

**Matching Theory to Data:  
Impulse Responses in Dynamic Macro Models  
versus Impulse Responses in Bayesian Vector  
Autoregression**

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## **Abstract**

An estimation method for the inference on parameters of a dynamic macro model originally developed by Christiano, Eichenbaum and Evans is examined, extended and implemented in MATLAB. Since a large amount of research deals with the dynamic implications of models, the idea is to minimize the distance between the impulse response function of a Bayesian VAR and the model's impulse response function. The results show that the method, with certain modifications, is applicable and can generate practically useful results.

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## Abbreviations

DSGE	dynamic stochastic general equilibrium
i.e.	id est (that is)
n.a.	not available
pdf	probability distribution function
RBC	real business cycle
RLOM	recursive law of motion
VAR	vector autoregression
VCV	variance covariance
w.r.t.	with respect to

# 1 Introduction

In 1982 Finn Kydland and Edward Prescott introduced a new way of employing economic models. By that time, economic models provided a framework mainly for econometric methods. Economic theory was used to derive a system of equations conditional on unknown parameters. Furthermore was an error term added to account for the gap between the theory and the data, and certain assumptions were made about it. Different estimation methods for the parameters were discussed depending on certain properties of the error term. With the help of hypothesis tests, theories got evaluated and falsified.

In Kydland and Prescott's opinion this approach was incorrect for two reasons: models fail to capture all characteristics of an economy and are likely to be misspecified. To put it differently, the model is false anyway and econometric methods to test if the model is right or not, are more or less useless. Instead, they suggested the following methodology to answer an economic question using a “false” model: choose a model, that bears some relevance to the question, solve for the endogenous variables depending on the exogenous ones and the parameters, select reasonable values for them and carry out an computational experiment. Then evaluate the outcome and propose an answer. This methodology is called calibration.

Even though this methodology and the idea of a computational experiment has been widely accepted, there still remain some controversial parts. One question of interest is how to select the parameters of the model and how to evaluate their choice. In the paper “Nominal rigidities and the dynamic effects of a shock to monetary policy” from 2001 Christiano, Eichenbaum and Evans made a new contribution to this matter. In their opinion, “most of our economic intuition about a model is concentrated on its implications for the dynamic effects” (Christiano, 2002). They proposed consequently to assimilate the dynamics of the model to the dynamics observable in the data. A group of parameters could be therefore estimated “by making model impulse responses look like the estimated impulse response functions of a VAR process” (Christiano, 2002), i.e. by minimising the distance between the two impulse response functions. The distance can be measured by the variances of the data. This enabled them to evaluate the choice of the parameters.

In this thesis I want to make a contribution to the selection of parameters for a Dynamic Stochastic General Equilibrium (DSGE) model by examining and extending the estimation procedure proposed by Christiano, Eichenbaum and Evans.

In order to analyse their approach an impulse response function for a Bayesian VAR process will be computed by a MATLAB program. As a benchmark model Hansen's Real Business Cycle (RBC) model will be solved in dependence on the model's parameter. The parameter will be estimated by employing the method above. The method will be supplemented by variation of the Variance Covariance matrix that is employed in the estimation criterion and by calculating standard deviations of the estimated parameter values. This will lead to a unified framework for the choice of parameters for a DSGE model.

The thesis is organised in the following way. An overview of the literature is given in section two. Section three, four and five describe the theoretical background and methods that are used, i.e. foundations of the Bayesian VAR are outlined, the benchmark model is solved and the estimation method is explained. In the subsequent section the implementation in MATLAB is explained, a manual for further usage is presented, the building blocks are introduced and potential problems are treated. In section seven and eight the results are presented and discussed. A conclusion is drawn in section nine.

## 2 Literature

When Kydland and Prescott first published their methodology in 1982, they also made the first suggestion of how to select parameters for a model. Parameters can be determined either on the basis of other studies in applied economics, i.e. studies in microeconomics about preferences, or by fitting the model's steady state to long run characteristics of the economy. In succeeding papers (Kydland and Prescott 1991, 1992; Kydland 1992) they went into more detail, explaining which long run averages they wanted to meet (i.e. the shares of inventories of GDP or share of consumption) and how to use studies from other economic fields to determine, for example, the elasticity of substitution. The values of common parameters among different areas should be identical, because the language is the same, they argued. Additionally they argued against the use of econometric estimation methods. The goal of using a model is to provide a clear answer to a question, not to fit the data. They even emphasize, "that the parameters values selected are not the ones that provide the best fit in some statistical sense"(Kydland and Prescott 1996). Given the simplicity and abstractions of a model anomalies and discrepancies will remain and provide the ground for further economic research.

