Brain and Risk Perception. Uncertainty and Complexity in Portfolio Decisions

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Portfolio Risk

Portfolio $\Pi = \sum_{i=1}^{n} X_i$ (Markowitz, 1952):

$$\sigma(\Pi) = \sqrt{\sum_{i=1}^{n} \sigma(X_i)^2 + \sum_{i \neq j}^{n} 2\,\mathrm{Cov}(X_i, X_j)}$$

Subject's Answers

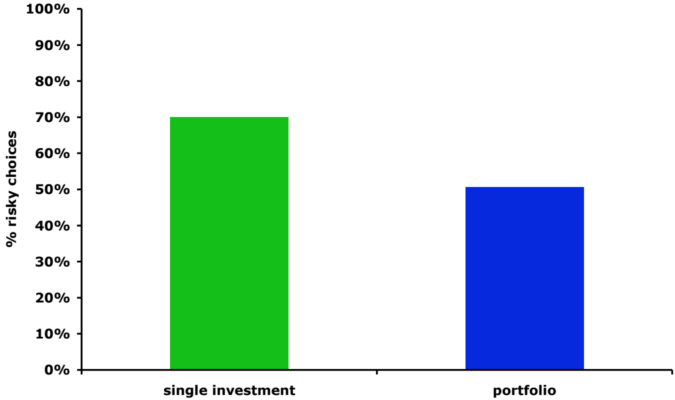


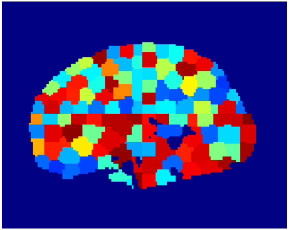
Figure 1: A proportion of risky choices selected by subjects for the single investment/portfolio (128/128 trials) setup averaged over all subjects.

Subject's Answers / Risk Perception

$$\text{Risk} = \text{Uncertainty} + \text{Complexity}$$

Investments and Brain Correlates

- How does individual perceive risk?
- Is risk perception reflected in brain activity?



σ_t

ID Experiment

- Survey by Department of Education and Psychology, FU Berlin
- 19 healthy volunteers payoff

- Investment Decision (ID) task ($\times 256$)
  safe vs. **random** (μ, σ) return
- fMRI images: 2 sec$\times$1400 $\approx$ 48 min
- Can one identify brain reactions?

Investment Decision

Choose between:

A) **Safe**, fixed return 5%

B) **Random**, investment return (3 types)

- Single Investment
- Portfolio of 2 (perfectly) correlated investments
- Portfolio of 2 uncorrelated investments

- Each type of portfolio $\times 64$, single $\times 128$
- Display and decision time: 7 sec; Answers

ID Experiment

Figure 2: Decide between **A)** 5% return and displayed **B)** portfolio/investment.

fMRI

- functional Magnetic Resonance Imaging

- Measuring Blood Oxygenation Level Dependent (BOLD) effect every 2 sec
  High-dimensional, high frequency & large data set

fMRI

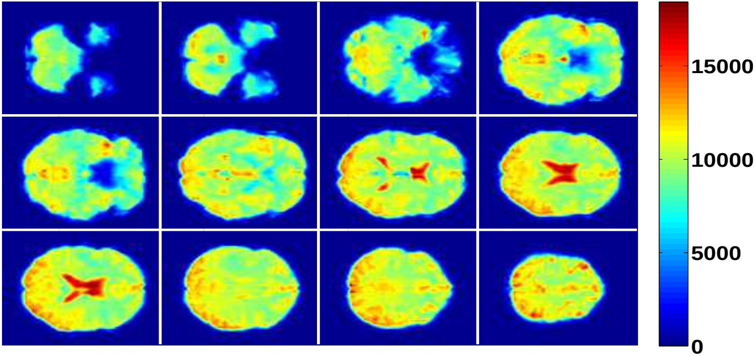
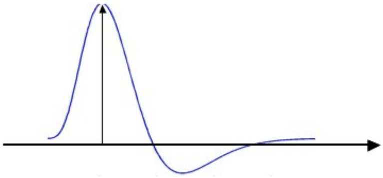


Figure 3: fMRI image observed every 2 sec, 12 horizontal slices of the brain's scan, $91 \times 109 \times 91 (x, y, z)$ data points of size 22 MB; voxel resolution: $2 \times 2 \times 2 mm^3$

fMRI Dynamics ▸ fMRI methods

Hemodynamic response (1 voxel) ▸ HRF



- Is there a significant reaction to specific stimuli?
- Is there any relation between perceived risk and complexity?

