Pricing Cryptocurrency options: the case of CRIX

Cathy Yi-Hsuan Chen
Wolfgang Karl Härdle
Ai Jun Hou
Weining Wang

Ladislaus von Bortkiewicz Chair of Statistics
C.A.S.E.-Center for Applied Statistics and Economics
Humboldt–Universität zu Berlin
http://lvb.wiwi.hu-berlin.de

Currencies - Cigarettes, USD, Cryptos

- Anything can be a currency

Figure 1: Cigarette trading in postwar Germany ([1])

- Anyone can offer a currency

Figure 2: Friedrich A. Hayek ([2])

Digital Economy

- Amazon
- Paypal
- Google Wallet
- Cryptocurrencies
- Ripple

Cryptocurrencies

- Decentralized, virtual, low transaction costs

- NYSE, Andreesen Horowitz, DFJ: Coinbase funding (75 M$)
- Nasdaq: company-wide utilization of blockchain technology
- Citigroup: own coin development
- PBOC: working on digital currency
- Switzerland Zug: first city accepts Bitcoin payments

Pokémon Go and Cryptocurrency

- Each creature could have an asset based crypto-tokens that could be traded in blockchain.
- Pokémon and BTC: PokéBits

Source: steemit, Bitcoin.com

Market Capitalization

CoinMarketCap

CRypto IndeX - CRIX

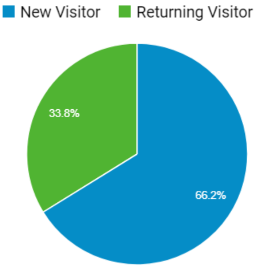
- high market capitalization
- covers approximately 30 cryptos
  - different liquidity rules
  - model selection criteria
- CRIX family
  - CRIX
  - ECRIX (Exact CRIX)
  - EFCRIX (Exact Full CRIX)

crix.hu-berlin.de

Reference: Trimborn, S. and Härdle, W. (2016)

CRypto IndeX - CRIX

- 1273 cryptos up to 20171108
- Prices, capitalization, volume
- As of 20160815, overview of CRIX: hu.berlin/crix
  - Users: 14500
  - Page views: 27300
  - average time: 00:01:01



Challenge

1. What's the dynamics of CRIX?
2. How stable is the CRIX model over time?
3. Consequence for pricing derivatives.

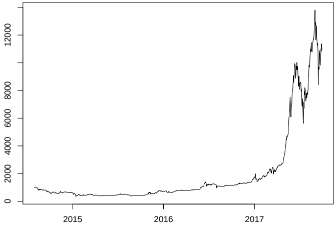
The Econometrics of CRIX

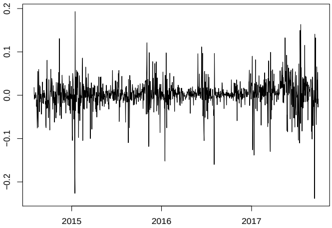
Outline

1. Motivation ✓
2. Data
3. Discrete-time GARCH Framework
4. Continuous-time Stochastic Volatility Model with Jumps
5. Pricing Cryptocurrency Options
6. Discussions

All QuantLets from www.quantlet.de

The price and log return of CRIX index





econ_crix

Distributional Property

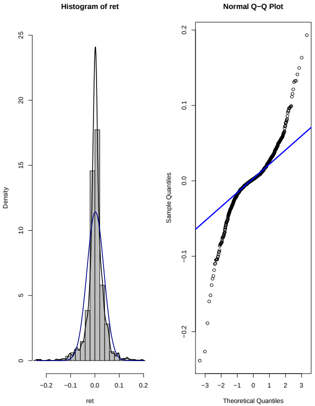


Figure 3: Histogram and QQ plot of CRIX returns from 01/08/2014 to 29/09/2017. econ_crix