The literature criticized this way of choosing parameters for the following reasons. At first parameters available from other microeconomic studies are sparse and cannot be applied directly in the DSGE framework (Hansen and Heckmann, 1996). The wide range of existing parameter estimates (for example the risk aversion parameter) can lead to selection bias and therefore to inconsistencies between studies. But the most striking point was the problem of evaluation. Kydland and Prescott pointed out, that the confidence we have in an answer does depend more on our confidence in the theory we use than on a computed measure, of how well the model economy mimics historical data. They suggested dealing only with "well tested theory" (Kydland and Prescott 1996). Many researchers did not agree and developed several methods to measure the fit of a model (for example Watson, 1993). Some argued, that the degree of uncertainty in the estimation of the parameters is "an important ingredient of model evaluation" (Christiano and Eichenbaum, 1992). They tried to apply a method to estimate parameters within the computational experiment framework, which allowed them to measure the uncertainty.

Two different methods became apparent, namely, the generalized method of moments (GMM) estimator and the simulated methods of moment (SMM) estimator. These methods produce not only measures of uncertainty that make it possible to evaluate the quality of the model, they also provide a coherent framework for estimating parameters and avoid thereby selecting parameters in a way that was criticised by Hansen and Heckmann. Besides the advantages they have also a range of disadvantages. Inconsistency across studies can arise, because different researchers select different moments or statistics to match. Estimation by GMM can be biased in small samples. Additionally, most of the variables are measured with error or unobservable facts that can cause serious problems. Furthermore, the tests performed on the parameter to indicate how to obtain a better fit fail. All these shortcomings encouraged researchers to calibrate their models rather than to estimate the parameters.

In 1995 Canova suggested a method, which responded to the criticism of calibration (Canova 1994 and 1995). He developed a coherent framework for the choice of parameters, which account for the uncertainty in the choice, and an evaluation process. Prior information about the parameter is used to form a prior distribution. In a Monte Carlo procedure, parameters are drawn from the distribution and the model is simulated for each draw. The received information makes it possible to perform a sensitivity analysis and to evaluate the model. This method was successfully applied, for example by Maffezoli (Maffezoli 2000).

The idea of comparing a model with VAR impulse responses had already been developed by Sims (1989) and Smith (1992). They also suggested measuring the distance between the impulse responses to judge the model. Finally, in 2001 Christiano, Eichenbaum and Evans proposed a criterion, which should “make models` impulse responses look like the estimated impulse responses” (Christiano, 2002). A formal estimation criterion was developed to minimize the distance between the models` impulse responses and impulse responses estimated from a VAR process. In their opinion, “most of our economic intuition about a model concentrated on its implications for the dynamic effects” (Christiano, 2002). Their main aim was, therefore, to fit their model to the dynamic effects of shocks. One advantage of the estimation procedure is that the estimated standard deviations of the VAR can be employed as measurement of the distance.

In this thesis I use the approach by Christiano, Eichenbaum and Evans to estimate the parameters in Hansen`s RBC model. I will change their estimation criterion by making

different assumptions about the variance-covariance-matrix and by computing standard deviations of the estimates. Different assumptions about the variance covariance matrix can improve the fit of the model. The report of standard deviations enables the researcher to reflect the uncertainty in the choice of the parameter values and may be used afterwards in an evaluation process.

### 3 Bayesian Vector Autoregression

In the first part of this section, the VAR is formally defined and some useful definitions are introduced. In the second and third part, Bayesian inference and the computation of the impulse response functions are described.

#### 3.1 Definition of the VAR process

A VAR process of order  $k$  contains a  $m$ -variate time series  $t = 1 \dots T$  and can be written as:

$$Y_t = B_1 * Y_{t-1} + B_2 * Y_{t-2} + \dots + B_k * Y_{t-k} + C_0 + C_1 * t + \varepsilon_t$$