Outline

1. Motivation ✓
2. fMRI Clustering
3. PEC
4. Subjective Complexity Measure
5. Empirical results
6. Appendix

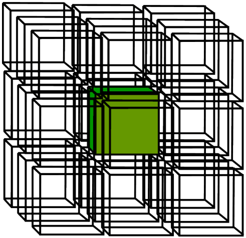
Clustering

- Find clusters (groups of voxels)
- A cluster has to be contiguous and homogeneous
- Data-driven (size,shape)
- Differences between clusters should be as large as possible

Proximity measure and group-building algorithm for fMRI?

Proximity between Voxels

▸ Correlation



- $Y_{t,j}$ - BOLD signal observed at voxel j with $3D$ coordinates $X_j = (x_j, y_j, z_j)$, $j = 1, \dots, J$
- Proximity measure $w(j,k)$ between Y_j and Y_k

$$w(j,k) = \begin{cases} \max\left\{\text{Corr}_t(Y_j, Y_k), 0\right\}, & \text{for } \|X_j - X_k\| < \mathbf{d} \\ 0, & \text{otherwise} \end{cases}$$

d - fixed distance, such that $\{\tilde{u} : \|X_{\tilde{u}} - X_k\| < d\}$ is a $3D$ neighborhood ($3\sqrt{3}$mm); Corr_t - Pearson correlation over 2×1400

Cut Cost and Normalized Cut

- Cost of partitioning $\mathcal{Y}$ into P and Q groups, $\mathcal{Y} = P + Q$

$$Cut(P, Q) = \sum_{Y_j \in P, Y_k \in Q} w(j, k)$$

sum of all "neglected" similarities between voxels in P and Q

minimizing the cut cost: singletons

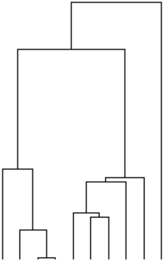
- Normalized cut:

$$N_{cut}(P, Q) = \frac{cut(P, Q)}{\sum_{Y_j \in P, Y_k \in \mathcal{Y}} w(j, k)} + \frac{cut(P, Q)}{\sum_{Y_j \in Q, Y_k \in \mathcal{Y}} w(j, k)}$$

Normalized cut (NCUT) spectral clustering

Hierarchically divide $\mathcal{Y}$ into pre-specified number of clusters K (top-down):

1. Find the division P^* and Q^*,
   $$(P^*, Q^*) = \operatorname*{argmin}_{Y=P+Q} N_{cut}(P, Q)$$
2. Decide if the current partition should be subdivided
3. Recursively partition the segmented parts, if necessary



Quantiles and Expectiles

For $Y \in \mathbb{R}^p$-valued r.v.:

τ-quantile:

$$q_\tau(Y) = \operatorname*{argmin}_{q \in \mathbb{R}^p} \mathsf{E}\, \| Y - q \|_{\tau,1},$$

τ-expectile

$$e_\tau(Y) = \operatorname*{argmin}_{e \in \mathbb{R}^p} \mathsf{E}\, \| Y - e \|_{\tau,2}.$$

where for $\alpha = 1, 2$

$$\|y\|_{\tau,\alpha} = \sum_{j=1}^{p} |y_j|^\alpha \cdot \left\{ \tau\, \mathbf{I}_{\{y_j \geq 0\}} + (1-\tau)\, \mathbf{I}_{\{y_j < 0\}} \right\}.$$

PEC as variance maximizers

Define the τ-variance for $Y \in \mathbb{R}^p$

$$\mathrm{Var}_\tau(Y) = \mathrm{E}\,\|Y - e_\tau(Y)\|^2_{\tau,2}$$

The principal expectile component(PEC)

$$\begin{aligned}\phi^*_\tau &= \underset{\phi\in\mathbb{R}^p,\phi^\top\phi=1}{\operatorname{argmax}} \mathrm{Var}_\tau(\phi^\top Y_i, i = 1, \ldots, n) \\ &= \underset{\phi\in\mathbb{R}^p,\phi^\top\phi=1}{\operatorname{argmax}} \frac{1}{n}\sum_{i=1}^{n}(\phi^\top Y_i - \mu_\tau)^2 w_i,\end{aligned}$$

where $\mu_\tau \in \mathbb{R}$ is the τ-expectile of $\phi^\top Y_1, \ldots, \phi^\top Y_n$, and

$$w_i = \begin{cases} \tau & \text{if } \sum_{j=1}^{p} Y_{ij}\phi_j > \mu_\tau, \\ 1-\tau & \text{otherwise.}\end{cases}$$

PEC is weighted PC!