Volatility Clustering

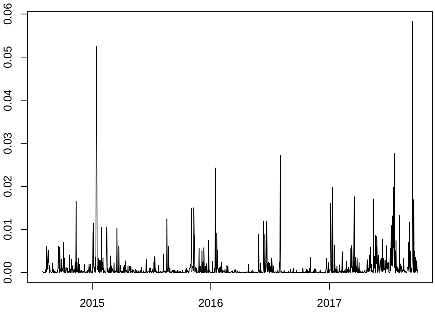


Figure 4: The squared ARIMA(2,0,2) residuals of CRIX returns.
econ_vola

ARIMA-t-GARCH specification

- The ARIMA($p,0,q$)-t-GARCH(p,q) model is

$$\begin{aligned} a(L)\Delta y_t &= b_L \varepsilon_t \\ \varepsilon_t &= Z_t \sigma_t, \quad Z_t \sim t(d) \\ \sigma_t^2 &= \omega + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 \end{aligned}$$

Parameter Estimation of ARIMA

Coefficients	Estimate	Standard deviation
intercept c	0.002	0.001
a_1	-0.819	0.188
a_2	-0.791	0.112
b_1	0.828	0.207
b_2	0.746	0.127
Log lik	2243.360	

Table 1: Estimation result of ARIMA(2,0,2) model. econ_arima

t-GARCH Estimation

- ξ controls the height and fat-tail of density function.

Coefficients	Estimates	Standard deviation	T test
ω	$4.93e-05$	$2.69e-05$	$1.83^{\cdot}$
α_1	$5.12e-01$	$2.30e-01$	2.23^{***}
β_1	$7.75e-01$	$3.38e-02$	22.88^{***}
ξ	$2.39e+00$	$2.18e-01$	10.96^{***}

Table 2: Estimation result of ARIMA(2,0,2)-t-GARCH(1,1) model. . represents significant level of 10% and $***$ of 0.1%. econ_tgarch

t-GARCH Model Residual

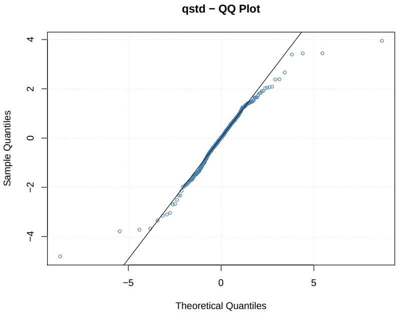


Figure 5: The QQ plots of model residuals of ARIMA-t-GARCH process.

econ_tgarch

Forecast based on ARIMA-t-GARCH

- With ARIMA-t-GARCH model, we predict CRIX returns for next 30 days.

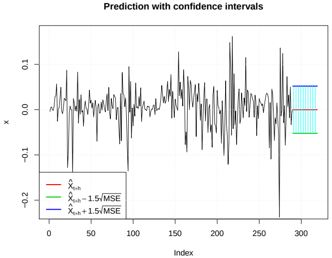


Figure 6: CRIX returns, predicted values, and confidence bands. econ_arima

Augment to joint price and volatility dynamics

- QQ plots are unsatisfactory
- Tail behavior not correctly reflected
- Move to SVCJ!

SVCJ (SV Correlated Jumps in price and volatility)

- Jumps in BTC
- Employ SVCJ, Duffie, Singleton and Pan (2000)
- $\{Y_t\}$ price process, $\{V_t\}$ volatility process:

$$d\log Y_t = \mu dt + \sqrt{V_t} dW_{y,t} + Z_{y,t} dN_t \quad (1)$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW_{v,t} + Z_{v,t} dN_t \quad (2)$$

SVCJ model - ctd

- κ, θ mean reversion rate, level
- W_y and W_v: ρ correlated Brownian motion
- N_t pure jump processes, jump-arrival rate λ, random jump size Z_y and Z_v

$$
\begin{aligned}
Z_y | Z_v &\sim \mathrm{N}(\mu_y + \rho Z_v, \sigma_y^2) \\
Z_v &\sim \exp(\mu_v)
\end{aligned}
$$

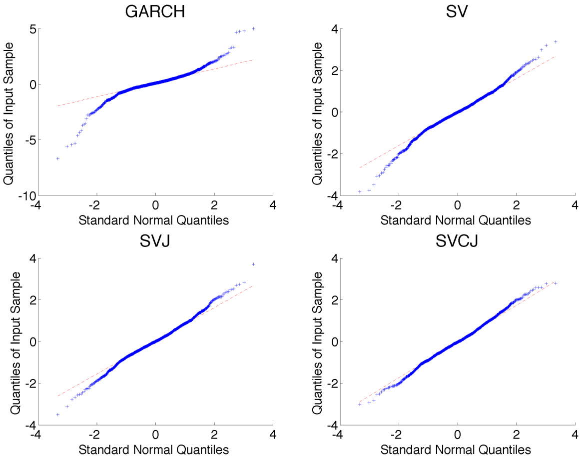
SVCJ remarks

- Put $Z_v = 0$, one obtains the SVJ model of Bates (1996)
- Taking $\lambda = 0$ reduces to the pure SV model(Heston,1993)
- SVJC encompasses these two model classes