Where the  $Y$  vectors are of size  $m \times 1$ , the coefficient matrices  $B$  of size  $m \times m$ , the vectors containing the constant  $C_0$ , the time trend  $C_1$  and the error term  $\varepsilon$  of size  $m \times 1$ . The error terms are normally identically and independently distributed:

$$\varepsilon_t \sim N(0, \Sigma), t = 1 \dots T$$

It is convenient to write the system more compactly:

$$\begin{bmatrix} Y'_{k+1} \\ Y'_{k+2} \\ \vdots \\ Y'_T \end{bmatrix} = \begin{bmatrix} Y'_k & Y'_{k-1} & \dots & Y'_1 & 1 & 1 \\ Y'_{k+1} & Y'_k & \dots & Y'_2 & 1 & 2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ Y'_{T-1} & Y'_{T-k} & \dots & Y'_{T-k} & 1 & T \end{bmatrix} * B + \begin{bmatrix} \varepsilon'_1 \\ \varepsilon'_2 \\ \vdots \\ \varepsilon'_T \end{bmatrix} \text{ with dimensions:}$$

$(T \times m) \quad (T \times (m * k + 2)) \quad ((m * k + 2) \times m) \quad (T \times m)$  or simply:

$Y = X * B + \varepsilon$ . This is the standard notation for a reduced form VAR model.

In some situations it can be useful to write a VAR ( $k$ ) process as a VAR (1) process. This can be achieved by "stacking" the process together:

$$\begin{bmatrix} Y_t \\ Y_{t-1} \\ \vdots \\ Y_{t-k+1} \end{bmatrix} = \begin{bmatrix} B_1 & B_2 & \dots & B_k \\ I_m & 0 & \dots & 0 \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & I_m & 0 \end{bmatrix} * \begin{bmatrix} Y_{t-1} \\ Y_{t-2} \\ \vdots \\ Y_{t-k} \end{bmatrix} + \begin{bmatrix} C_0 & C_1 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 \\ t \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \\ \vdots \\ 0 \end{bmatrix} \text{ with dimensions}$$

$(m * k \times 1) \quad m * k \times m * k \quad (m * k \times 1) \quad (m * k \times 2) \quad m * k \times 1$  or:

$$Y_t = P * Y_{t-1} + C * c_t + \varepsilon_t.$$

This form of writing a VAR process will be called the companion form. In combination with the companion form the  $m \times m * k$  matrix  $J$  is often defined as:  $J = [I_m \quad 0 \quad \dots \quad 0]$ .

Another useful definition is to denote the inverse of the variance covariance matrix  $\Sigma$  as precision matrix  $H$ .

### 3.2 Bayesian Inference

In Bayesian Inference researchers are interested in making probability statements about the unknown parameters, which are here  $B$  and  $H$ . We form the probability density function (pdf) by combining sample information from the dataset with prior information via Bayes' Theorem. The derived pdf is called the posterior pdf of the parameter and is defined by:

$$\pi_T(B, H) \propto \pi(B, H) * L(B, H; Y_{-k+1}, \dots, Y_T) \quad \text{where } \pi(B, H) \text{ denotes the prior pdf and } L(B, H; Y_{-k+1}, \dots, Y_T) \text{ the likelihood function conditional on the observations.}$$

To derive a posterior pdf we have to choose a prior pdf first. Using Uhlig (Uhlig, 1994) it is convincing to choose a Normal-Wishart-distribution  $\phi_{NW}(B, H | \bar{B}, N, S, \nu)$ . The precision matrix  $H$  follows a Wishart distribution  $W_m(S^{-1} / \nu, \nu)$  with mean  $E[H] = S^{-1}$  and  $\nu$  degrees of freedom. The vectorized matrix  $B$  is normally distributed with  $\text{vec}(B) = N(\text{vec}(\bar{B}), H^{-1} \otimes N^{-1})$ . To complete the prior distribution we have to choose values for  $N, S, \bar{B}$  and  $\nu$ . There are two reasonable priors suggested by Uhlig (Uhlig, 1994), the flat prior and the Minnesota prior. The first one is a non-informative prior, that simply sets all prior values equal to zero ( $B_0, N_0, \nu_0, S_0 = 0$ ), while the latter one is informative on the coefficient matrix  $B$ . It sets the first coefficient matrix equal to the identity matrix, predicting a random walk, and the other equal to zero. The prior covariance matrix is a diagonal matrix with variances that decrease with higher lags to exclude them “softly”. The specification of the “Minnesota”-prior follows Uhlig (Uhlig (1997), Appendix C).

