Local Adaptive Multiplicative Error Models for High-Frequency Forecasts

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Statistical Challenges

- Understanding high-frequency dynamics
  - Time-varying parameters
    - Parameter Dynamics
  - Regime shifts

- Modelling using Procrustean assumptions
  - Time-invariant parameters
  - Transition form, number of regimes, transition variable type

Objectives

(i) Localising Multiplicative Error Models (MEM)

- Local parametric approach (LPA)
- Balance between modelling bias and parameter variability
- Estimation windows with potentially varying lengths

(ii) Short-term forecasting

- Case study: trading volume
- Evaluation against standard approach - fixed estimation length on an ad hoc basis

Example

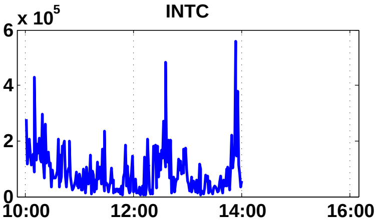


Figure 1: One-minute cumulated trading volume for Intel Corporation (INTC) on 20080902

Example

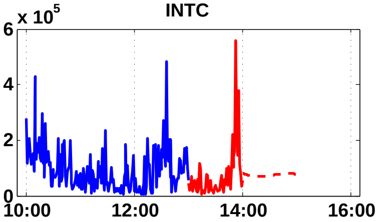


Figure 2: One-minute cumulated trading volume for Intel Corporation (INTC) on 20080902 with an EACD(1, 1) estimation window length of 60 with volume forecasts up to the next one hour (dashed)

Example

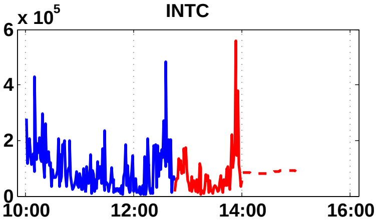


Figure 3: One-minute cumulated trading volume for Intel Corporation (INTC) on 20080902 with an EACD(1, 1) estimation window length of 75 with volume forecasts up to the next one hour (dashed)

Example

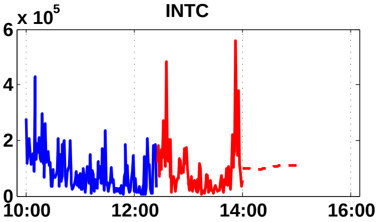


Figure 4: One-minute cumulated trading volume for Intel Corporation (INTC) on 20080902 with an EACD(1, 1) estimation window length of 95 with volume forecasts up to the next one hour (dashed)

Volume Weighted Average Price (VWAP)

VWAP - average price per share paid during a given time period (e.g., hourly, daily or weekly VWAP)

Example: A fund decides to sell 1% of their stock holdings over the next 1 week. A brokerage offers the fund manager to split such a large order at the hourly VWAP minus 1 cent.

Volume and VWAP prediction: The brokerage earns 1 cent per share if the shares are sold at the VWAP. Larger profit is achieved while trading at better prices. Important step: volume forecasts

Research Questions

- How strong is the variation of MEM parameters over time?
- What are typical interval lengths of parameter homogeneity?
- How good are LPA short-term forecasts?

Outline

1. Motivation ✓
2. Multiplicative Error Models (MEM)
3. Local Parametric Approach (LPA)
4. Forecasting Trading Volumes
5. Conclusions

Multiplicative Error Models (MEM)

- Engle (2002), MEM(p, q), $\mathcal{F}_i$ - information set up to i

$$y_i = \mu_i \varepsilon_i, \qquad \mathsf{E}\left[\varepsilon_i \,|\mathcal{F}_{i-1}\right] = 1$$

$$\mu_i = \omega + \sum_{j=1}^{p} \alpha_j y_{i-j} + \sum_{j=1}^{q} \beta_j \mu_{i-j}, \qquad \omega > 0, \alpha_j, \beta_j \geq 0$$

- Hautsch (2012) - comprehensive MEM literature overview
  - y_i - squared (de-meaned) log return: GARCH(p, q)
  - y_i - volume, bid-ask spread, duration: ACD(p, q)

Engle, Robert F. on BBI:

Autoregressive Conditional Duration (ACD)