Given the true weights w_i and

$\mathcal{I}_\tau^+ = \{i \in \{1, \ldots, n\} : w_i = \tau\}, \mathcal{I}_\tau^- = \{i \in \{1, \ldots, n\} : w_i = 1 - \tau\}$,

$n^+ = |\mathcal{I}_\tau^+|$ and $n^- = |\mathcal{I}_\tau^-|$, then the τ-expectile $e_\tau = e_\tau(Y) \in \mathbb{R}^p$ is:

$$e_\tau = \frac{\tau \sum_{i \in \mathcal{I}_\tau^+} Y_i + (1-\tau) \sum_{i \in \mathcal{I}_\tau^-} Y_i}{\tau n_+ + (1-\tau) n_-}.$$

ϕ_τ^* is the largest eigenvector of C_τ where

$$C_\tau = \frac{\tau}{n} \left\{ \sum_{i \in \mathcal{I}_\tau^+} (Y_i - e_\tau)(Y_i - e_\tau)^\top \right\} + \frac{1-\tau}{n} \left\{ \sum_{i \in \mathcal{I}_\tau^-} (Y_i - e_\tau)(Y_i - e_\tau)^\top \right\}.$$

Subjective Complexity Measures

- Complexity attitude as a risk factor
- Portfolio -averse, -neutral, -seeking subjects

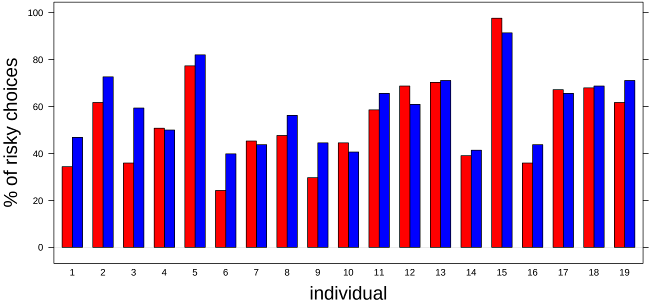
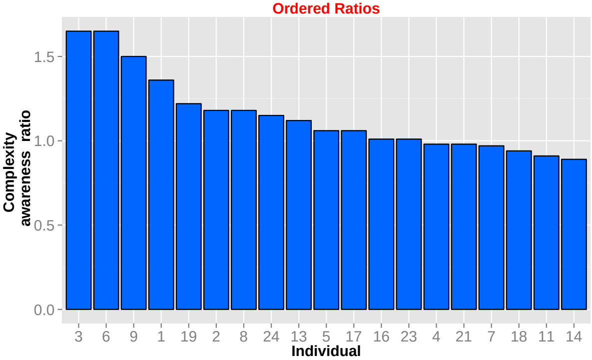


Figure 4: % of risky choices for single and portfolio inv. questions

Subjective Complexity Measures

$$ratio = \frac{\text{\% of risky choices for single investment questions}}{\text{\% of risky choices for portfolio question}}$$



Empirical Results: Clustering

- Number of clusters: 1000; cluster index $\mathfrak{s}$, $\mathfrak{s} = 1, \ldots, 1000$
  - 200: interpretability (anatomical atlases i.e. Talairach)
  - 1000: more accurate functional connectivity patterns

  NCut applied on brain initially divided into 8 subset (computationally feasible)

min	max	mean	median	Total
1	353	207.4	208	1000

Table 1: Descriptive statistics of clustering results averaged over subjects.
Computational time: 19×30 hours

