Cross Country Evidence for the Empirical Pricing Kernel Puzzle

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Motivation

- Understanding asset prices - highest need for more investments
- Pricing kernel (PK)
  - Market price for state-contingent payoffs
  - Physical state probabilities
- Example: booms and recessions

Motivation

- Pricing kernel, $\mathcal{K}$
  - Preference based models - marginal rate of substitution
  - Arbitrage free models - Radon-Nikodym derivative of the physical measure w.r.t. the risk neutral measure

Risk Neutral Valuation PK - Black-Scholes

Johann Radon on BBI:

Motivation

- Empirical pricing kernel (EPK)
  - $\widehat{\mathcal{K}}$ any estimate of the PK
  - Direct and indirect estimation ▸ PK Estimation
  - EPK puzzle - locally increasing EPK

EPK Puzzle

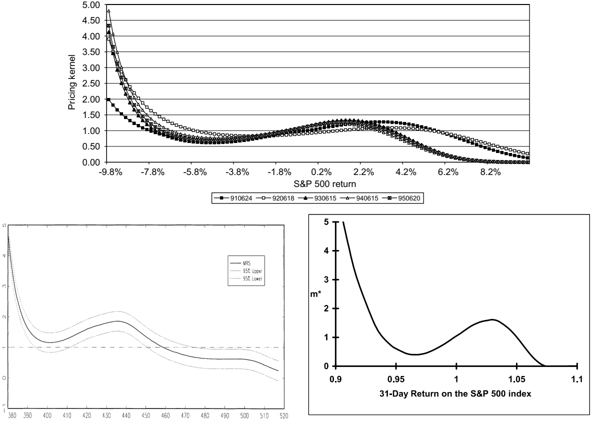


Figure 1: EPK's: Rosenberg and Engle (2002), Aït-Sahalia and Lo (2000), Brown and Jackwerth (2012)

EPK Puzzle

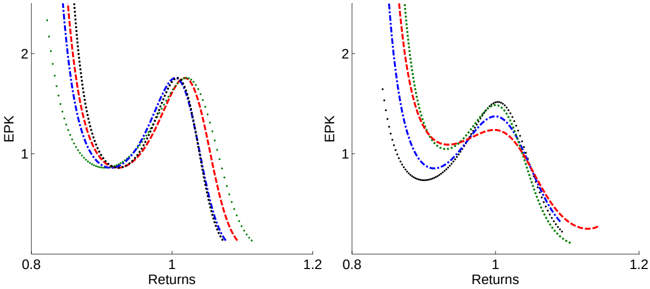


Figure 2: EPK's for various maturities (left) and different estimation dates for fixed maturity 1M (right), Grith et al. (2013)

EPK Puzzle

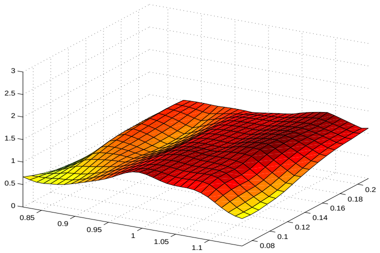


Figure 3: EPK's across moneyness κ and maturity τ for DAX from 20010101 – 20011231, Giacomini and Härdle (2008)

EPK Puzzle

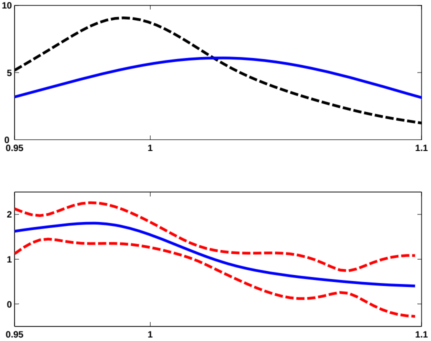


Figure 4: Upper panel: estimated risk neutral density $\widehat{q}$ and historical density $\widehat{p}$. Lower panel: EPK and 95% uniform confidence bands on 20060228, Härdle et al. (2014)

EPK Puzzle

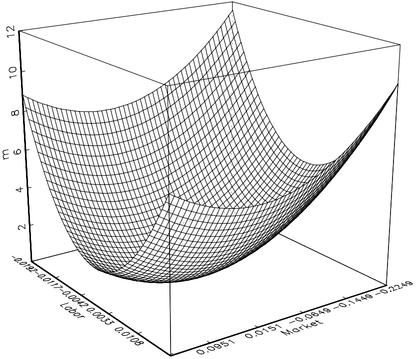
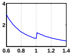