1. Exponential-ACD, Engle and Russel (1998) ▸ EACD

   $\varepsilon_i \sim Exp\,(1)$, $\boldsymbol{\theta}_E = (\omega, \boldsymbol{\alpha}, \boldsymbol{\beta})^\top$, $\boldsymbol{\alpha} = (\alpha_1, \dots, \alpha_p)$, $\boldsymbol{\beta} = (\beta_1, \dots, \beta_q)$

2. Weibull-ACD, Engle and Russel (1998) ▸ WACD

   $\varepsilon_i \sim \mathcal{G}\,(s, 1)$, $\boldsymbol{\theta}_W = (\omega, \boldsymbol{\alpha}, \boldsymbol{\beta}, s)^\top$

Weibull, E. H. Waloddi on BBI:

Parameter Estimation

- Consistent parameter estimation
- Data calibration with time-varying intervals
- Quasi maximum likelihood estimates (QMLEs) of $\boldsymbol{\theta}_E$ and $\boldsymbol{\theta}_W$

$$\widetilde{\boldsymbol{\theta}}_I = \arg\max_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} L_I(y;\boldsymbol{\theta}) \tag{1}$$

- $I = [i_0 - n, i_0]$ - interval of $(n+1)$ observations at i_0
- $L_I(\cdot)$ - log likelihood, EACD and WACD

Data

- NASDAQ Market in 2008, 250 trading days, 10:00-16:00
- 5 stocks: AAPL, CSCO, INTC, MSFT and ORCL
- $\breve{y}_{i,d}$ - one-minute cumulated trading volume (minute i, day d)
- $y_{i,d}$ - seasonally adjusted trading volume (assuming a multiplicative impact of intra-day periodicity effects)

$$y_{i,d} = \breve{y}_{i,d} / s_{i,d-1}$$

- $s_{i,d-1}$ - intraday periodicity component Details
  $d = 31, \ldots, 250$ $(20080214 - 20081231)$, $i = 1, \ldots, 360$

Intraday Periodicity

Figure 5: Estimated intraday periodicity components for INTC, order $M = 6$ selected by BIC

Parameter Dynamics

Statistical Challenges

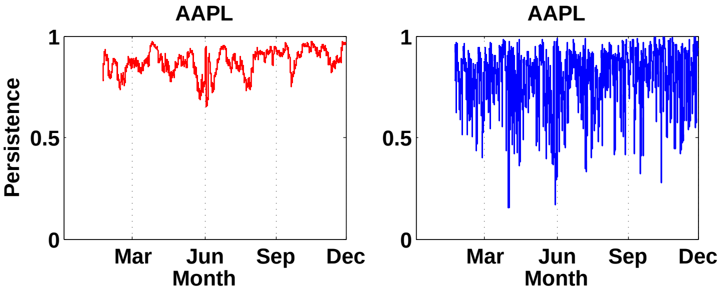


Figure 6: Estimated weekly $(n = 1800)$ and daily $(n = 360)$ persistence $\widetilde{\alpha}_i + \widetilde{\beta}_i$ for seasonally adjusted trading volume using an EACD$(1, 1)$ at each minute in 2008

Estimation Quality

- Mercurio and Spokoiny (2004), Spokoiny (2009)
- Quality of estimating *true* parameter vector $\boldsymbol{\theta}^*$ by QMLE $\widetilde{\boldsymbol{\theta}}_I$ in terms of Kullback-Leibler divergence; $\mathcal{R}_r(\boldsymbol{\theta}^*)$ - risk bound

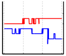
$$\mathsf{E}_{\boldsymbol{\theta}^*}\left|L_I(\widetilde{\boldsymbol{\theta}}_I) - L_I(\boldsymbol{\theta}^*)\right|^r \leq \mathcal{R}_r(\boldsymbol{\theta}^*)$$

▸ Gaussian Regression

- Likelihood based confidence sets

- 'Modest' risk, $r = 0.5$ (shorter intervals of homogeneity)
- 'Conservative' risk, $r = 1$ (longer intervals of homogeneity)

Kullback, Solomon and Leibler, Richard A. on BBI:



Local Parametric Approach (LPA)