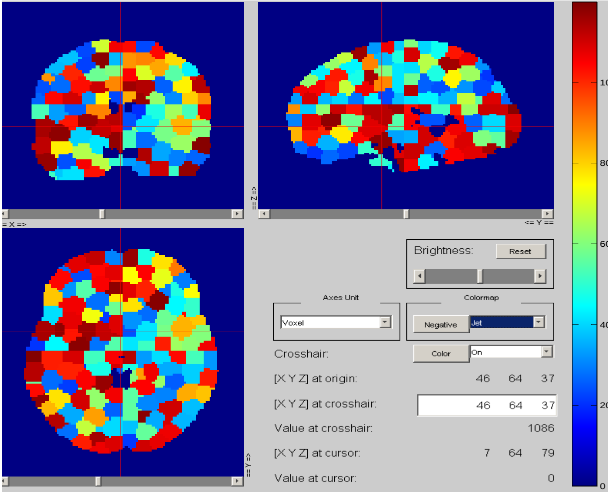


Figure 5: Parcellation results for the 1st subject's brain into 1000 clusters by NCut algorithm.

Cluster Activation: DMPFC

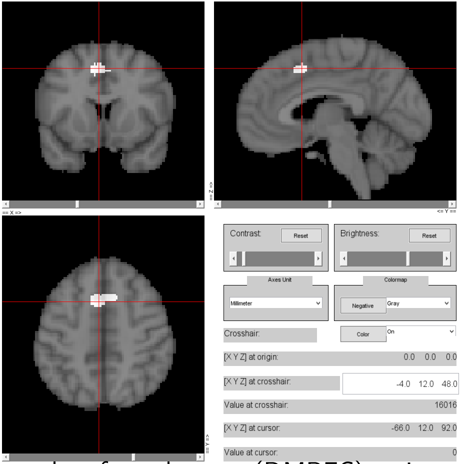


Figure 6: Dorsolateral prefrontal cortex (DMPFC) activated during all type of investment decisions in the group-level analysis. (▸ Z-scores)

Cluster Activation: aINS

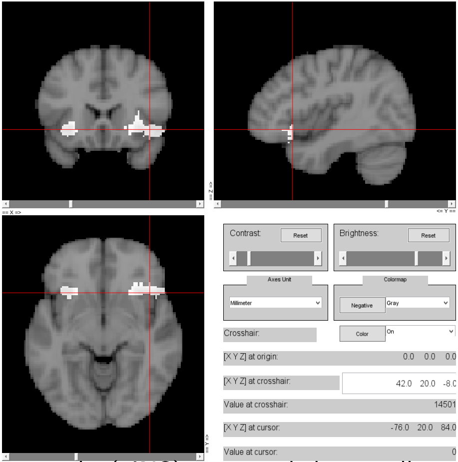


Figure 7: Anterior insula (aINS) activated during all type of investment decisions in the group-level analysis. ▸ Z-scores, ▸ aINS(l) ▸ aINS(r)

PEC

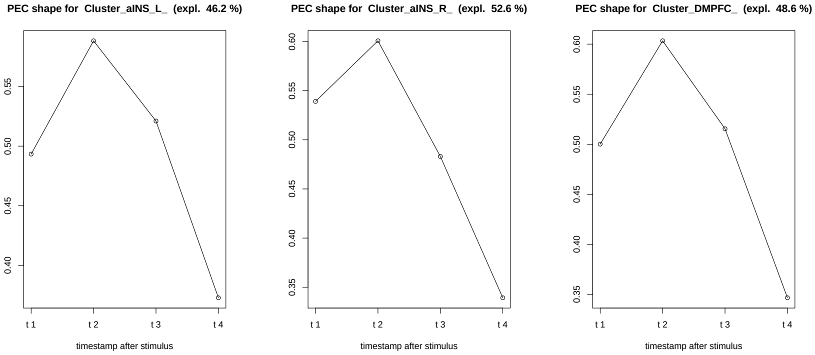


Figure 8: Estimated 1st 0.5-PEC of averaged cluster reaction for 4 time-points after stimulus common for all 19 subjects.

Complexity / Stimulus Response

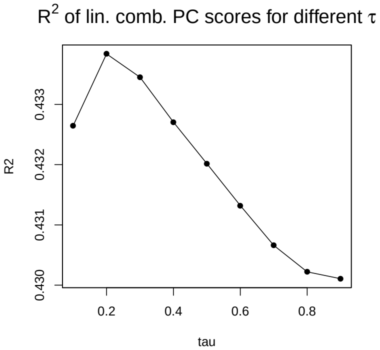
$$ratio = \beta_0 + \beta_1 \cdot \overline{score}^{\tau}_{ainsL} + \beta_2 \cdot \overline{score}^{\tau}_{ainsR} + \beta_3 \cdot \overline{score}^{\tau}_{DMPFC} \quad (1)$$

