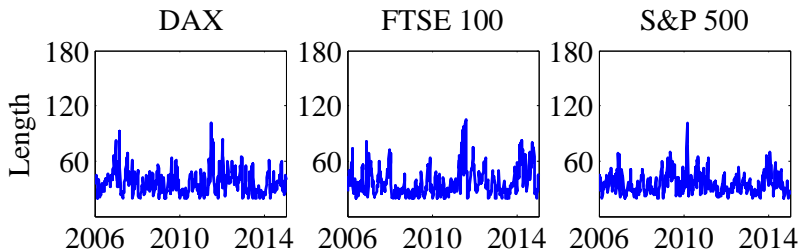


Figure 21: Weekly estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the modest risk case $r = 0.5$, at expectile level $\tau = 0.05$



Adaptive Estimation - Weekly

▶ Adaptive Estimation

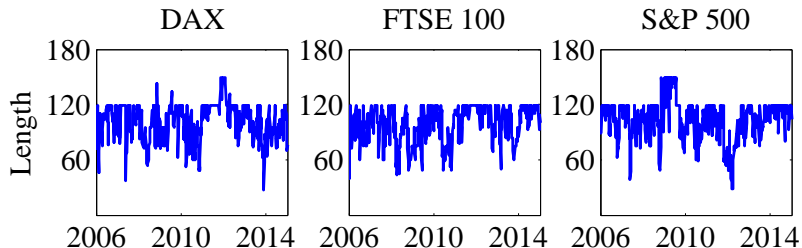


Figure 22: Weekly estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the conservative risk case $r = 1$, at expectile level $\tau = 0.05$



Adaptive Estimation - Weekly

▶ Adaptive Estimation

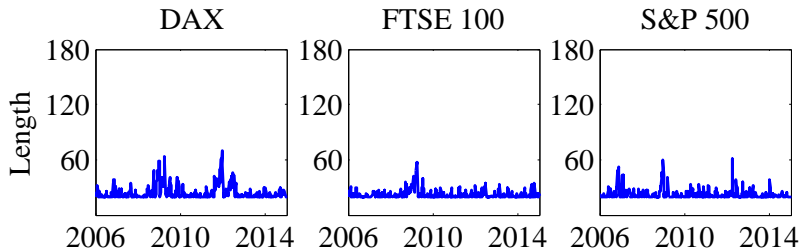


Figure 23: Weekly estimated length $n_{\hat{\kappa}}$ of *intervals of homogeneity* from 20060103-20141231 for the modest risk case $r = 0.5$, at expectile level $\tau = 0.01$



Adaptive Estimation - Weekly

▶ Adaptive Estimation

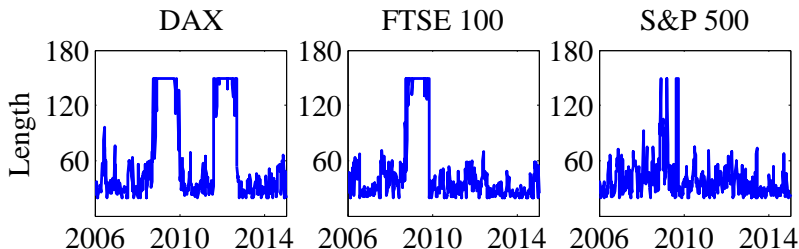


Figure 24: Weekly estimated length $n_{\hat{k}}$ of *intervals of homogeneity* from 20060103-20141231 for the conservative risk case $r = 1$, at expectile level $\tau = 0.01$