- LPA, Spokoiny (1998, 2009)
  - Time series parameters can be locally approximated
  - Finding the (longest) *interval of homogeneity*
  - Balance between modelling bias and parameter variability

- Time series literature
  - Volatility modelling - Mercurio and Spokoiny (2004)
  - GARCH$(1,1)$ models - Čížek et al. (2009)
  - Realized volatility - Chen et al. (2010)

Interval Selection

- $(K+1)$ nested intervals with length $n_k = |I_k|$

$$\begin{array}{ccccccc} I_0 & \subset & I_1 & \subset \cdots \subset & I_k & \subset \cdots \subset & I_K \\ \widetilde{\boldsymbol{\theta}}_0 & & \widetilde{\boldsymbol{\theta}}_1 & & \widetilde{\boldsymbol{\theta}}_k & & \widetilde{\boldsymbol{\theta}}_K \end{array}$$

Example: Trading volumes aggregated over 1-min periods

Fix i_0, $I_k = [i_0 - n_k, i_0]$, $n_k = \left[n_0 c^k\right]$, $c > 1$

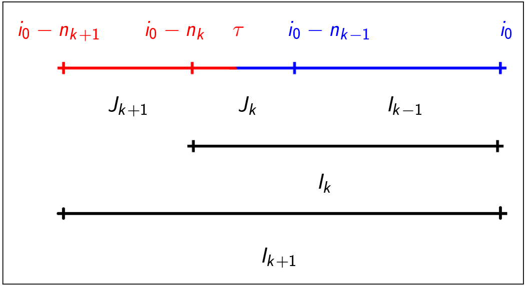
$\{n_k\}_{k=0}^{13} = \{60 \text{ min.}, 75 \text{ min.}, \ldots, 1 \text{ week}\}$, $c = 1.25$

Local Change Point Detection

Example

- Fix i_0, sequential test $(k = 1, \ldots, K)$

H_0 : parameter homogeneity within I_k vs. H_1 : $\exists$ change point within J_k



$$T_k = \sup_{\tau \in J_k} \left\{ L_{A_{k,\tau}} \left(\widetilde{\theta}_{A_{k,\tau}} \right) + L_{B_{k,\tau}} \left(\widetilde{\theta}_{B_{k,\tau}} \right) - L_{I_{k+1}} \left(\widetilde{\theta}_{I_{k+1}} \right) \right\},$$

with $J_k = I_k \setminus I_{k-1}$, $A_{k,\tau} = [i_0 - n_{k+1}, \tau]$ and $B_{k,\tau} = (\tau, i_0]$

Critical Values, $\mathfrak{z}_k$ ▸ Critical Values

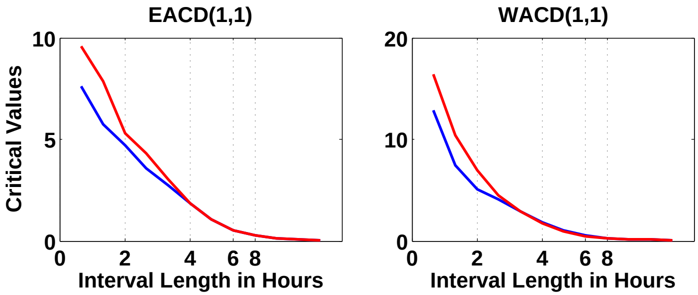


Figure 7: Critical values for low $(\widetilde{\alpha}+\widetilde{\beta}=0.84)$ and high $(\widetilde{\alpha}+\widetilde{\beta}=0.93)$ weekly persistence and 'modest' risk $(r=0.5)$ with $\rho=0.25$

Adaptive Estimation

- Compare T_k at every step k with $\mathfrak{z}_k$
- Data window index of the *interval of homogeneity* - $\widehat{k}$

- Adaptive estimate

$$\widehat{\boldsymbol{\theta}} = \widetilde{\boldsymbol{\theta}}_{\widehat{k}}, \quad \widehat{k} = \max_{k \leq K} \{k : T_\ell \leq \mathfrak{z}_\ell, \ell \leq k\}$$

- Note: rejecting the null at $k = 1$, $\widehat{\theta}$ equals QMLE at I_0
  If the algorithm goes until K, $\widehat{\theta}$ equals QMLE at I_K

