Adaptive Modelling of Price Duration and Flash Equity Dynamics

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Flash Crash

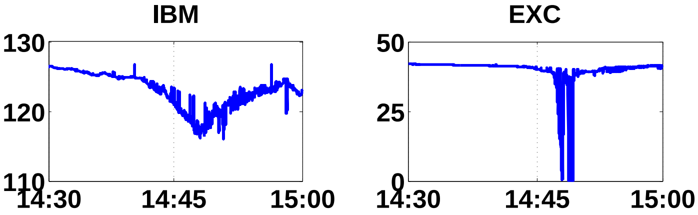


Figure 1: Stock prices of International Business Machines (IBM) and Excelon (EXC) during the flash crash on 20100506 at the NASDAQ stock market

Objectives

- Price Duration Dynamics
  - Local adaptive Multiplicative Error Model (MEM)
  - Balance between modelling bias and parameter variability
- Forecasting Flash Equity Dynamics
  - Estimation windows with potentially varying lengths
  - (Extreme) price movements, execution strategies

Statistics and Quantitative Finance

Statistics

- Modelling bias vs. parameter variability
- Flexible framework and predictive accuracy

Quantitative Finance Practice

- Irregularly spaced data
- Execution strategies

Example 1

Price Duration

A fund manager is trading a large number of shares of a company.

How long does it take (in seconds) that the price goes down/up by some fixed amount, say by 1%?

Example 2

Price Reversion

A fund manager observes that the price (suddenly) goes down/up.

Is a price reversion predicted? If yes, how long does the predicted recovery last (in seconds)?

Outline

1. Motivation ✓
2. Multiplicative Error Model (MEM)
3. Local Parametric Approach (LPA)
4. Price Duration and Flash Equity Dynamics
5. Conclusions

Multiplicative Error Model (MEM)

- Engle (2002), MEM(p, q), $\mathcal{F}_i$ - information set up to i

$$y_i = \mu_i \varepsilon_i, \qquad \mathsf{E}\left[\varepsilon_i \,|\mathcal{F}_{i-1}\right] = 1$$

$$\mu_i = \omega + \sum_{j=1}^{p} \alpha_j y_{i-j} + \sum_{j=1}^{q} \beta_j \mu_{i-j}, \qquad \omega > 0, \alpha_j, \beta_j \geq 0$$

- Hautsch (2012) - comprehensive MEM literature overview
  - y_i - squared (de-meaned) log return: GARCH(p, q)
  - y_i - volume, bid-ask spread, duration: ACD(p, q)

Engle, Robert F. on BBI:

Autoregressive Conditional Duration (ACD)

1. Exponential-ACD, Engle and Russel (1998) ▸ EACD

   $\varepsilon_i \sim Exp\,(1)$, $\boldsymbol{\theta}_E = (\omega, \boldsymbol{\alpha}, \boldsymbol{\beta})^\top$, $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_p)$, $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_q)$

2. Weibull-ACD, Engle and Russel (1998) ▸ WACD

   $\varepsilon_i \sim \mathcal{G}\,(s, 1)$, $\boldsymbol{\theta}_W = (\omega, \boldsymbol{\alpha}, \boldsymbol{\beta}, s)^\top$

Weibull, E. H. Waloddi on BBI:

Parameter Estimation

- Consistent parameter estimation
- Data calibration with time-varying intervals
- Quasi maximum likelihood estimates (QMLEs) of $\boldsymbol{\theta}_E$ and $\boldsymbol{\theta}_W$

$$\widetilde{\boldsymbol{\theta}}_I = \arg\max_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} L_I\left(y;\boldsymbol{\theta}\right) \quad (1)$$

  - $I = [i_0 - m, i_0]$ - interval of $(m+1)$ observations at i_0
  - $L_I\left(\cdot\right)$ - log likelihood, ▸ EACD and ▸ WACD

