Investing with cryptocurrencies - A liquidity constrained investment approach

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The emergence of cryptocurrencies

- Satoshi Nakamoto found Bitcoin in 2009
- 1178 cryptos (20.10.2017)
- Market cap: 170 billion USD
- 24h trading volume: 3.5 billion USD
- Community driven currencies
- Source codes public

Low correlation: Diversification

	BTC	ETH	XRP	LTC	DASH
USD/EUR	−0.05	−0.04	0.04	−0.06	−0.01
JPY/USD	0.02	−0.04	−0.03	−0.04	0.09
USD/GBP	−0.06	−0.09	0.04	−0.09	−0.01
Gold	0.05	0.04	0.04	0.05	−0.01
SP500	0.00	−0.05	0.05	−0.05	0.02
XWD	0.01	−0.03	0.02	−0.07	0.03
EEM	0.00	−0.09	0.04	−0.09	0.00
REIT	0.03	−0.09	0.04	0.05	0.00
DTB3	0.02	0.09	0.00	0.02	0.03
DGS10	−0.02	−0.08	0.00	−0.02	0.01

Table 1: Correlations between cryptos and conventional financial assets: 3 exchange rates, gold, 3 stock indices, real estate and the US Treasury Bills Rates.

Source: ▸ Elendner et al. (2017)

Benefit of crypto investment: high return

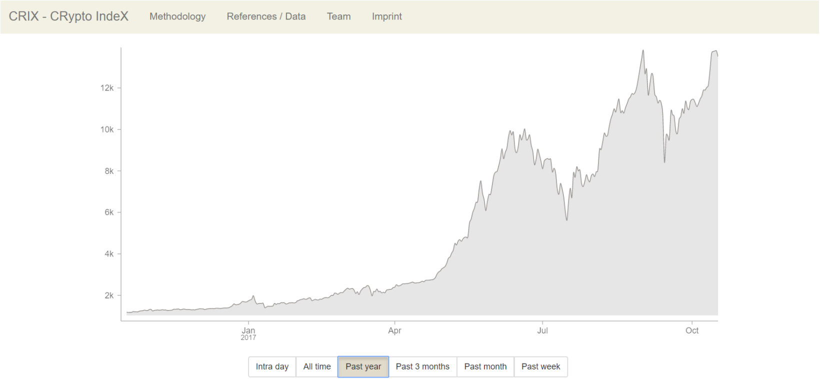


Figure 1: hu.berlin/crix

Reference: ▸ Trimborn and Härdle (2017)

Challenge I: high risk

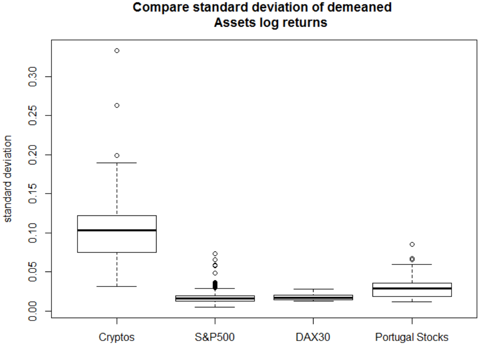


Figure 2: Cryptocurrencies have higher volatilities than stocks, highlighting the importance of risk management when investing on them LIBRObox1

Challenge II: low trading volume

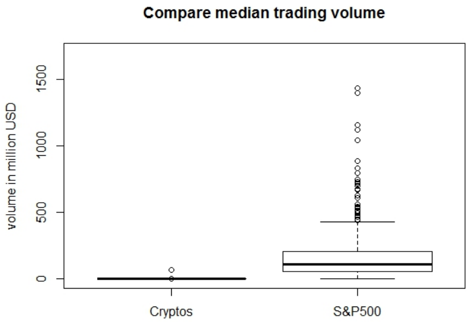


Figure 3: Cryptocurrencies have much lower trading volume compared to traditional assets LIBRObox2

Investment strategies

- Volatility based: Markowitz
- Quantile & shrinkage: TEDAS
- LASSO: Smaller tracking portfolios

But:

- Perfect liquidity is assumed
- Might not hold in crypto markets

▸ Liquidity measures

▸ Cryptocurrency investment literature

Challenges

- Adding low liquidity cryptocurrencies into standard portfolio
- Investment portfolios under liquidity restrictions
- How to measure liquidity?
- Errors due to chosen liquidity measure?