	Estimate	SE	tStat	pValue
C	1.138	0.049	23.0941	$3.89 \cdot 10^{-13}$
$score^{\tau}_{ainsL}$	0.005	0.007	0.644	0.529
$score^{\tau}_{ainsR}$	-0.015	0.006	2.541	0.023
$score^{\tau}_{DMPFC}$	-0.007	0.012	-0.539	0.598

Table 2: Complexity measure regressed on the average response for $\tau = 0.5$; $R^2 = 0.43$, adj.$R^2 = 0.32$.

Complexity / Stimulus Response

$$ratio = \beta_0 + \beta_1 \cdot \overline{score}^{\tau}_{ainsL} + \beta_2 \cdot \overline{score}^{\tau}_{ainsR} + \beta_3 \cdot \overline{score}^{\tau}_{DMPFC}$$



Complexity / Stimulus Response

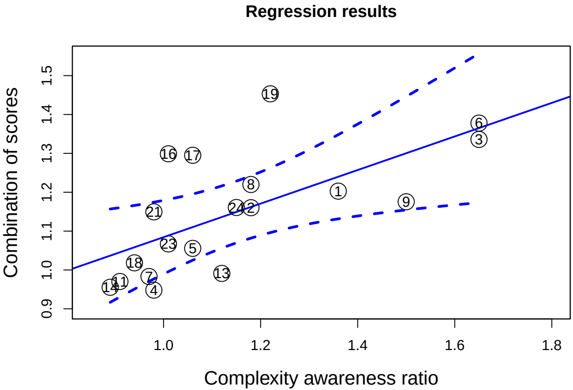


Figure 9: Added variable plot for model 1. Vertical axis denotes the best linear combination of scores that fit *ratio*.

Conclusion

- Local dynamic representation of the brain data
- Complexity as a factor in risk perception

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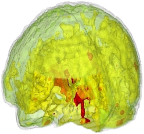

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fMRI Methods ▸ fMRI Dynamics

- Voxel-wise GLM ▸ Voxel-wise GLM
  - linear model for each voxel separately
  - strong a priori hypothesis
- Tensor probabilistic independent component analysis (T-PICA)
  - factors in spatial, temporal and subject domain
- Dynamic Semiparametric Factor Model (DSFM)
  - Use a “time & space” dynamic approach
  - Low dim time series exploratory analysis

Voxel-wise GLM ▸ fMRI methods, ▸ Cluster Activation, ▸ Simulations

- FEAT - FMRI Expert Analysis Tool by Department of Clinical Neurology, University of Oxford

- GLM framework

$$Y = X\mathfrak{b} + \eta, \qquad (2)$$

  Y - single voxel BOLD time series, X - design matrix (predicted response to stimulus i.e. **ID**, visual, auditory), $\mathfrak{b}$ - effect size

- Significant, active areas ($\mathfrak{b} >> 0$) selected by *z-scores* $\equiv \frac{\mathfrak{b}_i - 0}{\sqrt{\mathrm{Var}(\mathfrak{b}_i)}}$ and grouping (i.e. 20 neighbors) scheme

HRF ▸ fMRI methods ▸ fMRI dynamics

- Hemodynamic response function e.g. Double Gamma function

$$h(t) = (\frac{t}{5.4})^6 \exp(-\frac{t-5.4}{0.9}) - 0.35(\frac{t}{10.8})^{12} \exp(-\frac{t-10.8}{0.9}),\ t \geq 0\text{-time [sec]}$$

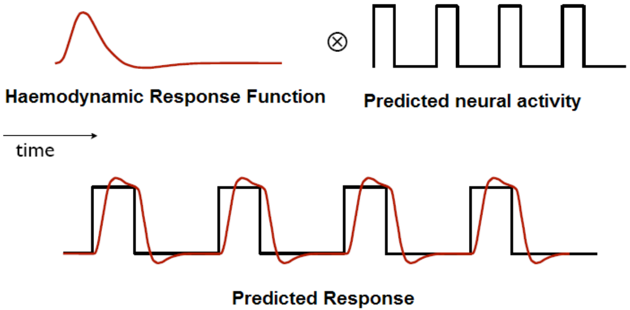

Figure 10: Predicted response as a convolution of a stimulus signal and a HRF. Figure modified from FEAT - FMRI.