Adaptive Estimation - Results

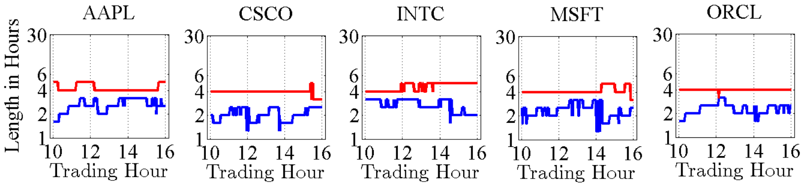


Figure 8: Estimated length $n_{\hat{k}}$ of *intervals of homogeneity* given the modest $(r = 0.5)$ and the conservative $(r = 1)$ risk case on 20080222 using the EACD$(1,1)$ model with $\rho = 0.25$

Adaptive Estimation - Results

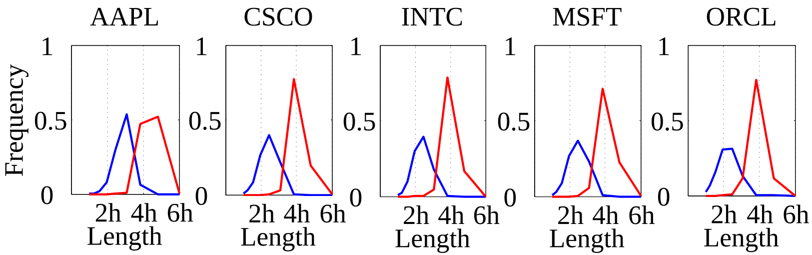


Figure 9: Relative frequency of the estimated interval length $n_{\hat{k}}$ (discrete variable, in hours) given the modest $(r = 0.5)$ and conservative $(r = 1)$ risk case from 20080222 to 20081231 using the EACD(1, 1) model with $\rho = 0.25$

Forecasting Trading Volumes

Setup

- 5 stocks, forecasting period: 20080222 - 20081222 (210 days)
- Forecasts at each minute, horizon $h = 1, \dots, 60$ min.
- EACD(1,1) and WACD(1,1), $r \in \{0.5, 1\}$, $\rho \in \{0.25, 0.5\}$

Strategies

- LPA technique - prediction $\widehat{y}_{i+h}$, error $\widehat{\varepsilon}_{i+h} = \breve{y}_{i+h} - \widehat{y}_{i+h}$
- 'Standard' method: 360 (1 day) or 1800 observations (1 week) - prediction $\widetilde{y}_{i+h}$, error $\widetilde{\varepsilon}_{i+h} = \breve{y}_{i+h} - \widetilde{y}_{i+h}$

Forecasting Measures

- Diebold and Mariano (1995) tests, loss differential

$$d_h = \{d_{i+h}\}_{i=1}^{n} = \widetilde{\varepsilon}^2_{i+h} - \widehat{\varepsilon}^2_{i+h} \tag{2}$$

- Ratio of root mean squared errors

$$\sqrt{n^{-1}\sum_{i=1}^{n}\widehat{\varepsilon}^2_{i+h}} \Big/ \sqrt{n^{-1}\sum_{i=1}^{n}\widetilde{\varepsilon}^2_{i+h}} \tag{3}$$

Forecasting Measures

- Qualitative test ▸ EACD ▸ WACD

$$T_{ST,h} = \left\{ \sum_{i=1}^{n} \mathrm{I}\left(d_{i+h} > 0\right) - 0.5n \right\} / \sqrt{0.25n} \xrightarrow{\mathcal{L}} \mathrm{N}(0,1) \qquad (4)$$

- Quantitative test, $H_0 : \mathrm{E}\left[d_h\right] = 0$

$$T_{DM,h} = \bar{d}_h / \sqrt{2\pi \widehat{f}_{d_h}(0)/n} \xrightarrow{\mathcal{L}} \mathrm{N}(0,1) \qquad (5)$$

$\bar{d}_h = n^{-1} \sum_{i=1}^{n} d_{i+h}$, $\widehat{f}_{d_h}(0)$ - spectral density estimate at frequency zero