After the specification of the prior pdf, the parameters are estimated by ordinary least squares and the posterior pdf is calculated, which is also a Normal-Wishart-distribution and given by:

$$\phi_{NW}(B, H | B_T, N_T, S_T, \nu_T), \text{ where:}$$

$$\nu_T = T + \nu_0 \quad (T \text{ is the number of observations})$$

$$N_T = N_0 + X' * X$$

$$\bar{B}_T = N_T^{-1} * (N_0 * B_0 + X' * X * \hat{B})$$

$$S_T = \frac{\nu_0}{\nu_T} * S_0 + \frac{T}{\nu_T} * \hat{\Sigma} + \frac{1}{\nu_T} * (\hat{B} - B_0)' * N_0 * N_T^{-1} * X' * X * (\hat{B} - B_0).$$

The precision matrix  $H$  follows a Wishart distribution  $W_m(S_T^{-1}/v_T, v_T)$  with mean  $E[H] = S_T^{-1}$  and  $v_T$  degrees of freedom; the vectorized matrix  $B$  follows a Normal distribution  $vec(B) \sim N(vec(B_T), H^{-1} \otimes N_T^{-1})$ . With these results a sample of values for  $B$  and  $H$  can be generated directly from their probability distributions.

### 3.3 Impulse response function for the data

In this section, I describe how the program computes the impulse response function for the data. The main idea is to sample directly  $n$  draws of  $B$  and  $H$ , compute impulse responses for each draw and use the mean of these functions as an estimator for the impulse response function. The first task is done by the function “ziehen”, the second by the function “impulsedata”.

The function “ziehen” uses the following theoretical background: Consider a  $m * m$  Matrix  $X$  with  $X \sim N(0, I_m)$ . The matrix  $S = X * X' / v$  follows then a Wishart distribution with  $E[S] = I_m$  and  $v$  degrees of freedom. If we are interested in changing the VCV matrix of  $X$  from  $I_m$  to  $\Sigma$ , we have to multiply  $X$  first with the Cholesky factorisation of  $\Sigma$  and compute  $S$  afterwards. The so obtained matrix  $S$  follows then a Wishart distribution with  $E[S] = \Sigma$ .

We draw one VCV  $\Sigma$  and one coefficient matrix  $B$  each time. Because of  $\Sigma = H^{-1}$  we draw first a precision matrix with the desired properties and invert it afterwards. Given the posterior specification for  $S_T$  and  $v_T$  the function draws a matrix  $X$  of size  $S_T$  with standard normally distributed entries. It then uses the theoretical background from above to compute first  $H$  and  $\Sigma$  afterwards. To draw  $B$  the function draws a vector of size  $vec(B)$  with  $vec(B) \sim N(0,1)$ . It then computes the VCV matrix of  $B$  ( $H^{-1} \otimes N_T^{-1}$ ) and their Cholesky factorisation. In some cases, the VCV matrix is not positive definite. To prevent the program from a breakdown, it uses the expected value of  $B$  as a substitute. Since it happens only once in a thousand draws, it should not lead to a biased estimation. Employing the Cholesky factorisation of the VCV matrix the function transforms the vector into a normally distributed vector with  $vec(B) \sim N(vec(B_T), H^{-1} \otimes N_T^{-1})$  and reshapes it into the coefficient matrix  $B$  afterwards.

The uncertainty about the coefficients can lead to draws of  $B$ , which contain explosive roots. These coefficients should be excluded from the impulse response computation. It is therefore necessary to check every coefficient matrix for stability.

A VAR (1) process is stable, if  $p(z) = \det(I - B_1 * z) \neq 0$  for  $|z| \leq 1$ , i.e. the solution to its reverse characteristic polynomial does not lie in and on the complex unit circle. As pointed out before, can every VAR (k) process be written as a VAR (1) process using companion form. This process is then stable, if  $p(z) = \det(I - P * z) \neq 0$  for  $|z| \leq 1$ . The reverse characteristic polynomial can be written as:

$$p(z) = \det((-z) * (P - I * z^{-1})) \Leftrightarrow p(z) = (-z^m) * \det(P - I * z^{-1}) \Leftrightarrow p(z) = (-z^m) * q(z^{-1})$$

The solution to  $q(z^{-1}) = 0$  are the eigenvalues of  $P$ . If they are smaller than 1, the process will be stable.