Design Matrix

▸ fMRI methods

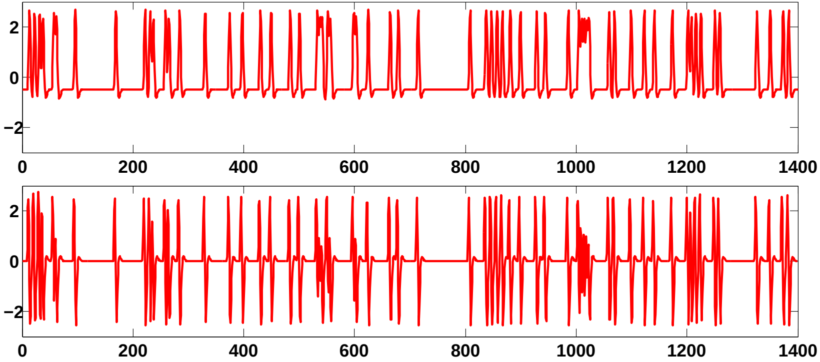

Figure 11: Predicted reaction to the stimulus (upper panel) and its derivative (lower panel) as an example of the elements of design matrix X 2).

Experiment

▸ ID Experiment

- Incentive to be **rational**
  - Draw 1 ID task and multiply subject's choice by 100 EUR
    $9\% \times 100 = 9$ EUR
- Gaussian returns:
  - $\mu = 5\%, 7\%, 9\%, 11\%$
  - $\sigma = 2\%, 4\%, 6\%, 8\%$

Single Investment

fMRI Experiment

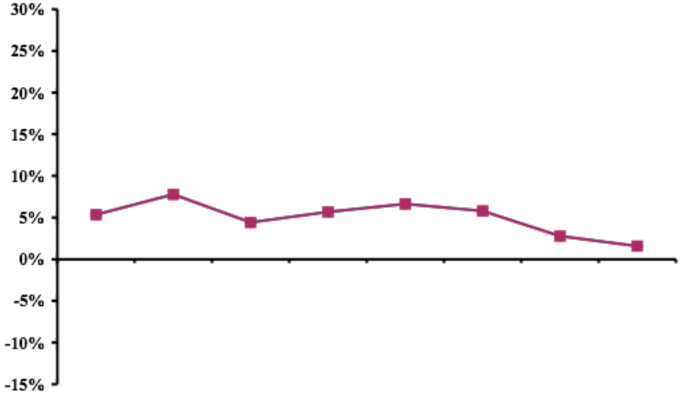

Figure 12: An example of return stream from single investment displayed to the subject during the experiment for 7 sec.; returns $r_i \sim \mathrm{N}(\mu, \sigma^2)$, here $\mu = 5\%, \sigma = 2\%$

Correlated Portfolio

▸ fMRI Experiment

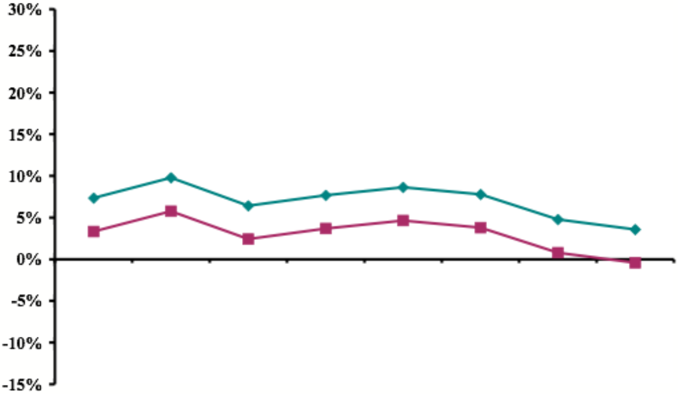

Figure 13: An example of return streams from correlated portfolio displayed to the subject during the experiment for 7 sec.; returns $r_i \sim N(\mu, \sigma^2)$, here $\mu_1 = 5\%$,$\mu_2 = 9\%$ and $\sigma = 2\%$