Forecasting Superiority

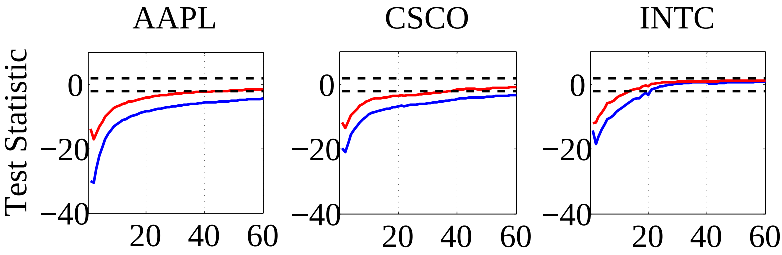


Figure 10: Test statistic $T_{DM,h}$ across all 60 forecasting horizons from 20080222 to 20081222 (210 trading days): LPA against a fixed-window scheme using 360 and 1800 observations using the EACD(1,1) model, $r = 0.5$ and $\rho = 0.25$

Forecasting Superiority

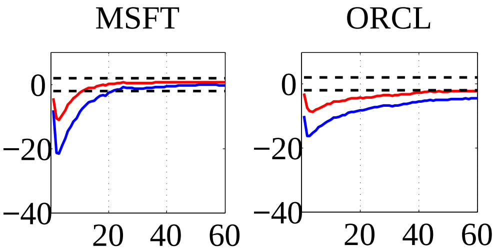


Figure 11: Test statistic $T_{DM,h}$ across all 60 forecasting horizons from 20080222 to 20081222 (210 trading days): LPA against a fixed-window scheme using 360 and 1800 observations using the EACD$(1,1)$ model, $r = 0.5$ and $\rho = 0.25$

Forecasting Superiority

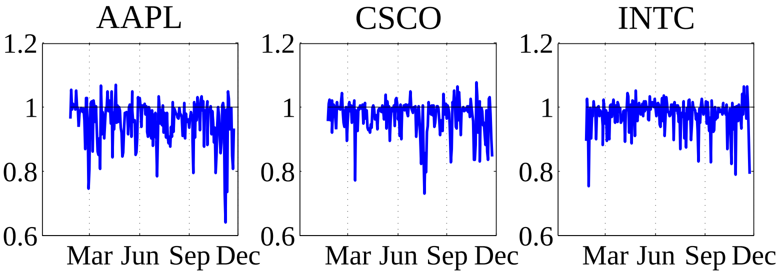


Figure 12: Ratio between the RMSPEs (averaged over all forecasting horizons) of the LPA and of a fixed-window approach (6 trading hours) over the sample from 20080222 to 20081222 (210 trading days) using an EACD(1,1) model, $r = 0.5$ and $\rho = 0.25$

Forecasting Superiority

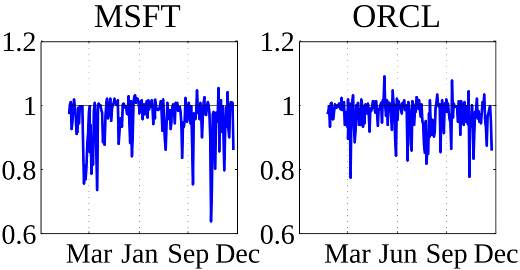


Figure 13: Ratio between the RMSPEs (averaged over all forecasting horizons) of the LPA and of a fixed-window approach (6 trading hours) over the sample from 20080222 to 20081222 (210 trading days) using an EACD(1, 1) model, $r = 0.5$ and $\rho = 0.25$

Forecasting Superiority

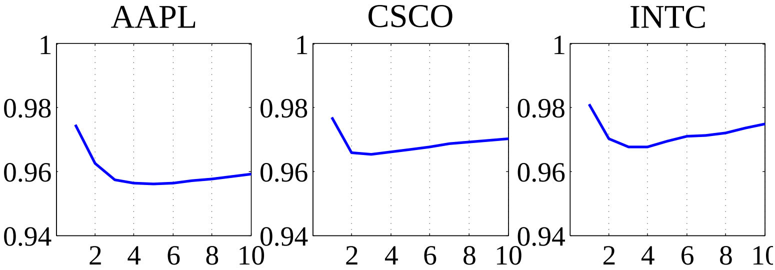


Figure 14: Ratio between the RMSPEs of the LPA and of a fixed-window approach (6 trading hours) across horizon from 20080222 to 20081222 (210 trading days) using an EACD$(1,1)$ model, $r = 0.5$ and $\rho = 0.25$