Outline

1. Motivation ✓
2. Optimization method
3. Data
4. Empirical results
5. Appendix
    - Bibliography
    - Effects of constraints

Optimization problem

Target optimization problem:

$$\min \; w^\top \hat{\Sigma} w \tag{1}$$
$$\text{s.t. } \mathbf{1}_p^\top w = 1, \; ||w||_1 \leq 1,$$
$$w \leq \frac{1}{M} \cdot \widehat{Liq} = \widehat{a},$$

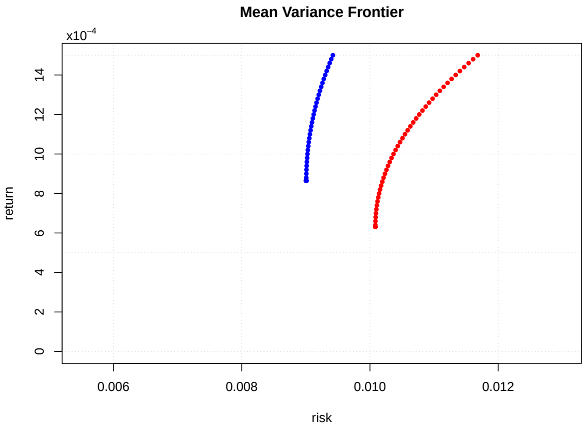
- $\widehat{Liq} = (TV_1 \cdot f_1, \cdots, TV_N \cdot f_N)^\top$
- $\widehat{\Sigma}$: estimated covariance matrix
- $w = (w_1, w_2, \cdots, w_p)^\top$: weight on assets
- $\mathbf{1}_p^\top = (1, 1, \cdots, 1)_{(1 \times p)}$
- M: investment amount

▸ Effect of constraints

Data Information

- 39 crypto currencies
- S&P100 component (102 stocks)
- DAX30 component (30 stocks)
- Portugal stock index (PSI) component (46 stocks)
- 2014-04-01 to 2017-03-20

DAX30 & Cryptos



▸ S&P and PSI Figure 4: —— with cryptos —— without cryptos

Weights on cryptos under target returns

$\bar{r}$	with S&P100		with DAX30		with PSI	
($\times 10^{-4}$)	BTC	ALT	BTC	ALT	BTC	ALT
0	0.007	0.046	0.000	0.137	0.000	0.068
1	0.007	0.046	0.000	0.137	0.000	0.068
2	0.007	0.046	0.000	0.137	0.000	0.068
3	0.007	0.047	0.000	0.137	-0.000	0.070
4	0.006	0.052	0.000	0.137	-0.000	0.073
5	0.006	0.056	0.000	0.137	0.000	0.075
6	0.005	0.061	0.000	0.137	0.000	0.078
7	0.004	0.068	0.000	0.137	-0.000	0.081
8	0.002	0.075	0.000	0.137	0.000	0.085
9	-0.000	0.081	0.000	0.138	0.000	0.088
10	0.000	0.087	0.000	0.142	-0.000	0.091

Table 2: Weights on cryptocurrencies in-sample given different target return. "ALT" refers to total weight on altcoins.

DAX30 & Cryptos

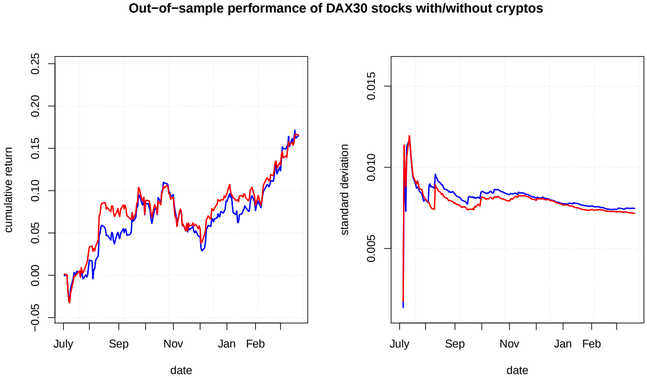


Figure 5: —— with cryptos —— without cryptos, no liquidity constraint applied LIBROoutsample