Uncorrelated Portfolio

▸ fMRI Experiment

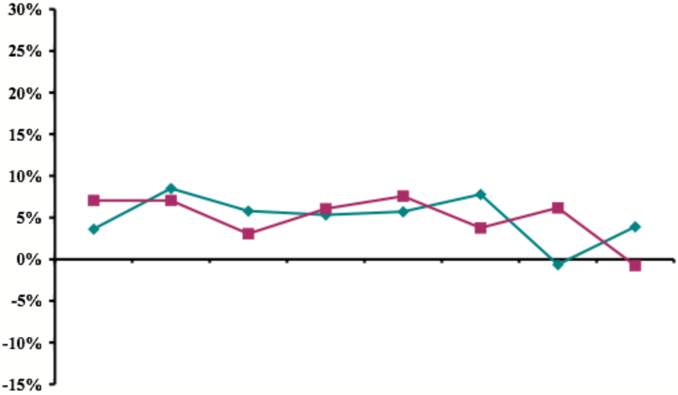

Figure 14: An example of return streams from uncorrelated portfolio displayed to the subject during the experiment for 7 sec.; returns $r_i \sim N(\mu, \sigma^2)$, here $\mu = 7\%, \sigma = 2\%$

Subject's Answers

▸ fMRI Experiment

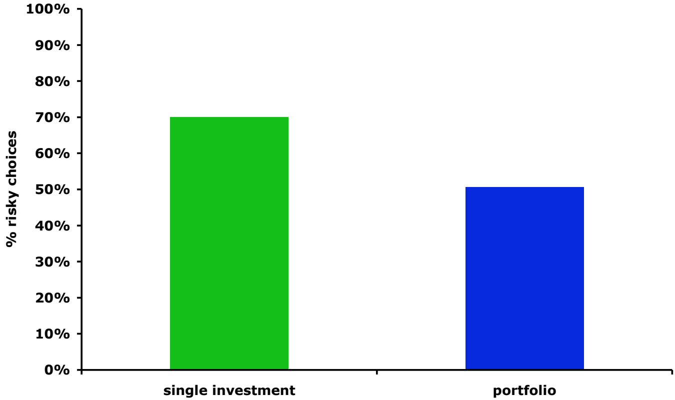

Figure 15: A proportion of risky choices selected by subjects for the single investment/portfolio (128/128 trials) setup averaged over all subjects.

aINS(left)

▸ aINS

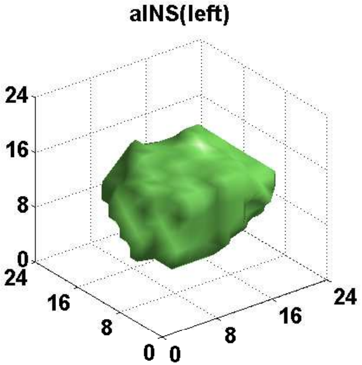

Figure 16: Derived aINS(l) regions for subject 1 (left) and 19 (right); axis are scaled in millimeters.

aINS(right)

aINS

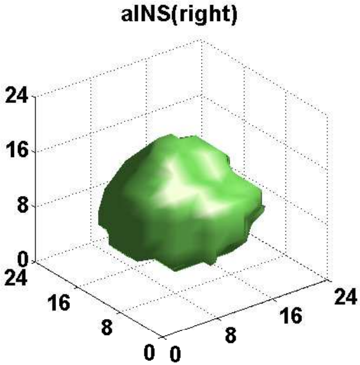

Figure 17: Derived aINS(r) regions for subject 1 (left) and 19 (right); axis are scaled in millimeters.

Cluster Activation: Results

	DSFM		Average		GLM	
aINS(l)	4.13	$(-34, 18, -8)$	4.08	$(-36, 18, -8)$	4.58	$(-32, 22, -12)$
		3×10^{-4}		4×10^{-4}		3×10^{-3}
aINS(r)	4.39	$(34, 24, -4)$	4.21	$(36, 18, -6)$	5.24	$(40, 22, -16)$
		6×10^{-6}		6×10^{-7}		3×10^{-7}
DMPFC	4.43	$(6, 24, 42)$	3.88	$(4, 24, 42)$	4.56	$(4, 24, 24)$
		2×10^{-9}		1×10^{-8}		3×10^{-7}

Table 3: Z-scores and p-values of activated "risk" clusters during the ID stimuli. The position of the cluster local maximum is denoted in the MNI (Montreal Neurological Institute) standard at 2mm resolution. Average stands for a mean value of each cluster (results of the Ncut parcellation with $K = 1000$). Analysis done in the FSL (FEAT/FLAME) software.