Forecasting Superiority

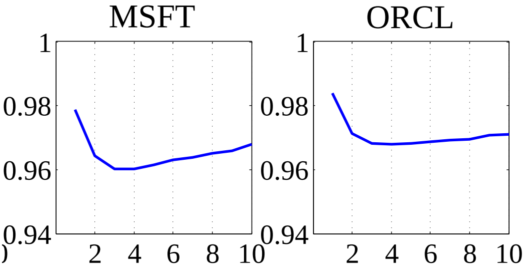


Figure 15: Ratio between the RMSPEs of the LPA and of a fixed-window approach (6 trading hours) across horizon from 20080222 to 20081222 (210 trading days) using an EACD$(1,1)$ model, $r = 0.5$ and $\rho = 0.25$

Conclusions

(i) Localising MEM

- Time-varying parameters and estimation quality
- NASDAQ blue chips: AAPL, CSCO, INTC, MSFT and ORCL
- 'Conservative' adaptive estimation ($r = 1$) requires 4-5 hours of data, modest risk approach ($r = 0.5$) requires 2-3 hours

(ii) Forecasting Trading Volumes

- LPA outperforms the 'standard' method
- Overall performance, horizon, trading day, tuning parameters

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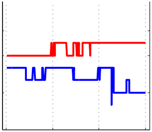

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Multiscale Local Change Point Detection with Applications to Value-at-Risk
The Annals of Statistics **37**(3): 1405–1436, 2009

Intra-day periodicity

Data

- Flexible Fourier Series (FFS) approximation, Gallant (1981)
  Intraday periodicity components $\left(s_{1,d-30}, \ldots, s_{360,d-1}\right)^\top$

- Estimation: 30-day rolling window with $s_{i,d-1} = \ldots = s_{i,d-30}$, Engle and Rangel (2008)

$$s_{i,d-1} = \delta \bar{\imath}_i + \sum_{m=1}^{M} \{\delta_{c,m} \cos\left(\bar{\imath}_i \cdot 2\pi m\right) + \delta_{s,m} \sin\left(\bar{\imath}_i \cdot 2\pi m\right)\}$$

$\bar{\imath} = (\bar{\imath}_1, \ldots, \bar{\imath}_{360})^\top = (1/360, \ldots, 360/360)^\top$ - intraday time trend

Gaussian Regression

▸ Estimation Quality

$$Y_i = f(X_i) + \varepsilon_i, i = 1, \dots, n, \text{ weights } W = \{w_i\}_{i=1}^n$$

$$L(W, \theta) = \sum_{i=1}^{n} \ell\{Y_i, f_\theta(X_i)\} w_i, \text{ log-density } \ell(\cdot), \; \widetilde{\theta} = \arg \max_{\theta \in \Theta} L(W, \theta)$$

1. Local constant, $f(X_i) \approx \theta^*$, $\varepsilon_i \sim \mathrm{N}\left(\theta^*, \sigma^2\right)$

$$\mathrm{E}_{\theta^*} \left| L(W, \widetilde{\theta}) - L(W, \theta^*) \right|^r \le \mathrm{E}\left|\xi\right|^{2r}, \quad \xi \sim \mathrm{N}(0, 1)$$

2. Local linear, $f(X_i) \approx \theta^{*\top} \Psi_i$, $\varepsilon_i \sim \mathrm{N}\left(0, \sigma^2\right)$, basis functions $\Psi = \{\psi_1(X_1), \dots, \psi_p(X_p)\}$

$$\mathrm{E}_{\theta^*} \left| L(W, \widetilde{\theta}) - L(W, \theta^*) \right|^r \le \mathrm{E}\left|\xi\right|^{2r}, \quad \xi \sim \mathrm{N}(0, \mathcal{I}_p)$$

Exponential-ACD (EACD)

▸ ACD ▸ Parameter Estimation

- Engle and Russel (1998), $\varepsilon_i \sim Exp(1)$

$$L_I(y;\boldsymbol{\theta}_E) = \sum_{i=max(p,q)+1}^{n} \left(-\log \mu_i - \frac{y_i}{\mu_i} \right) \mathbf{I}\{i \in I\} \tag{6}$$