If the matrix is stable the function proceeds with the next step, the computation of the impulse response functions. Impulse response functions describe the response of variable  $y_{i,t+s}$  to a shock in  $\varepsilon_t$ . Following Lütkepohl (1991) each period  $I_i$  can be computed as:  $I_i = J * P^i * J' * A * u$ , where  $J, P$  were defined above, matrix  $A$  is the Cholesky decomposition of  $\Sigma$  and  $u$  a vector containing the shock. The Cholesky decomposition creates uncorrelated error terms, which are necessary in order to calculate meaningful impulse response functions, but also involves the assumption of a recursive identification scheme. For the VAR process I use the assumption is justified. In other application this assumption does not necessarily hold.

After the calculation of all impulses, averages for each period and the variance covariance matrix are calculated and saved as ID and VCV.

## 4 A Dynamic Macro Model

As a benchmark model, I use Hansen's Real Business Cycle model. In the model a representative agent maximizes the following utility function:

$$\max E \left[ \sum_{t=0}^{\infty} \beta^t (\log c_t - A * n_t) \right] \text{ subject to the budget constraints:}$$

$$c_t + k_t = \gamma * e^{z_t} * k_{t-1}^{\theta} * n_t^{1-\theta} + (1 - \delta) * k_{t-1} \text{ and } z_t = \rho * z_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \sim N(0, \sigma^2) \text{ iid.}$$

Here  $\beta$  denotes the discount factor,  $c_t$  consumption,  $n_t$  labour,  $k_t$  capital and  $\gamma_t = \gamma * e^{z_t}$  total factor productivity. The productivity shock  $z_t$  follows an AR(1) process. It is common to define further equations for output  $y_t$  and return on capital investments  $R_t$ :

$$y_t = \gamma * e^{z_t} * k_{t-1}^{\theta} * n_t^{1-\theta}; R_t = \theta \frac{y_t}{k_{t-1}} + 1 - \delta.$$

Because the purpose of the thesis is to estimate the parameters, they should be described in more detail.

$\theta$  defines the share of capital in the production function. In case of a Cobb-Douglas production function it is also the production elasticity, a one percentage increase in capital will increase the production by  $\theta$  percent. If the model would be calibrated,  $\theta$  is often set around 0,3.

$\delta$  is the depreciation rate of capital and should lie inside an interval between  $[0,003;0,1]$ .

$\rho$  defines the persistence of the shock, the higher the persistence of the shock the longer last the impulse. For  $\rho < 1$  the model will always return to its steady state, if  $\rho > 1$  the model can enter a new long run equilibrium or produce an unstable, explosive solution. Therefore  $\rho$  should lie between  $[0,01;1]$ .

How much an agent discounts the utility he will receive in further periods is measured by the discount factor  $\beta$ .  $\beta$  is often defined between  $[0,9;0,99]$ . A low value of  $\beta$  means a high discount of further periods and vice versa.

The parameter  $\gamma$  defines the increase in total factor productivity after a shock in technology,  $A$  represents the discomfort of work and  $\bar{n}$  the steady state level of employment. The steady state solution to the model has an equation left for one endogenous variable. In this case either  $\bar{n}$  is set and the steady state is solved for  $A$  or  $A$  is set and the equation is solved for  $\bar{n}$ . I decided to solve the steady state for  $A$  and set  $\bar{n}$ .  $\gamma$  and  $\bar{n}$  are then normalised and do not have to lie inside certain intervals.

For the RBC model, a solution strategy already exist in MATLAB, but I had to solve the model "by hand" in order to implement it into the program. The solution strategy is (see Uhlig, 1995):

1. Find the First-order-conditions
2. Calculate the steady state
3. Loglinearize around the steady state
4. Solve for the recursive law of motion
5. Calculate impulse responses

#### **4.1 Find the First-order-conditions**

I form the Lagrangian and differentiate afterwards with respect to  $c_t, n_t, k_t$ :

$$L = \max E \left[ \sum_{t=0}^{\infty} \beta^t \left( (\log c_t - A * n_t) - \lambda_t (c_t + k_t - \gamma * e^{\bar{z}_t} * k_{t-1}^{\theta} * n_t^{1-\theta}) - (1 - \delta) * k_{t-1} \right) \right]$$

$$\frac{\delta L}{\delta c_t} : \frac{1}{c_t} = \lambda_t$$

$$\frac{\delta L}{\delta n_t} : A = -\lambda_t * (1 - \theta) * \frac{y_t}{n_t}$$

$$\frac{\delta L}{\delta k_t} : \lambda_t = \beta * E_t [\lambda_{t+1} * R_{t+1}]$$