Figure 5: EPK for the US stock market. Data: returns of 20 industry-sorted portfolios from 19630731 to 19951231 with human capital (lagged labor income), Dittmar (2002)



Objectives

- Pricing Kernel (PK) estimation
  - State-dependent utility
  - Generalized Method of Moments (GMM)

- EPK puzzle across equity markets
  - Six worldwide largest markets
  - Time-varying EPK, statistical properties

Research Questions

- Does empirical evidence from equity data suggest a locally increasing EPK?
- Are the estimation results significant?
- How do PK estimates vary across countries and over time?

Outline

1. Motivation ✓
2. Pricing Kernel (PK)
3. Generalized Method of Moments (GMM)
4. Empirical Results
5. DACH Project Proposal
6. Conclusion

Preference Based Model

- Representative agent with exogenous income ω_t
- Budget constraints

$$c_t = \omega_t - q_t^\top S_t \quad (1)$$

$$c_{t+1} = \omega_{t+1} + q_t^\top S_{t+1} \quad (2)$$

with consumption c_t, k assets, prices $S_t = \left(S_{1,t}, \ldots, S_{k,t}\right)^\top$, asset holdings $q_t = \left(q_{1,t}, \ldots, q_{k,t}\right)^\top$

- Returns $R_{t+1} = \left(S_{1,t+1}/S_{1,t}, \ldots, S_{k,t+1}/S_{k,t}\right)^\top$

State-Independent Preferences

- Utility maximization

$$\max_{c_{t+1}} \mathsf{E}_t \left[u\left(c_{t+1}\right)\right], \text{under constraints (1) and (2)}$$

with utility function of the representative agent $u(\cdot)$,
$\mathsf{E}_t[\cdot] = \mathsf{E}[\cdot \mid \mathcal{F}_t]$, $\mathcal{F}_t$ - information set up to t

State-Independent Preferences

- State-independent PK

$$\mathcal{K}_t\left(c_{t+1}\right) = \beta \frac{u'\left(c_{t+1}\right)}{u'\left(c_t\right)}$$

where β is the discount factor

State-Dependent Preferences

- Reference dependent utility function, Grith et al. (2013)

  $$u\left(c_{t+1},x\right)=u\left(c_{t+1}\right)\mathbf{I}\left\{x\in\left[0,x_0\right)\right\}+bu\left(c_{t+1}\right)\mathbf{I}\left\{x\in\left[x_0,\infty\right)\right\},$$

  State variable x, reference point x_0 and parameter b

- Utility maximization

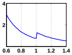
  $$\max_{c_{t+1}} \mathrm{E}_t\left[u\left(c_{t+1},x\right)\right],$$ under constraints (1) and (2)

State-Dependent Preferences

- State-dependent PK

$$\mathcal{K}_t(c_t, c_{t+1}) = \beta_1 \frac{u'(c_{t+1})}{u'(c_t)} \mathbf{I}\{x \in [0, x_0)\} + \\ + \beta_2 \frac{u'(c_{t+1})}{u'(c_t)} \mathbf{I}\{x \in [x_0, \infty)\},$$

$\beta_1 = \beta$, $\beta_2 = \beta b$



Setup

- Consumption growth is linear in the market portfolio gross return, Cochrane (2001); $c_{t+1} \stackrel{\text{def}}{=} r_{m,t+1} = S_{m,t+1}/S_{m,t}$
- State variable x is $r_{m,t+1}$

- Parameter $\theta = (\beta_1, \beta_2, x_0)^\top$
- Log utility, $u(y) = \log y$, $u'(y) = 1/y$

Pricing Kernel

- State-dependent PK

$$\mathcal{K}_{\theta,t}\left(r_{m,t+1}\right) = \beta_1 r_{m,t+1}^{-1} \mathbf{I}\left\{r_{m,t+1} \in [0, x_0)\right\} + \\ + \beta_2 r_{m,t+1}^{-1} \mathbf{I}\left\{r_{m,t+1} \in [x_0, \infty)\right\}$$

Asset Pricing

- k assets, prices $S_t = \left(S_{1,t}, \ldots, S_{k,t}\right)^\top$

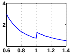
$$
\begin{aligned}
S_t &= \mathsf{E}_t\left[\mathcal{K}_{\theta,t}\left(r_{m,t+1}\right) S_{t+1}\right] \\
1_k &= \mathsf{E}_t\left[\mathcal{K}_{\theta,t}\left(r_{m,t+1}\right) R_{t+1}\right]
\end{aligned}
$$