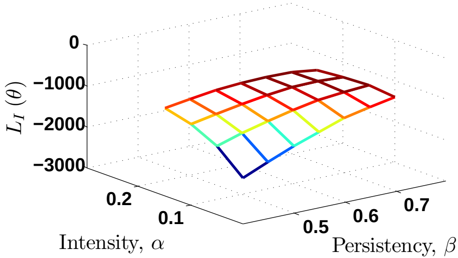

Figure 16: Log likelihood - EACD(1,1), $\boldsymbol{\theta}_E^* = (0.10, 0.20, 0.65)^\top$

Weibull-ACD (WACD)

▸ ACD ▸ Parameter Estimation

- Engle and Russel (1998), $\varepsilon_i \sim \mathcal{G}(s, 1)$

$$L_I(y; \boldsymbol{\theta}_W) = \sum_{i \in I} \left[\log \frac{s}{y_i} + s \log \frac{\Gamma(1+1/s)\, y_i}{\mu_i} - \left\{ \frac{\Gamma(1+1/s)\, y_i}{\mu_i} \right\}^s \right] \mathbf{I}\{i \in I\} \tag{7}$$

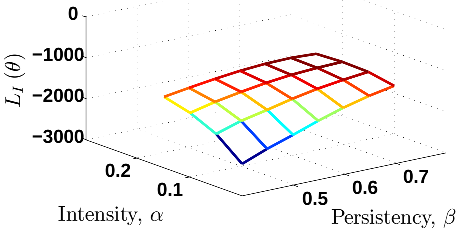

Figure 17: Log likelihood - WACD(1,1), $\boldsymbol{\theta}^*_W = (0.10, 0.20, 0.65, 0.85)^\top$

Parameter Dynamics

Estimation window	EACD(1,1)			WACD(1,1)		
	Q25	Q50	Q75	Q25	Q50	Q75
1 week	0.85	0.89	0.93	0.82	0.88	0.92
2 days	0.77	0.86	0.92	0.74	0.84	0.91
1 day	0.68	0.82	0.90	0.63	0.79	0.89
3 hours	0.54	0.75	0.88	0.50	0.72	0.87
2 hours	0.45	0.70	0.86	0.42	0.67	0.85
1 hour	0.33	0.58	0.80	0.31	0.57	0.80

Table 1: Quartiles of estimated persistence levels $\left(\widetilde{\alpha}+\widetilde{\beta}\right)$ for all five stocks at each minute from 20080222-20081231 (215 trading days). Calibration period: 20080102 - 20080221

Parameter Dynamics

▸ Critical Values

Model	Low Persistence			Moderate Persistence			High Persistence		
	Q25	Q50	Q75	Q25	Q50	Q75	Q25	Q50	Q75
EACD, $\widetilde{\alpha}$	0.28	0.22	0.18	0.30	0.23	0.19	0.31	0.24	0.20
EACD, $\widetilde{\beta}$	0.56	0.62	0.67	0.59	0.66	0.71	0.62	0.68	0.73
WACD, $\widetilde{\alpha}$	0.28	0.21	0.17	0.30	0.23	0.18	0.32	0.24	0.19
WACD, $\widetilde{\beta}$	0.54	0.60	0.65	0.58	0.65	0.70	0.60	0.68	0.74

Table 2: Quartiles of 774,000 estimated ratios $\widetilde{\beta}/\left(\widetilde{\alpha}+\widetilde{\beta}\right)$ (estimation windows covering 1800 observations) from 20080222-20081231 conditional on the persistence level: low {EACD (0.85), WACD (0.82)}, moderate {EACD (0.89), WACD (0.88)} or high {EACD (0.93), WACD (0.92)}. 774,000 ratios = 215 days $\times$ 360 minutes/day $\times$ 5 stocks $\times$ 2 models.