DAX30 & Cryptos

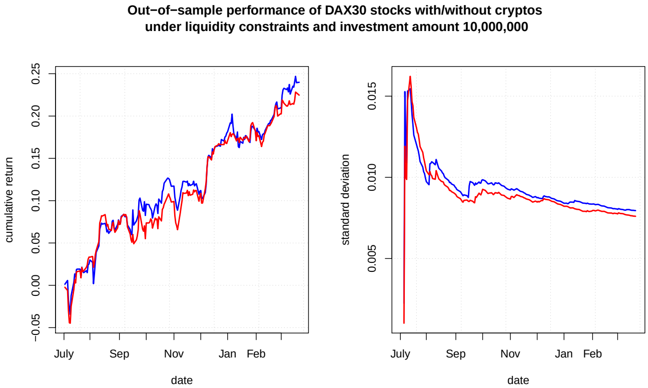


Figure 6: —— with cryptos —— without cryptos, liquidity constraint with $M = 10,000,000$ LIBROoutsample

S&P and PSI

Conclusion

- We propose LIBRO: LIquidity Bounded Risk-return Optimization
- Including cryptos can provide better risk-return trade off
- Cryptos beside Bitcoin matter for portfolio optimization
- Better on less developed markets (see Appendix)
- LIBRO even enhances results in traditional markets

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Oberwolfach Report No. 42/2015 "The Mathematics and Statistics of Quantitative Risk".

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Constrained Portfolios I

Define

$$R(w) = w^\top \Sigma w \quad R_n(w) = w^\top \widehat{\Sigma} w$$

Let

$$w_{opt,a} = \underset{w^\top 1=1, \mu^\top w \geq \bar{r}, ||w||_1 \leq c, w \leq a}{\arg\min} R(w)$$

$$\widehat{w}_{opt,\hat{a}} = \underset{w^\top 1=1, \mu^\top w \geq \bar{r}, ||w||_1 \leq c, w \leq \hat{a}}{\arg\min} R_n(w)$$

$$b_n = ||\widehat{\Sigma} - \Sigma||_\infty$$

$$\widehat{a} = a + \varepsilon$$

▸ Back

Constrained Portfolios II

Fan et al. (2012):

$$|R(w_{opt,a}) - R_n(\widehat{w}_{opt,a})| \leq b_n c^2$$

Define:

$$\begin{aligned}
w_{opt,a} &\leq a \\
w_{opt,\widehat{a}} &\leq \widehat{a} \\
a &= w_{opt,a} + \delta_1, \quad \delta_1 \text{ slack variable} \\
\widehat{a} &= w_{opt,\widehat{a}} + \delta_2, \quad \delta_2 \text{ slack variable} \\
w_{opt,\widehat{a}} &= w_{opt,a} + \delta_1 - \delta_2 + \varepsilon
\end{aligned}$$

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Constrained Portfolios III

Consider $\delta_2 = 0$:

- All estimated weights are at boundary

$$
\begin{aligned}
|R_n(\widehat{w}_{opt,a}) - R_n(\widehat{w}_{opt,\hat{a}})| &= |\widehat{w}_{opt,\widehat{a}}^\top \widehat{\Sigma} \varepsilon| \\
|R(w_{opt,a}) - R_n(\widehat{w}_{opt,\hat{a}})| &\leq |\widehat{w}_{opt,\widehat{a}}^\top \widehat{\Sigma} \varepsilon| + b_n c^2 \\
A &= |\widehat{w}_{opt,\widehat{a}}^\top \widehat{\Sigma} \varepsilon|
\end{aligned}
$$

- A bigger when $\varepsilon > 0$ than if $\varepsilon < 0$
- Overestimation of a causes smaller upper bound for error
- ε bounded by the choice of c

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Constrained Portfolios IV

$$
\begin{aligned}
&|R_n(\widehat{w}_{opt,a}) - R_n(\widehat{w}_{opt,\hat{a}})| \\
&= |\widehat{w}_{opt,a}^\top \widehat{\Sigma} \widehat{w}_{opt,a} - \widehat{w}_{opt,\hat{a}}^\top \widehat{\Sigma} \widehat{w}_{opt,\hat{a}}| \\
&= |(\widehat{w}_{opt,a} + \delta_1)^\top \widehat{\Sigma} (\widehat{w}_{opt,a} + \delta_1) \\
&\quad - (\widehat{w}_{opt,a} + \delta_1 - \delta_2 + \varepsilon)^\top \widehat{\Sigma} (\widehat{w}_{opt,a} + \delta_1 - \delta_2 + \varepsilon)| \\
&= |(\widehat{w}_{opt,a} + \delta_1)^\top \widehat{\Sigma} (\delta_2 - \varepsilon) - \delta_2^\top \widehat{\Sigma} (\delta_2 - \varepsilon) + \varepsilon^\top \widehat{\Sigma} (\delta_2 - \varepsilon)|
\end{aligned}
$$