Estimation Quality

- Mercurio and Spokoiny (2004), Spokoiny (2009)
- Quality of estimating *true* parameter vector $\boldsymbol{\theta}^*$ by QMLE $\widetilde{\boldsymbol{\theta}}_I$ in terms of Kullback-Leibler divergence; $\mathcal{R}_r(\boldsymbol{\theta}^*)$ - risk bound

$$\mathsf{E}_{\boldsymbol{\theta}^*} \left| L_I(\widetilde{\boldsymbol{\theta}}_I) - L_I(\boldsymbol{\theta}^*) \right|^r \leq \mathcal{R}_r(\boldsymbol{\theta}^*)$$

▸ Gaussian Regression

- 'Modest' risk, $r = 0.5$ (shorter intervals of homogeneity)
- 'Conservative' risk, $r = 1$ (longer intervals of homogeneity)

Kullback, Solomon and Leibler, Richard A. on BBI:

Local Parametric Approach (LPA)

- LPA, Spokoiny (1998, 2009)
  - Time series parameters can be locally approximated
  - Finding the (longest) *interval of homogeneity*
  - Balance between modelling bias and parameter variability

- Time series literature
  - GARCH$(1,1)$ models - Čížek et al. (2009)
  - Realized volatility - Chen et al. (2010)
  - Multiplicative Error Models - Härdle et al. (2014)

Interval Selection

- $(K+1)$ nested intervals with length $n_k = |I_k|$

$$\begin{array}{ccccccc} I_0 & \subset & I_1 & \subset \cdots \subset & I_k & \subset \cdots \subset & I_K \\ \widetilde{\boldsymbol{\theta}}_0 & & \widetilde{\boldsymbol{\theta}}_1 & & \widetilde{\boldsymbol{\theta}}_k & & \widetilde{\boldsymbol{\theta}}_K \end{array}$$

Example: Price durations

Fix i_0, $I_k = [i_0 - n_k, i_0]$, $n_k = \left[n_0 c^k\right]$, $c > 1$

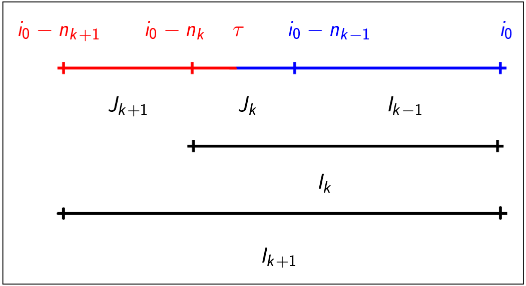
$\{n_k\}_{k=0}^{14} = \{60 \text{ obs.}, 75 \text{ obs.}, \ldots, 360 \text{ obs.}\}$, $c = 1.25$

Local Change Point Detection

▸ Example

- Fix i_0, sequential test $(k = 1, \ldots, K)$

H_0 : parameter homogeneity within I_k vs. H_1 : $\exists$ change point within J_k



$$T_k = \sup_{\tau \in J_k} \left\{ L_{A_{k,\tau}}\left(\widetilde{\theta}_{A_{k,\tau}}\right) + L_{B_{k,\tau}}\left(\widetilde{\theta}_{B_{k,\tau}}\right) - L_{I_{k+1}}\left(\widetilde{\theta}_{I_{k+1}}\right) \right\},$$

with $J_k = I_k \setminus I_{k-1}$, $A_{k,\tau} = [i_0 - n_{k+1}, \tau]$ and $B_{k,\tau} = (\tau, i_0]$

Critical Values, $\mathfrak{z}_k$ ▸ Critical Values

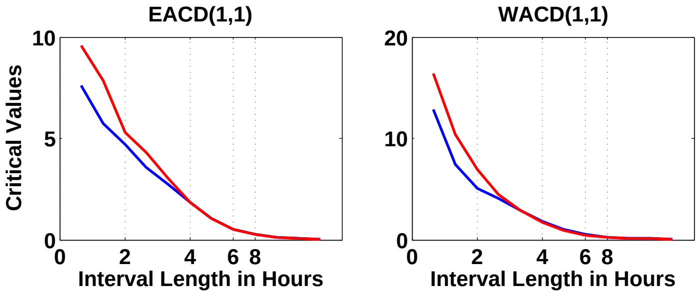


Figure 2: Critical values for low $(\tilde{\alpha}+\tilde{\beta}=0.84)$ and high $(\tilde{\alpha}+\tilde{\beta}=0.93)$ weekly persistence and 'modest' risk $(r=0.5)$ with $\rho=0.25$, Härdle et al. (2014)