Generalized Method of Moments

- Hansen (1982), expectation of k moment conditions

$$\mathsf{E}_t \left[\mathcal{K}_{\theta,t}\left(r_{m,t+1}\right) R_{t+1} - 1_k\right] = 0_k$$

- Define $g\left(\theta\right) = \mathcal{K}_{\theta,t}\left(r_{m,t+1}\right) R_{t+1} - 1_k, \quad \mathsf{E}_t\left[g\left(\theta\right)\right] = 0_k$
- Sample: $g_n\left(\theta\right) = n^{-1}\sum_{t=0}^{n-1}\left\{\mathcal{K}_{\theta,t}\left(r_{m,t+1}\right) R_{t+1} - 1_k\right\}$



GMM Estimation

1. Iterated GMM

   Hansen and Singleton (1982), Ferson and Foerster (1994)

$$\widetilde{\theta}_n \stackrel{\text{def}}{=} \arg\min_{\theta} \left\{ g_n^\top(\theta)\, g_n(\theta) \right\}, \quad \widetilde{W}_n = n^{-1} \sum_{t=0}^{n-1} g(\widetilde{\theta}_n) g(\widetilde{\theta}_n)^\top$$

$$\widehat{\theta}_n \stackrel{\text{def}}{=} \arg\min_{\theta} \left\{ g_n^\top(\theta)\, \widetilde{W}_n^{-1} g_n(\theta) \right\}, \quad \widehat{W}_n = n^{-1} \sum_{t=0}^{n-1} g(\widehat{\theta}_n) g(\widehat{\theta}_n)^\top$$

GMM Estimation

2. GMM with Hansen-Jagannathan (HJ) weighting matrix Jagannathan and Wang (1996), Hansen and Jagannathan (1997), $\widetilde{W}_n = n^{-1} \sum_{t=0}^{n-1} R_t R_t^\top$

GMM Hypothesis Testing

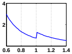
- Newey and West (1987) - "D-test"
- Test statistic

$$D = n g_n^\top(\widetilde{\theta}_n)\widetilde{W}_n^{-1} g_n(\widetilde{\theta}_n) - n g_n^\top(\check{\theta}_n)\check{W}_n^{-1} g_n(\check{\theta}_n) \xrightarrow{\mathcal{L}} \chi^2_j,$$

with j parameter restrictions, two estimates, e.g., $\widetilde{\theta}_n$ and $\check{\theta}_n$ with weighting matrices $\widetilde{W}$ and $\check{W}$, respectively

Data

- Markets: Australian Securities Exchange (AUS), Deutsche Börse (GER), Tokyo Stock Exchange (JPN), SIX Swiss Exchange (SUI), LSE (UK), NYSE (US)
- Span: 1 January 1990 - 31 May 2012 (daily data)
- Series: stock market indices, 20 largest blue chips per market
- Windows: $n \in \{250 \text{ (1 year)}, 500 \text{ (2 years)}, 1250 \text{ (5 years)}\}$



PK Estimation

- Scenarios

  Case 1. $\beta_1, \beta_2 > 0$ - state-dependent, unconstrained

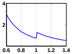
  Case 2. $\beta_2 > \beta_1 > 0$ - state-dependent, constrained

  Case 3. $\beta_1 = \beta_2 = \beta > 0$ - state-independent

GMM Estimation

	GMM with HJ matrix			Iterated GMM		
	(1)	(2)	(3)	(1)	(2)	(3)
AUS	1.19	1.31	1.39	1.08	1.25	1.42
GER	0.85	0.91	1.00	0.81	0.90	1.01
JPN	0.68	0.71	0.80	0.66	0.71	0.81
SUI	1.01	1.06	1.15	0.88	0.96	1.17
UK	0.84	0.89	0.95	0.79	0.86	0.97
US	0.91	0.96	1.01	0.84	0.95	1.03

Table 1: Average optimal objective function for two competing techniques and three scenarios: (1) $\beta_1, \beta_2 > 0$, (2) $\beta_1 > \beta_2 > 0$ and (3) $\beta_1 = \beta_2 = \beta = 0$. The estimation window covers $n = 500$ observations (2 years).