$$\frac{\delta L}{\delta \lambda_t} : c_t + k_t = y_t + (1 - \delta) * k_{t-1}$$

## 4.2 Calculate the steady state

Here I calculate explicit solutions for the steady state for the variables. For eight variables we have four first order conditions and the four constraints from above. In the RBC literature it is often a value for  $\bar{n}$  assumed. To prevent the model from over identification, it is then solved for the parameter  $A$  instead of  $\bar{n}$  (see discussion above). The solution for the model is:

$$\begin{aligned}
 1. \quad \bar{R} &= \frac{1}{\beta} & 2. \quad \frac{\bar{y}}{\bar{k}} \kappa &= \frac{1}{\theta} \left( \frac{1}{\beta} - 1 + \delta \right) & 3. \quad \bar{y} &= (1 - \bar{u})^* \gamma^* \kappa^{-\theta} \\
 4. \quad \bar{k} &= \frac{\bar{y}}{\kappa} & 5. \quad \bar{c} &= \bar{k}^* (\kappa - \delta) & 6. \quad \bar{\lambda} &= \frac{1}{c} & 7. \quad \bar{z} &= 0 \\
 8. \quad A &= \bar{\lambda}^* (1 - \theta)^* \frac{\bar{y}}{\bar{n}}
 \end{aligned}$$

## 4.3 Loglinearize around the steady state

In this section I replace the non-linear equations by dynamic linear equations. The equations are expressed in percentage deviations from the steady state. The loglinearized equations are:

$$\begin{aligned}
 \text{I.} \quad 0 &= \hat{\lambda}_t + \hat{c}_t \\
 \text{II.} \quad 0 &= \hat{\lambda}_t + \hat{y}_t - \hat{n}_t \\
 \text{III.} \quad 0 &= -\hat{\lambda}_t + E_t [\hat{\lambda}_{t+1} + \hat{R}_{t+1}] \\
 \text{IV.} \quad 0 &= -\bar{c}^* \hat{c}_t + \bar{y}^* \hat{y}_t - \bar{k}^* \hat{k}_t + (1 - \delta)^* \bar{k}^* \hat{k}_{t-1} \\
 \text{V.} \quad 0 &= -\bar{R}^* \hat{R}_t + \theta^* \kappa^* \hat{y}_t - \theta^* \kappa^* \hat{k}_{t-1} \\
 \text{VI.} \quad 0 &= -\hat{y}_t + z_t + \theta^* \hat{k}_{t-1} + (1 - \theta)^* \hat{n}_t \\
 \text{VII.} \quad z_t &= \rho^* z_{t-1} + \varepsilon_t
 \end{aligned}$$

## 4.4 Solve for the recursive law of motion

For the RBC model, it is possible to formulate the dynamics of the model in dependence on the two state variables and  $\lambda$  only. After I formulated and solved recursive laws of motion (RLOM) for them, I can solve for the endogenous variables. RLOM can be postulated to be linear in log-deviations:

$$\begin{bmatrix} z_t \\ k_t \\ \lambda_t \end{bmatrix} = \begin{bmatrix} \rho & 0 & 0 \\ \eta_{kz} & \eta_{kk} & 0 \\ \eta_{\lambda z} & \eta_{\lambda k} & 0 \end{bmatrix} * \begin{bmatrix} z_{t-1} \\ k_{t-1} \\ \lambda_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} c_t \\ y_t \\ R_t \\ n_t \end{bmatrix} = \begin{bmatrix} 0 & 0 & \eta_{c\lambda} \\ \eta_{yz} & 0 & \eta_{y\lambda} \\ \eta_{Rz} & 0 & \eta_{R\lambda} \\ \eta_{nz} & 0 & \eta_{n\lambda} \end{bmatrix} * \begin{bmatrix} z_t \\ k_t \\ \lambda_t \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \eta_{yk} & 0 \\ 0 & 0 & 0 \\ 0 & \eta_{nk} & 0 \end{bmatrix} * \begin{bmatrix} z_{t-1} \\ k_{t-1} \\ \lambda_{t-1} \end{bmatrix}$$