Adaptive Estimation

- Compare T_k at every step k with $\mathfrak{z}_k$
- Data window index of the *interval of homogeneity* - $\widehat{k}$
- Adaptive estimate

$$\widehat{\boldsymbol{\theta}} = \widetilde{\boldsymbol{\theta}}_{\widehat{k}}, \quad \widehat{k} = \max_{k \le K} \{k : T_\ell \le \mathfrak{z}_\ell, \ell \le k\}$$

- Note: rejecting the null at $k = 1$, $\widehat{\theta}$ equals QMLE at I_0
  If the algorithm goes until K, $\widehat{\theta}$ equals QMLE at I_K

Data

- NASDAQ Stock Market
  - Trading days: 20100503, 20100506 (Flash crash), 20100510
  - Transactions at every 25 miliseconds
  - Daily trading period: 09:30-16:00
  - Duration series: price moves up or down (0.95 quantile of the series)

Data

- Companies
  - Stocks without crash
    Goldman Sachs (GS), Zions Bancorporation (ZION)
  - Normal crash
    International Business Machines (IBM), Procter&Gamble (PG)
  - Penny stocks
    Accenture (ACN), Excelon (EXC)

Price Duration

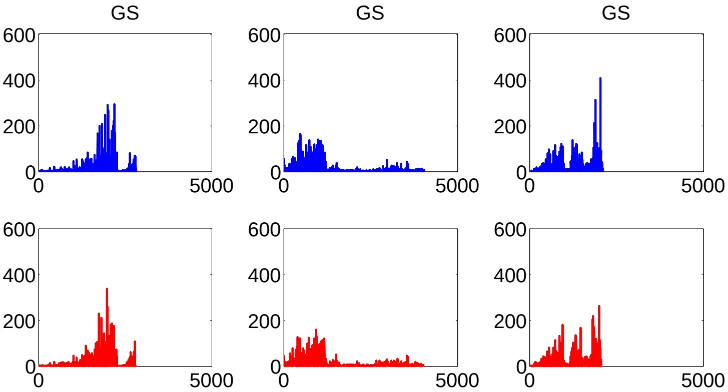


Figure 3: Price duration of GS on 20100503 (left), 20100506 (middle) and 20100510 (right). Price movement: down or up

Price Duration

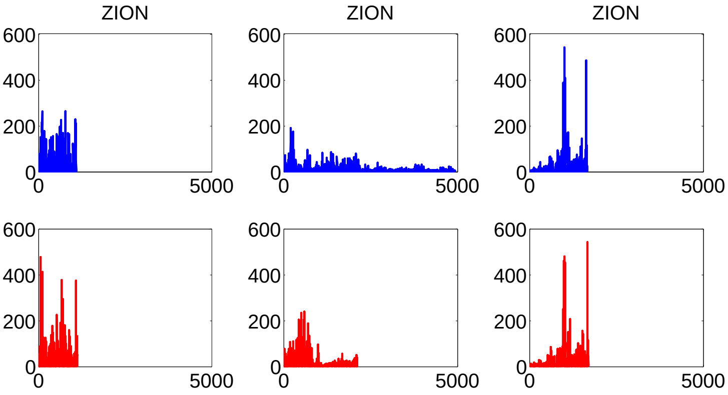


Figure 4: Price duration of ZION on 20100503 (left), 20100506 (middle) and 20100510 (right). Price movement: down or up