Situation of interest:

- Any $\varepsilon_i > 0$ and any $\delta_{2,i} = 0$
- Then a weight is at a non optimal point

▸ Back

Constrained Portfolios V

Consider $\delta_2 = 0$:

$$
\begin{aligned}
&|-(\widehat{w}_{opt,a} + \delta_1)^\top \widehat{\Sigma}\varepsilon - \varepsilon^\top \widehat{\Sigma}\varepsilon| \\
=&|-(a + \varepsilon)^\top \widehat{\Sigma}\varepsilon| \\
=&|-\widehat{w}_{opt,\widehat{a}}^\top \widehat{\Sigma}\varepsilon|
\end{aligned}
$$

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Constrained Portfolios VI

$$
\begin{aligned}
&|R(w_{opt,a}) - R_n(\widehat{w}_{opt,\widehat{a}})| \\
=&|R(w_{opt,a}) - R_n(\widehat{w}_{opt,a}) + R_n(\widehat{w}_{opt,a}) - R_n(\widehat{w}_{opt,\widehat{a}})| \\
\leq&|R(w_{opt,a}) - R_n(\widehat{w}_{opt,a})| + |R_n(\widehat{w}_{opt,a}) - R_n(\widehat{w}_{opt,\widehat{a}})| \\
\leq&b_n c^2 + |(\widehat{w}_{opt,a} + \delta_1)^\top \widehat{\Sigma}(\delta_2 - \varepsilon) - \delta_2^\top \widehat{\Sigma}(\delta_2 - \varepsilon) + \varepsilon^\top \widehat{\Sigma}(\delta_2 - \varepsilon)|
\end{aligned}
$$

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Implications for application

- Range of ε bounded by c
- Proper choice of c sets upper bound
- Divergence in volatility risk increases, but liquidity risk controlled for
- Trade-off between two risk sources increases volatility risk of portfolio

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S&P100 & Cryptos

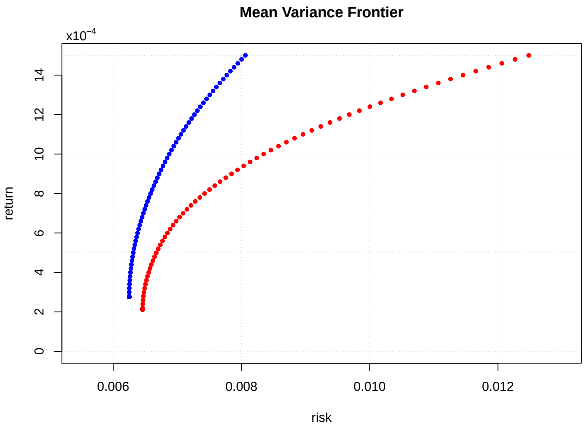

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Figure 7: —— with cryptos —— without cryptos

PSI component & Cryptos

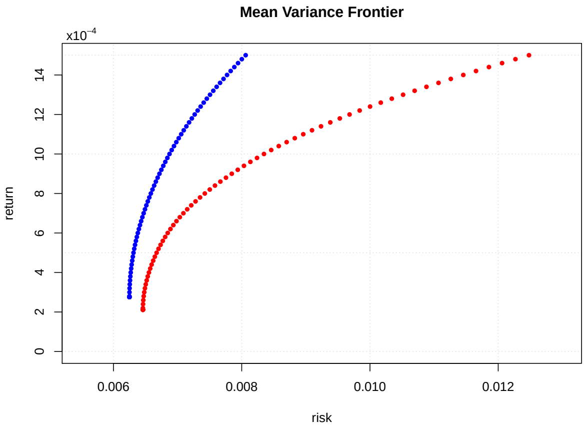

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Figure 8: — with cryptos — without cryptos