I can solve for the system using the method of undetermined coefficients. Writing the equation III. And IV. In dependence on  $\hat{k}$ ,  $\hat{\lambda}$  and  $z$  yields:

$$0 = -\hat{k}_t + (\kappa + (1 - \delta)) * \hat{k}_{t-1} + \left( \frac{\bar{c}}{\bar{k}} + \kappa * \frac{(1 - \theta)}{\theta} \right) * \hat{\lambda}_t + \frac{\bar{y}}{\theta * \bar{k}} * z_t$$

$$\Leftrightarrow 0 = -\hat{k}_t + \alpha_1 * \hat{k}_{t-1} + \alpha_2 * \hat{\lambda}_t + \alpha_3 * z_t$$

$$0 = -\hat{\lambda}_t + E_t \left[ \left( \frac{\kappa * (1 - \theta)}{\bar{R}} + 1 \right) * \hat{\lambda}_{t+1} + \frac{\kappa}{\bar{R}} * z_{t+1} \right]$$

$$\Leftrightarrow 0 = -\hat{\lambda}_t + E_t [\alpha_5 * \lambda_{t+1} + \alpha_6 * z_{t+1}]$$

In these equations I plug the earlier formulated RLOM in and receive:

$$0 = (-\eta_{\lambda k} + \alpha_4 * \eta_{kk} + \alpha_5 * \eta_{\lambda k} * \eta_{kk}) * \hat{k}_{t-1} + (-\eta_{\lambda z} + \alpha_4 * \eta_{kz} + \alpha_5 * \eta_{\lambda k} * \eta_{kz} + (\alpha_5 * \eta_{\lambda z} + \alpha_6) * \rho) * z_t$$

$$0 = (-\eta_{\lambda k} + \alpha_4 * \eta_{kk} + \alpha_5 * \eta_{\lambda k} * \eta_{kk}) * \hat{k}_{t-1} + (-\eta_{\lambda z} + \alpha_4 * \eta_{kz} + \alpha_5 * \eta_{\lambda k} * \eta_{kz} + (\alpha_5 * \eta_{\lambda z} + \alpha_6) * \rho) * z_t$$

By comparing coefficients I can derive solutions for  $\eta_{kk}$ ,  $\eta_{kz}$ ,  $\eta_{\lambda z}$  and  $\eta_{\lambda k}$  and afterwards use the equations, which are left, to solve for the dynamics of the endogenous variables.

#### 4.5 Calculate impulse responses

The dynamics of the system are sufficiently described by the matrix systems above. Impulse response functions can now be calculated by setting  $\hat{k}_0, z_0 = 0$ , the shock  $\varepsilon_1 = 1$  and solving the system recursively.

## 5 The Estimation Method

Let  $\theta$  be the vector of model parameters and  $\psi(\theta)$  the model impulse response function depending on the parameter vector. The estimated impulse response function will be noted as  $\hat{\psi}$  and their variance covariance matrix as  $\Sigma$ . In this section, I will analyse three different estimation criteria conditional on the form of the VCV matrix. The first criterion will only use the diagonal of  $\Sigma$ , the second the complete matrix without weights and the third will put weights on the different covariances. For simplicity I will first describe each criterion with impulse responses computed for only one variable and one shock. The criterion will be extended to several variables later.

### 5.1 Diagonal Variance-covariance matrix

This is the estimation criterion as Christiano and Eichenbaum originally applied it. They defined their estimation criterion as:  $J = \min_{\theta} (\hat{\psi} - \psi(\theta))' * V^{-1} * (\hat{\psi} - \psi(\theta))$  where  $V$  is the sampling uncertainty in  $\hat{\psi}$  and not equal to  $\Sigma$ . They constructed  $V$  the following way (Christiano, Eichenbaum, Evans, 2001): Assume  $g(\hat{\theta}, \hat{\psi}) = 0$  as the first order condition associated with the solution to the minimization problem above, where  $\hat{\theta}, \hat{\psi}$  are the estimates of the true values  $\theta^0, \psi^0$ . This function defines implicitly a mapping from  $\psi^0$  to  $\theta^0$ . They then used a Taylor series expansion to derive  $g_1(\hat{\theta} - \theta^0) + g_2(\hat{\psi} - \psi^0) = 0 \Leftrightarrow D = g_1^{-1} * g_2$  where  $g_i$  are the partial derivatives with respect to  $\theta, \psi$  and derived:  $\sqrt{T}(\hat{\theta} - \theta^0) \sim N(0, DWD')$  with  $W$  as the asymptotic variance covariance matrix of  $\sqrt{T}(\hat{\psi} - \psi^0)$ .  $V$  is then:  $V = (diag(DWD'))^{0.5} / \sqrt{T}$ .  $\theta^0, \psi^0$  were evaluated at the point estimates.

In contrast to Christiano, Eichenbaum and Evans I employ the estimated VCV matrix from the VAR impulse response function  $V = diag(\Sigma)$ .