Price Duration

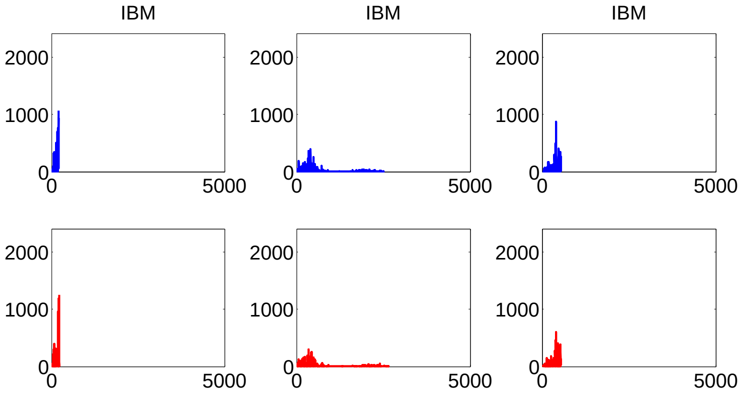


Figure 5: Price duration of IBM on 20100503 (left), 20100506 (middle) and 20100510 (right). Price movement: down or up

Price Duration

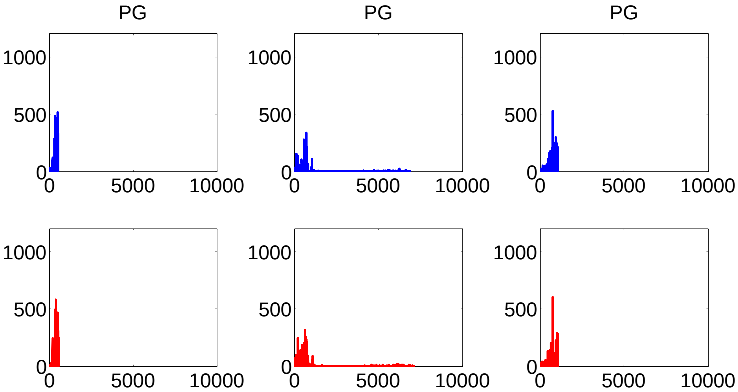


Figure 6: Price duration of PG on 20100503 (left), 20100506 (middle) and 20100510 (right). Price movement: down or up

Price Duration

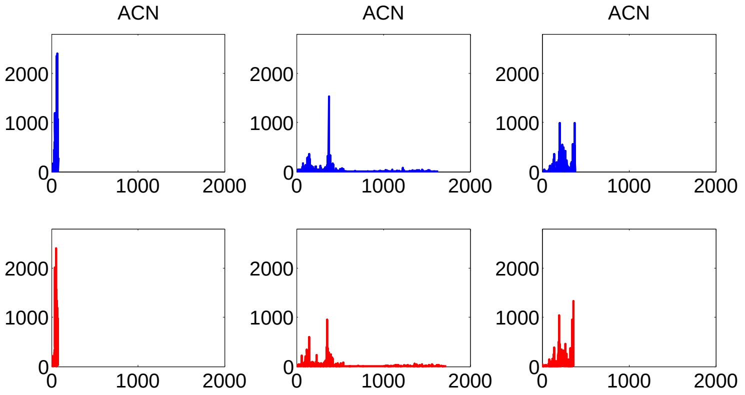


Figure 7: Price duration of ACN on 20100503 (left), 20100506 (middle) and 20100510 (right). Price movement: down or up

Price Duration

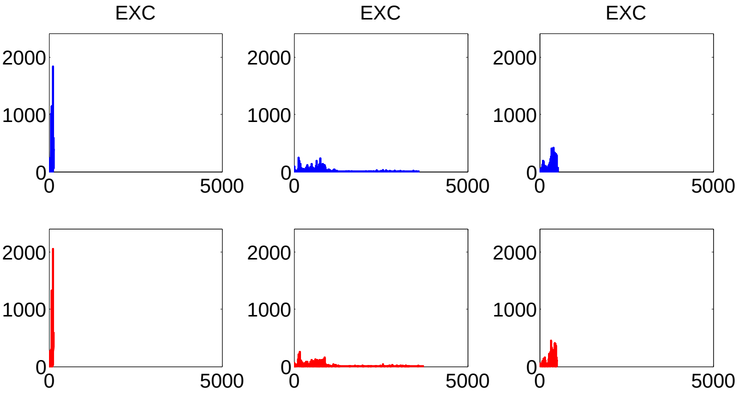


Figure 8: Price duration of EXC on 20100503 (left), 20100506 (middle) and 20100510 (right). Price movement: down or up