Parameter Dynamics

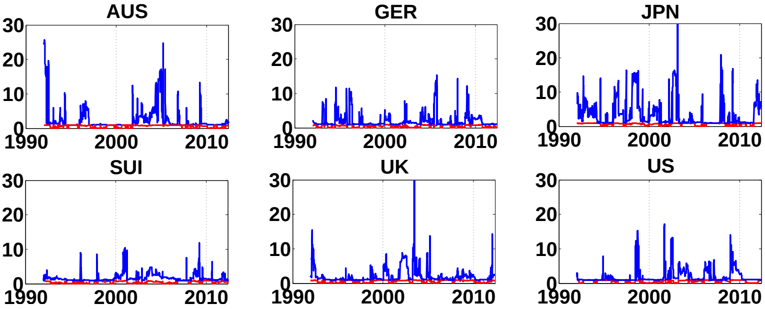


Figure 6: Time series of the estimated parameters β_1 and β_2 across six worldwide largest stock markets for case 2 ($\beta_2 > \beta_1 > 0$). We employ the iterated GMM estimation technique with $n = 500$ (2 years).

Reference Point Analysis

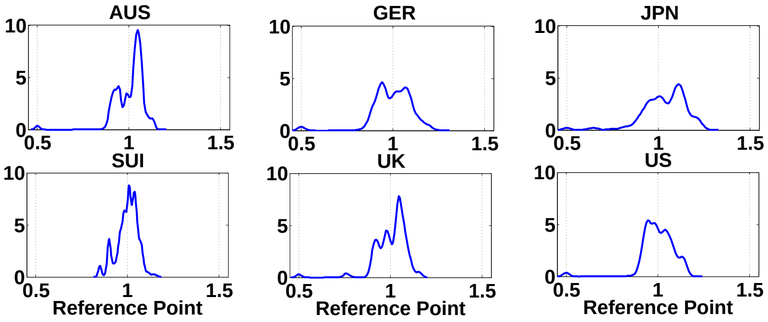


Figure 7: Kernel density plots (Gaussian kernel with optimal bandwidth) of optimal reference point x_0 for case 2 ($\beta_2 > \beta_1 > 0$). We employ the iterated GMM estimation technique with $n = 500$ (2 years).

Empirical Pricing Kernels

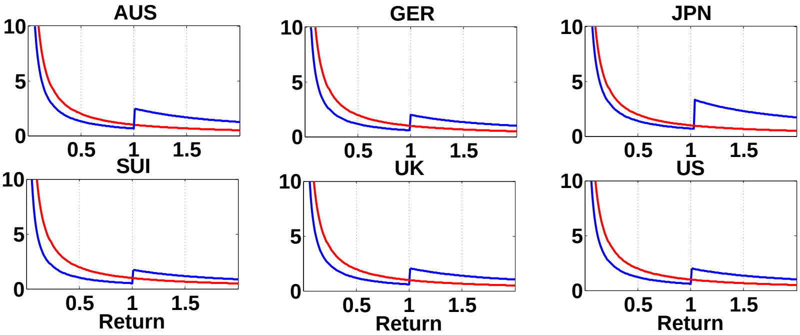
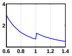


Figure 8: Empirical pricing kernels across six worldwide largest stock markets (for average parameter values): case 1, $\beta_1, \beta_2 > 0$ and case 2, $\beta_1 = \beta_2 = \beta$.



Hypothesis Testing

	Iterated GMM			GMM with HJ matrix		
	1 year	2 years	5 years	1 year	2 years	5 years
AUS	76.32	79.49	67.76	68.64	69.88	70.21
GER	89.94	88.99	81.76	81.55	84.27	86.30
JPN	84.22	83.02	83.15	83.60	84.67	76.93
SUI	92.06	88.47	87.14	85.21	79.77	80.62
UK	82.13	86.43	79.26	86.20	73.61	81.32
US	78.16	75.92	74.85	70.44	52.64	54.81

Table 2: Percentage of rejections of the null hypothesis of the D-test (H_0 : $\beta_1 = \beta_2 = \beta$) as indicator for the existence of the EPK puzzle across the worldwide largest six stock markets.

Germany: EPK Dynamics

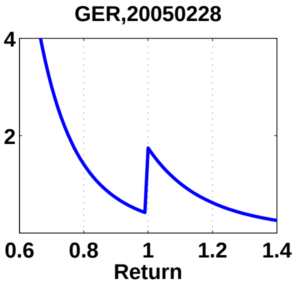
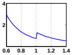


Figure 9: EPK on the German stock market in 2005.