S&P100 & Cryptos

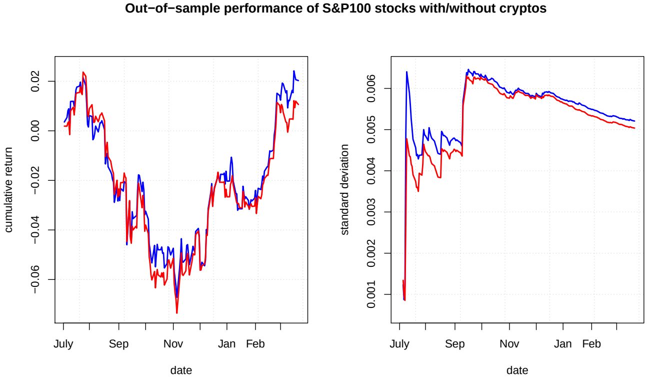

Figure 9: —— with cryptos —— without cryptos, assets on the right of the black dash line are cryptos LIBROoutsample

S&P100 & Cryptos

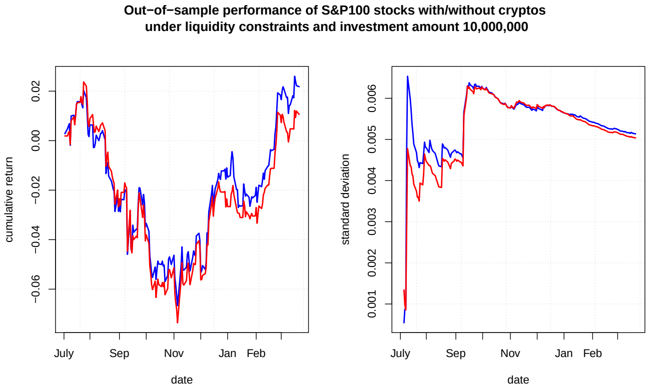

Figure 10: —— with cryptos —— without cryptos, assets on the right of the black dash line are cryptos LIBROoutsample

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PSI component & Cryptos

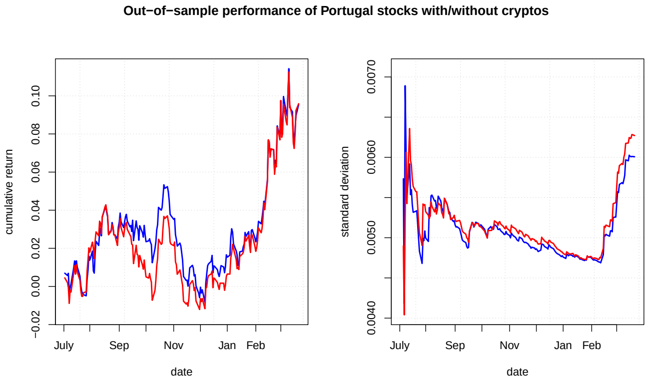

Figure 11: — with cryptos — without cryptos, assets on the right of the black dash line are cryptos LIBROoutsample

PSI component & Cryptos

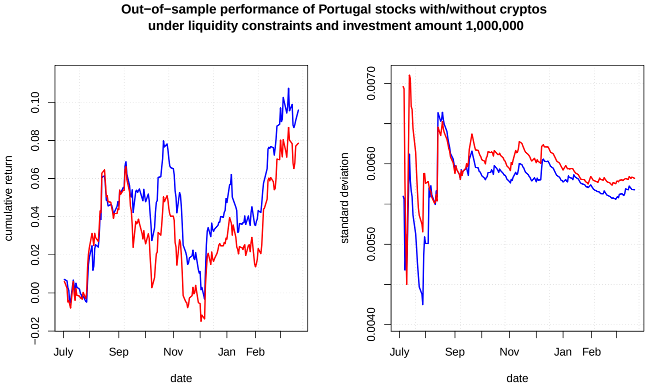

Figure 12: —— with cryptos —— without cryptos, assets on the right of the black dash line are cryptos LIBROoutsample

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Liquidity restrictions

- Assets have different liquidity
- Trading volume on 20.10.2017
  - Apple: 6,643,129,896 USD
  - Bitcoin: 1,684,650,000 USD
  - Ethereum: 332,016,000 USD
- But liquidity is unobservable
- Existing measures:
  - Trading volume
  - Amihud measure
  - ...

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Cryptos from an investment viewpoint

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