Conclusions

Price Duration Dynamics

- Different price duration dynamics across stocks
- Stocks without crash share stable dynamics

Forecasting Flash Equity Dynamics

- Stocks affected by the flash crash exhibit three different regimes
- Evidence for price reversion

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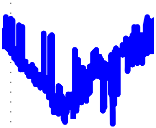

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The Annals of Statistics **37**(3): 1405–1436, 2009

Exponential-ACD (EACD)

▸ ACD ▸ Parameter Estimation

- Engle and Russel (1998), $\varepsilon_i \sim Exp(1)$

$$L_I(y;\theta_E) = \sum_{i=max(p,q)+1}^{n} \left(-\log \mu_i - \frac{y_i}{\mu_i}\right) \mathbf{I}\{i \in I\} \qquad (2)$$

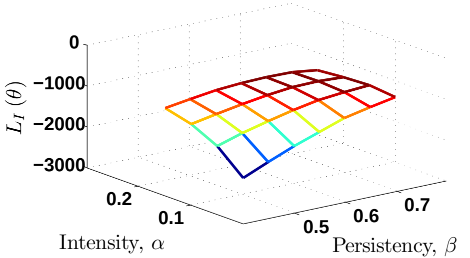

Figure 9: Log likelihood - EACD(1,1), $\theta_E^* = (0.10, 0.20, 0.65)^\top$

Weibull-ACD (WACD)

▸ ACD ▸ Parameter Estimation

- Engle and Russel (1998), $\varepsilon_i \sim \mathcal{G}(s, 1)$

$$L_I(y; \boldsymbol{\theta}_W) = \sum_{i \in I} \left[\log \frac{s}{y_i} + s \log \frac{\Gamma(1 + 1/s)\, y_i}{\mu_i} - \left\{ \frac{\Gamma(1 + 1/s)\, y_i}{\mu_i} \right\}^s \right] \mathbf{I}\{i \in I\} \tag{3}$$

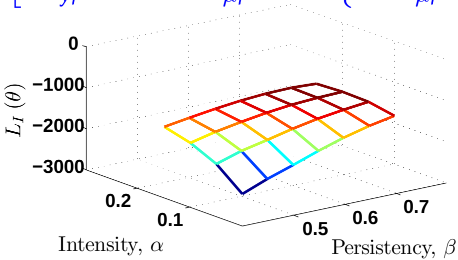

Figure 10: Log likelihood - WACD(1,1), $\boldsymbol{\theta}_W^* = (0.10, 0.20, 0.65, 0.85)^\top$

Gaussian Regression

Estimation Quality

$$Y_i = f(X_i) + \varepsilon_i, i = 1, \ldots, n, \text{ weights } W = \{w_i\}_{i=1}^n$$

$$L(W, \theta) = \sum_{i=1}^{n} \ell\{Y_i, f_\theta(X_i)\} w_i, \text{ log-density } \ell(\cdot), \ \widetilde{\theta} = \arg\max_{\theta \in \Theta} L(W, \theta)$$

1. Local constant, $f(X_i) \approx \theta^*$, $\varepsilon_i \sim \mathrm{N}(0, \sigma^2)$

$$\mathrm{E}_{\theta^*} \left| L(W, \widetilde{\theta}) - L(W, \theta^*) \right|^r \le \mathrm{E}\,|\xi|^{2r}, \quad \xi \sim \mathrm{N}(0, 1)$$