DACH Project Proposal

Country Specific and Cross Country Evidence for the Empirical Pricing Kernel Puzzle

- DACH - Deutschland (D), Österreich (A), Schweiz (CH)
- Principal investigators
  - Wolfgang Karl Härdle, Humboldt-Universität zu Berlin
  - Thorsten Hens, Universität Zürich
  - Nikolaus Hautsch, Universität Wien

DACH Project Proposal

- Duration: 3 years
- Keywords: asset pricing, financial markets, pricing kernels, cross country study
- Research
  - Non-/semiparametric modelling and financial statistics
  - Behavioural finance and asset pricing
  - Liquidity and volatility modelling

DACH Project Proposal

- Objectives
  - PK estimation using market data - leading financial markets
  - Connecting behavioural finance and market dynamics (investor behaviours, welfare impact of speculation)
  - EPK and market characteristics (e.g., liquidity, volatility)
  - Pricing and portfolio optimization in risk management practice

DACH Project Proposal

- Highlights
  - Promoting young researchers - 3 postdoctoral positions
  - Research Data Center RDC
    Databases of the International Test of Risk Attitudes
  - High quality research
  - Academic exchange: visiting researchers, two workshops

Conclusion

- Pricing Kernel (PK) estimation
    - State-dependent utility admits PK nonmonotonicity
    - GMM successfully used for estimation and hypothesis testing

- EPK puzzle across equity markets
    - Time-varying preferences
    - Optimal reference point slightly above 1, statistically significant results
    - DACH project proposal

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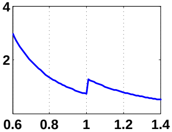

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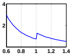

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National Centre of Competence in Research Financial Valuation and Risk Management Working Paper 732, 2011

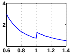
Risk Neutral Valuation

Motivation

- Present value of the payoffs $\psi(S_T)$

$$P_0 = \mathsf{E}_Q\left[e^{-Tr}\psi(s_T)\right] = \int_0^\infty e^{-Tr}\psi(s_T)\ \mathcal{K}(s_T)p(s_T)\ ds_T$$

r risk free interest rate, $\{S_t\}_{t\in[0,T]}$ stock price process,
p pdf of S_T, Q risk neutral measure, $\mathcal{K}(\cdot)$ pricing kernel



PK under the Black-Scholes Model

▸ Motivation

- Geometric Brownian motion for S_t

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

μ mean, σ volatility, W_t Wiener process

Fischer Black and Myron S. Scholes on BBI:

PK under the Black-Scholes Model

▸ Motivation

- Physical density p is log-normal, $\tau = T - t$

$$p_t(S_T) = \frac{1}{S_T\sqrt{2\pi\sigma^2\tau}} \exp\left[-\frac{1}{2}\left\{\frac{\log(S_T/S_t) - \left(\mu - \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}\right\}^2\right]$$

- Risk neutral density q is log-normal: replace μ by r

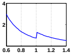
PK under the Black-Scholes Model

▸ Motivation

- PK is a decreasing function in S_T for fixed S_t

$$\mathcal{K}(S_t, S_T) = \left(\frac{S_T}{S_t}\right)^{-\frac{\mu - r}{\sigma^2}} \exp\left\{\frac{(\mu - r)(\mu + r - \sigma^2)\tau}{2\sigma^2}\right\}$$
$$= b\left(\frac{S_T}{S_t}\right)^{-\delta}$$

$b = \exp\left\{\frac{(\mu - r)(\mu + r - \sigma^2)\tau}{2\sigma^2}\right\}$ and $\delta = \frac{\mu - r}{\sigma^2} \geq 0$ constant relative risk aversion (CRRA) coefficient



PK Estimation

▸ Motivation

- Indirect estimation, $\widehat{\mathcal{K}} = \dfrac{\widehat{q}}{\widehat{p}}$
  - q risk neutral density; p physical density
  - Aït-Sahalia and Lo (2000), Rosenberg and Engle (2002), Giacomini and Härdle (2008), Brown and Jackwerth (2012), Grith et al. (2013), Härdle et al. (2014)

- Direct estimation, $\widehat{\mathcal{K}} = G_{\widehat{\theta}}$
  - Given function G, parameter θ
  - Dittmar (2002), Schweri (2011)

Robert F. Engle on BBI: