# Time-varying Volatility Estimation with high Frequency Cryptocurrencies

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# High Frequency (HF) Cryptocurrency Trading

Spread (BCH/EUR)



#### Recent Trades (BCH/EUR)

1 - 15 of 250 trades

Time	•	Order (	Price	\$	Vo	lume 👙
18:37:05	+00:00	buy	€1,061	.0	0.25	i820000
18:37:05	+00:00	buy	€1,060	.8	0.21	540000
18:37:05	+00:00	buy	€1,054	.8	0.21	540000
18:37:05	+00:00	buy	€1,052	.0	0.14	500000
18:36:53	+00:00	sell	€1,050	.0	17.91	719153
18:36:53	+00:00	sell	€1,050	.0	0.32	000000
18:36:53	+00:00	sell	€1,050	.0	4.00	000000
18:36:53	+00:00	sell	€1,050	.0	0.00	201000
18:36:53	+00:00	sell	€1,050	.0	2.00	000000
18:36:53	+00:00	sell	€1,050	.0	1.00	000000
18:36:53	+00:00	sell	€1,050	.0	1.00	000000
18:36:53	+00:00	sell	€1,050	.0	0.02	000000
18:36:53	+00:00	sell	€1,050	.0	1.18	571428
18:36:53	+00:00	sell	€1,050	.0	0.25	000000
18:36:53	+00:00	sell	€1,050	.0	0.7	000000
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## HF Cryptocurrency Trading

Spread (BCH/EUR)



#### Recent Trades (BCH/EUR)

1 - 15 of 250 trades

Time 🔻	Order	Price 🔶	Volume	
18:39:11 +00:00	sell	€1,075.0	0.06900000	
18:39:11 +00:00	sell	€1,075.0	0.50000000	
18:39:11 +00:00	buy	€1,088.9	1.73518517	
18:39:11 +00:00	buy	€1,088.8	7.04000000	
18:39:11 +00:00	buy	€1,080.0	0.20000000	
18:39:11 +00:00	buy	€1,075.0	0.12587000	
18:39:10 +00:00	buy	€1,074.9	0.89894483	
18:39:05 +00:00	buy	€1,074.8	0.06940000	
18:39:05 +00:00	buy	€1,074.7	0.09260000	
18:39:05 +00:00	buy	€1,074.6	0.09260000	
18:39:05 +00:00	buy	€1,060.8	1.09224540	
18:39:05 +00:00	buy	€1,060.0	2.88410934	
18:39:05 +00:00	buy	€1,060.0	0.10000000	
18:39:04 +00:00	buy	€1,060.0	0.01269066	
18:38:58 +00:00	sell	€1,052.3	0.02742651	
< 1 <b>2</b>	3 4	5	17 >	





#### Event = surprise element?



HF CC

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## **Jump Detection**

- In the case where a large jump occurs, a simple glance at the dataset might be sufficient to decide this issue.
- Such large jumps are usually infrequent, small frequent jumps should also be considered.
- Characterize jumps both theoretically and empirically.
- Need efficient tests available for jumps that are sufficiently robust to withstand misspecification and small sample bias.
- Literature: Xue, Genday and Fagan (2014), Alt-Sahalia and Jacod (2009) etc.

#### **CC** Market

 Cryptocurrencies (CC) still represent an emerging market that suffers many changes because of updating regulatory requirements and contradictory attitudes from institutions and influential people



#### Data Source

# mkraken



- Data collected by Prof. Dr. Hermann Elendner.
- □ Trading period: trading never stops, 24/7, every single day.
- Cryptocurrency/Fiat exchange rates
  - Source: kraken.com
  - Largest Bitcoin exchange in euro volume and liquidity.





#	Name	<ul> <li>Market Cap</li> </ul>	Price	Volume (24h)	Circulating Supply	Change (24h)
1	B Bitcoin	€95,721,192,940	€5739.54	€5,747,818,097	16,677,500 BTC	7.66%
2	Ethereum	€25,900,477,737	€270.64	€1,194,641,852	95,701,059 ETH	3.79%
3	3 Bitcoin Cash	€16,290,313,750	€969.65	€3,513,114,092	16,800,275 BCH	-28.91%
4	- Ripple	€6,701,057,015	€0.173911	€145,051,449	38,531,538,922 XRP *	2.43%
5	Litecoin	€2,831,427,640	€52.62	€257,057,548	53,810,507 LTC	4.37%
6	Dash	€2,733,118,150	€355.60	€484,397,807	7,686,039 DASH	16.05%

Source: coinmarketcap.com

#### Data Structure



2017-06-28 BTC/EURO

- □ 1st level bid and ask price of CC/EUR exchange rate.
- ⊡ Timespan: 23.06.2017 30.07.2017, 24/7 every single day.

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#### **Basic Model**

 $\Box$  Observe a microstruct-noise contaminated  $Y_t$  with latent  $X_t$ ,

$$Y_t = X_t + \varepsilon_t, \quad t \ge 0$$

with  $E(\varepsilon_t|X) = 0$ .

 Efficient log price X<sub>t</sub> is semi-martingale, Delbaen and Schachermayer (1994),

$$X_t = X_0 + \int_0^t a_s ds + \int_0^t \sigma_s dW_s$$

- ▶  $(a_s)_{s\geq 0}$  càdlàg drift process,  $(\sigma_s)_{s\geq 0}$  càdlàg volatility process.
- $\int_0^t \sigma_s^2 ds$  integrated volatility.

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#### **Robust Integrated Volatility Estimator**

#### ⊡ Realized kernel estimator: weighted autocovariances.

- Barndorff-Nielsen, Hansen, Lunde, and Shephard (2008)
- Two-scale/Multi-scale estimator: weighted subsampled RVs.
  - Zhang, Mykland, Podolskij and Aït-Sahalia (2005)
  - Zhang (2006, 2011)
- Pre-averaging estimator: take weighted local averages before taking squares.
  - Jacod, Li, Mykland, Podolskij, and Vetter (2009)
  - Podolskij and Vetter (2009)





#### Price process with Jumps

When

$$X_t = X_0 + \int_0^t a_s ds + \int_0^t \sigma_s dW_s + \sum_{j=1}^{N_t} J_j$$

: The limit of RV  $\sum_{i=2}^{N} (X_{t_i} - X_{t_{i-1}})^2$  is  $\int_0^t \sigma_s^2 ds + \sum_{j=1}^{N_t} J_i^2$ 

- $\boxdot$  In practice, it is necessary to distinguish the  $\int_0^t \sigma_s^2 ds$  from  $\sum_{j=1}^{N_t} J_j^2$
- □ The bipower variation,  $\sum_{i=2}^{N_i} |X_{t_{i+1}} X_{t_i}| |X_{t_i} X_{t_{i-1}}|$ , converges to  $\lambda_1 \int_0^t \sigma_s^2 ds$  as  $max|t_i t_{i-1}| \to 0$ .
- Barndorff-Nelson and Shephard (2004, 2006) etc.

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## Non-synchronicity

□ Suppose covariation of two price process  $X_t^1, X_t^2$  as  $\langle X^1, X^2 \rangle$ , its realized volatility estimator is,

$$V_{\Delta_n} = \sum_{i=1}^{\left[\frac{t}{\Delta_n}\right]} \left( \overline{X}_{i\Delta_n}^1 - \overline{X}_{(i-1)\Delta_n}^1 \right) \left( \overline{X}_{i\Delta_n}^2 - \overline{X}_{(i-1)\Delta_n}^2 \right)$$

► Actual transaction are recorded at random times.

A portion of data missing at pre-specified grid.
 Based on Hayashi and Yoshida (2005),

$$\mathsf{E}[V_{\Delta_n}] = \mathsf{E}\left[\sum_{i=1}^{\lfloor \frac{t}{\Delta_n} \rfloor} \left\{ \langle X^1, X^2 \rangle_{\tau^1(i\Delta_n) \wedge \tau^2(i\Delta_n)} - \langle X^1, X^2 \rangle_{\tau^1((i-1)\Delta_n) \vee \tau^2((i-1)\Delta_n)} \right\} \mathsf{I}_{G^1 \wedge G^2} \right]$$

HF CC —

#### Jump Detection - BTC/EURO



Image: Interpretending of the state of t

#### Jump Detection - BTC/EURO



 $\boxdot$  h=200: unstable since 12th of July. The impacts last longer. HF CC

## Jump Detection - Ripple/EURO



➡ h=100: similar pattern as Bitcoin HF CC

Empirical Finding

#### Event?



 $\hfill Jumps$  may be caused by the exogenous events. source: HF Cbitcoinmagazine.com

#### Outlook

- □ Co-movement across different CC/Fiat exchange.
- Efficient tests for jumps that are sufficiently robust to withstand misspecification and small sample bias.
- Combine with sentiment analysis.



#### References

YI XUE, RAMAZAN GENCAY and STEPHEN FAGAN Jump detection with wavelets for high-frequency financial time series

Quantitative Finance Vol. 14, No. 8, 1427-1444.

🔋 N Hautsch, M Podolskij

Preaveraging-based estimation of quadratic variation in the presence of noise and jumps: theory, implementation, and empirical evidence

Journal of Business & Economic Statistics 31 (2), 165-183.



#### References

- YACINE AÏT-SAHALIA1 AND JEAN JACOD Testing for jumps in a discretely observed process The Annals of Statsistics Vol. 37, No. 1, 184-222.
- 🔋 N Hautsch, M Podolskij

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