2. Local linear, $f(X_i) \approx \theta^{*\top}\Psi_i$, $\varepsilon_i \sim \mathrm{N}(0, \sigma^2)$, basis functions $\Psi = \{\psi_1(X_1), \ldots, \psi_p(X_p)\}$, multivariate ξ

$$\mathrm{E}_{\theta^*} \left| L(W, \widetilde{\theta}) - L(W, \theta^*) \right|^r \le \mathrm{E}\,|\xi|^{2r}, \quad \xi \sim \mathrm{N}(0, \mathcal{I}_p)$$

Local Change Point Detection ▸ LCP

Example: Price duration

- Scheme with $(K+1) = 14$ intervals and fix i_0
- Assume $I_0 = 60$obs. is homogeneous

- H_0 : parameter homogeneity within $I_1 = 75$min.
  - Define $J_1 = I_1 \setminus I_0$ - observations from y_{i_0-75} up to y_{i_0-60}
  - For each $\tau \in J_1$ fit log likelihoods over $A_{1,\tau}$, $B_{1,\tau}$ and I_2
  - Find the largest likelihood ratio - T_{I_1,J_1}

Critical Values, $\mathfrak{z}_k$

Critical Values

- Simulate $\mathfrak{z}_k$ - homogeneity of the interval sequence $I_0, \dots, I_k$
- 'Propagation' condition (under H_0)

$$\mathsf{E}_{\theta^*} \left| L_{I_k}(\widetilde{\theta}_k) - L_{I_k}(\widehat{\theta}_k) \right|^r \leq \rho_k \mathcal{R}_r(\theta^*), \quad k = 1, \dots, K \qquad (4)$$

$\rho_k = \rho k / K$ for given significance level ρ, $\widehat{\theta}_k$ - adaptive estimate

- Check $\mathfrak{z}_k$ for (nine) different θ^* Parameter Dynamics - Quartiles
  - EACD and WACD, $K \in \{8, 13\}$, $r \in \{0.5, 1\}$, $\rho \in \{0.25, 0.50\}$
  - Findings: $\mathfrak{z}_k$ are virtually invariable w.r.t. θ^* given a scenario
    Largest differences at first two or three steps

Parameter Dynamics

Estimation window	EACD(1,1) Q25	EACD(1,1) Q50	EACD(1,1) Q75	WACD(1,1) Q25	WACD(1,1) Q50	WACD(1,1) Q75
1 week	0.85	0.89	0.93	0.82	0.88	0.92
2 days	0.77	0.86	0.92	0.74	0.84	0.91
1 day	0.68	0.82	0.90	0.63	0.79	0.89
3 hours	0.54	0.75	0.88	0.50	0.72	0.87
2 hours	0.45	0.70	0.86	0.42	0.67	0.85
1 hour	0.33	0.58	0.80	0.31	0.57	0.80

Table 1: Quartiles of estimated persistence levels $\left(\widetilde{\alpha}+\widetilde{\beta}\right)$, Härdle et al. (2014)

Parameter Dynamics

▸ Critical Values

Model	Low Persistence			Moderate Persistence			High Persistence		
	Q25	Q50	Q75	Q25	Q50	Q75	Q25	Q50	Q75
EACD, $\widetilde{\alpha}$	0.28	0.22	0.18	0.30	0.23	0.19	0.31	0.24	0.20
EACD, $\widetilde{\beta}$	0.56	0.62	0.67	0.59	0.66	0.71	0.62	0.68	0.73
WACD, $\widetilde{\alpha}$	0.28	0.21	0.17	0.30	0.23	0.18	0.32	0.24	0.19
WACD, $\widetilde{\beta}$	0.54	0.60	0.65	0.58	0.65	0.70	0.60	0.68	0.74

Table 2: Quartiles of estimated ratios $\widetilde{\beta}/\left(\widetilde{\alpha}+\widetilde{\beta}\right)$ (estimation windows covering 1800 observations) conditional on the persistence level: low {EACD (0.85), WACD (0.82)}, moderate {EACD (0.89), WACD (0.88)} or high {EACD (0.93), WACD (0.92)}, Härdle et al. (2014)