aINS, DMPFC

ACF: DMPFC

▸ DMPFC $\widehat{Z}$

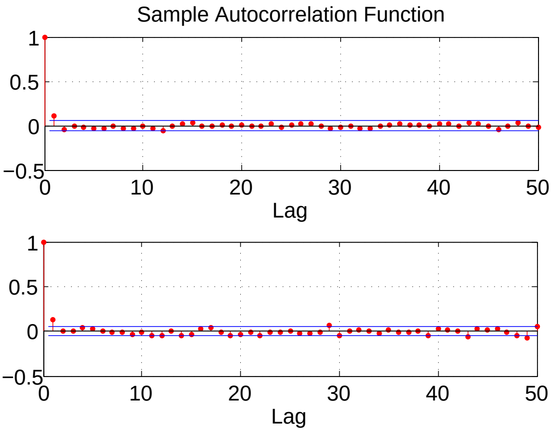

Figure 18: Sample autocorrelation function of **DMPFC** $\widehat{Z}$ for subjects 1 (top) and 19 (bottom), respectively.

ACF: aINS(l)

▸ aINS(left) $\widehat{Z}$

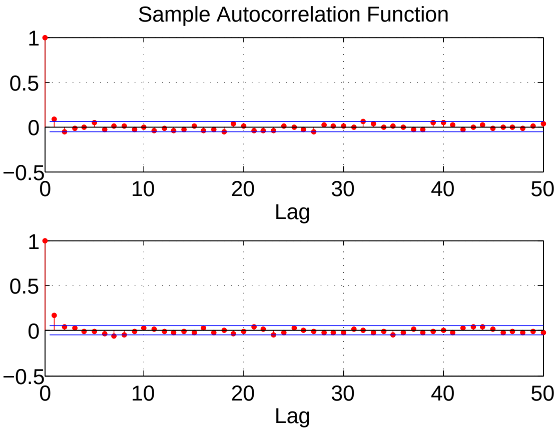

Figure 19: Sample autocorrelation function of **aINS**(left) $\widehat{Z}$ for subjects 1 (top) and 19 (bottom), respectively.

ACF: aINS(r)

▸ aINS(right) $\widehat{Z}$

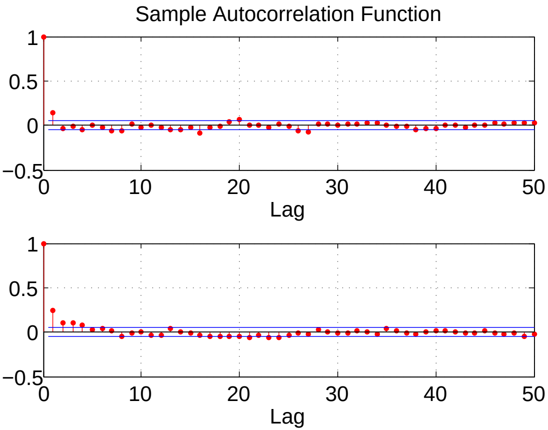

Figure 20: Sample autocorrelation function of **aINS**(right) $\widehat{Z}$ for subjects 1 (top) and 19 (bottom), respectively.

Correlation

▸ Proximity Measure

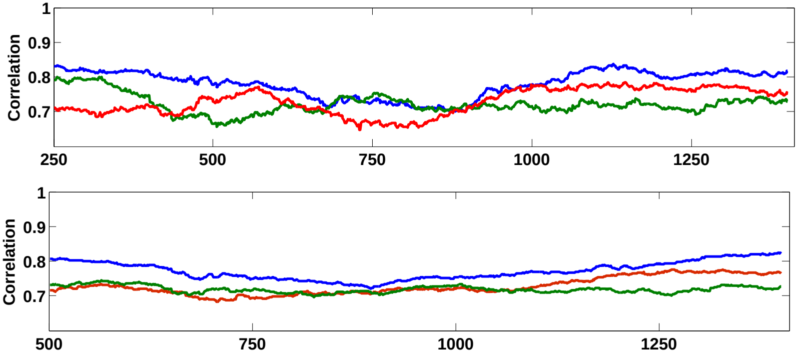

Figure 21: Time series of the correlation coefficient derived by the rolling window (250 top, 500 bottom) for the center voxel and: horizontal, vertical diagonal neighboring voxel for aINS(right) of subject 1.