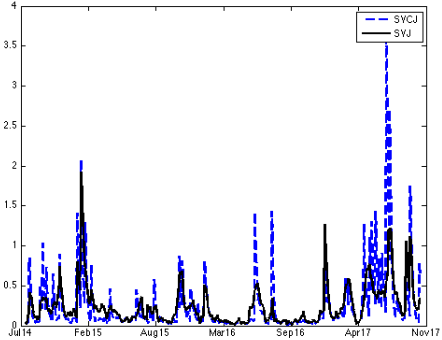
Table 3: Parameters estimated for SVCJ, SVJ and SV model

	SVCJ		SVJ		SV	
	Mean	*Std.Dev*	*Mean*	*Std.Dev*	*Mean*	*Std.Dev*
μ	0.0421***	0.006	0.044***	0.009	0,023***	0,009
μ_y	-0.049	0.371	-0.515	0.303	-	-
σ_y	2.061***	0.432	2.851***	0.767	-	-
λ	0.051***	0.007	0.035***	0.009	-	-
α	0.010***	0.001	0.026	0.019	0,010***	0.001
β	-0.19***	0.009	-0.24***	0.073	-0.04***	0,009
ρ	0.275***	0.069	0.214**	0.102	0,003	0,068
σ_v	0.007***	0.001	0.016*	0.009	0.018	0.002
ρ_j	-0.210	0.364	-	-	-	-
μ_v	0.709***	0.089	-	-	-	-
MSE	0.673	-	0.702	-	0.736	-

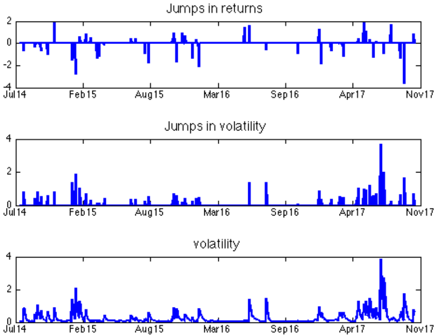
Figure 7: QQ Plots for varies models



Estimated V_t from SVCJ and SVJ



SVCJ jumps in returns and volatility



SVCJ model under Risk Neutral

- Under the “risk neutral” Q the spot price and volatility dynamics are given by

$$\frac{dy_t}{y_t} = (r - \mu^*)\mathrm{d}t + \sqrt{V_t} dW^Q_{y,t} + Z_{y,t} dN^Q_{y,t}$$
$$V_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW^Q_{v,t} + Z^v_{v,t} dN^Q_{v,t}$$

- where $W^Q_{y,t}$, $W^Q_{v,t}$ and $N^Q_{y,t}$, $N^Q_{v,t}$ are the corresponding Brownian motions and Poisson process under Q.
- Absence of arbitrage requires that the drift of S under Q is given by $(r - \mu^*)$ where r is the risk free interest rate and $\mu^* = \mathrm{E}\, Q[\exp(Z^Y)] - 1$ is the jump compensator.
- Since our purpose is to explore the impact of model choice on option values we follow Eraker et al. (2003) and ignore the possible existence of risk premia. This is not an uncommon assumption in the empirical option literature.

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COGARCH Model

- Irregularly spaced data: continuous-time GARCH model.
- The GARCH(1,1) model diffusion limit satisfies,

$$
\begin{aligned}
dG_t &= \sigma_t dW_t^{(1)} \\
d\sigma_t^2 &= \theta(\gamma - \sigma_t^2) + \rho\sigma_t^2 dW_t^{(2)}
\end{aligned}
$$

- G_t is the log return r_t to estimate.
- $\left\{W_t^{(1)}\right\}_{t\geq 0}$ and $\left\{W_t^{(2)}\right\}_{t\geq 0}$ are two independent Brownian motions.
- θ, γ and ρ are parameters.

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