Parameter Dynamics - Summary

- MEM parameters, their variability and distribution properties change over time
- Longer local estimation windows increase estimation precision and the misspecification risk
- Tradeoff between estimation (in)efficiency and local flexibility

Critical Values, $\mathfrak{z}_k$

Critical Values

- Simulate $\mathfrak{z}_k$ - homogeneity of the interval sequence $I_0, \dots, I_k$
- 'Propagation' condition (under H_0)

$$\mathrm{E}_{\theta^*} \left| L_{I_k}(\widetilde{\theta}_k) - L_{I_k}(\widehat{\theta}_k) \right|^r \leq \rho_k \mathcal{R}_r(\theta^*), \quad k = 1, \dots, K \qquad (8)$$

$\rho_k = \rho k / K$ for given significance level ρ, $\widehat{\theta}_k$ - adaptive estimate

- Check $\mathfrak{z}_k$ for (nine) different θ^* Parameter Dynamics - Quartiles
  - EACD and WACD, $K \in \{8, 13\}$, $r \in \{0.5, 1\}$, $\rho \in \{0.25, 0.50\}$
  - Findings: $\mathfrak{z}_k$ are virtually invariable w.r.t. θ^* given a scenario
    Largest differences at first two or three steps

Local Change Point Detection LCP

Example: Trading volumes aggregated over 1-min periods

- Scheme with $(K+1) = 14$ intervals and fix i_0
- Assume $I_0 = 60$min. is homogeneous

- H_0 : parameter homogeneity within $I_1 = 75$min.
  - Define $J_1 = I_1 \setminus I_0$ - observations from y_{i_0-75} up to y_{i_0-60}
  - For each $\tau \in J_1$ fit log likelihoods over $A_{1,\tau}$, $B_{1,\tau}$ and I_2
  - Find the largest likelihood ratio - T_{I_1,J_1}

Forecasting Measures

▸ Forecasting Measures

	EACD(1,1)				
	AAPL	CSCO	INTC	MSFT	ORCL
1 week					
$r = 0.5,\ \rho = 0.25$	-38.9	-28.6	-24.1	-33.8	-31.4
$r = 0.5,\ \rho = 0.50$	-38.7	-28.7	-24.2	-33.8	-31.4
$r = 1.0,\ \rho = 0.25$	-40.5	-31.4	-23.3	-39.1	-32.8
$r = 1.0,\ \rho = 0.50$	-40.4	-31.3	-23.3	-39.0	-32.9
1 day					
$r = 0.5,\ \rho = 0.25$	-10.8	-6.0	-13.1	-5.7	-15.1
$r = 0.5,\ \rho = 0.50$	-10.6	-6.0	-12.8	-5.5	-15.0
$r = 1.0,\ \rho = 0.25$	-6.9	-8.6	-8.7	-4.4	-12.9
$r = 1.0,\ \rho = 0.50$	-7.1	-8.6	-8.8	-4.4	-13.0

Table 3: Largest (in absolute terms) test statistic $T_{ST,h}$ across all 60 forecasting horizons from 20080222-20081222 (210 trading days). LPA against a fixed-window scheme using 360 and 1800 observations

Forecasting Measures

▸ Forecasting Measures

	WACD(1,1)				
	AAPL	CSCO	INTC	MSFT	ORCL
1 week					
$r = 0.5,\ \rho = 0.25$	-22.6	-25.7	-20.2	-26.7	-26.6
$r = 0.5,\ \rho = 0.50$	-22.7	-25.5	-20.3	-26.7	-26.6
$r = 1.0,\ \rho = 0.25$	-27.9	-30.8	-21.5	-31.3	-29.8
$r = 1.0,\ \rho = 0.50$	-28.1	-30.8	-21.5	-31.5	-29.7
1 day					
$r = 0.5,\ \rho = 0.25$	-6.4	-3.5	-6.1	-4.9	-12.6
$r = 0.5,\ \rho = 0.50$	-6.3	-3.2	-6.2	-4.8	-12.7
$r = 1.0,\ \rho = 0.25$	-4.1	-5.1	-6.5	-4.2	-11.5
$r = 1.0,\ \rho = 0.50$	-3.9	-5.2	-6.5	-4.1	-11.4

Table 4: Largest (in absolute terms) test statistic $T_{ST,h}$ across all 60 forecasting horizons from 20080222-20081222 (210 trading days). LPA against a fixed-window scheme using 360 and 1800 observations