## 5.2 Unweighted Variance-covariance matrix

The second estimation criterion take the covariances into account. Since I include more information the estimation of the parameter should be improved. Incorporated covariances have an additional effect: it enables the estimation criterion to take correlations between different periods into consideration. If for example one estimated period deviates from the trend and is highly correlated with the following periods, these also necessarily deviate. The first estimation criterion will be surprised by each deviation; the second one will not and therefore lead to a different estimation for the model's impulse response function.

The estimation criterion is defined by:

$$J = \min_{\theta} (\hat{\psi} - \psi(\theta))' * \Sigma^{-1} * (\hat{\psi} - \psi(\theta)).$$

## 5.3 Weighted Variance-covariance matrix

This criterion is due to the idea that it is often more important to fit the first periods of the model and the VAR impulses rather than the latter ones. This might be, because scientists mostly try to predict the nearest periods ahead or because most model impulses die out at the end. Adding weights to the covariances can reflect this priority. These weights are chosen intuitively rather than with deduced mathematical reasoning. The results for the third criterion will therefore be only reported and not interpreted. The main focus will be on the first and second estimation criterion.

I define the weighted variance covariance matrix the following way:

$$V = \begin{bmatrix} \sigma_1^2 & \frac{\sigma_{12}}{2} & \dots & \frac{\sigma_{1,im-1}}{im-1} & \frac{\sigma_{1,im}}{im} \\ \frac{\sigma_{12}}{2} & \sigma_2^2 & & & \frac{\sigma_{2,im}}{im} \\ \vdots & & \ddots & & \vdots \\ & & & \sigma_{m-1}^2 & \frac{\sigma_{im-1,im}}{im} \\ \frac{\sigma_{1,im}}{im} & \dots & & & \sigma_{im}^2 \end{bmatrix}$$

where  $im$  are the number of periods impulse

responses are computed.

While the variances enter the matrix unweighted, the covariances are divided by the higher period of each covariance. Covariances of higher periods are systematically excluded. The estimation criterion is then defined by:

$$J = \min_{\theta} (\hat{\psi} - \psi(\theta))' * V^{-1} * (\hat{\psi} - \psi(\theta)).$$

#### 5.4 Extension to several variables

In the case of more variables the matrix  $\hat{\psi}$ , which contains the estimated impulse response functions for the VAR process of  $i$  variables with the horizon  $im$ , is defined as:  $\hat{\psi} = [\mu_{1,1} \dots \mu_{1,im} \quad \mu_{2,1} \dots \mu_{2,im} \dots \mu_{i,1} \dots \mu_{i,im}]$  with the dimensions  $(1 \times i * im)$ . The model's impulse responses are defined equivalent. The variance covariance matrix  $\Sigma$  is a diagonal matrix containing the variance covariance matrix for each variable on the diagonal:

$$\Sigma = \begin{bmatrix} \Sigma_1 & 0 & 0 & 0 \\ 0 & \Sigma_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \Sigma_i \end{bmatrix} \text{ and has the dimensions } (i * im \times i * im).$$

#### 5.5 Standard deviation for the parameters

The standard deviations for the parameter vector are deduced from non-linear estimation theory. Every draw from the VAR is collected in a vector  $y$ :

$$y_j = \begin{bmatrix} y_{1,1,1} \\ \vdots \\ y_{1,1,im} \\ y_{2,1,1} \\ \vdots \\ y_{i,k,l} \end{bmatrix}$$

$i * z * l \times 1$

where the subscript  $i$  denote the variable,  $z$  the shock,  $l$  the impulse period and  $j$  the draw. In the case of one variable of interest, one shock and a horizon of 20 periods,  $y_j$  is  $20 \times 1$  vector.

Given a parameter vector  $\theta$ , the model's impulse response function is:  $\psi(\theta)$ , where  $\psi(\theta)$  is a  $(i * z * l \times 1)$  vector. The relationship between  $y_j$  and  $\psi(\theta)$  can be described by:

$y_j = \psi(\theta) + e_j$  or  $y = \Psi(\theta) + e$  where all the  $j$  draws are stacked together, and the model's impulse responses are  $j$  times replicated.

In order to derive a linear model a first order Taylor series expansion is developed at  $\theta^*$ :  $\psi(\theta) \cong \psi(\theta^*) + Z(\theta^*) * (\theta - \theta^*)$ , where  $Z(\theta^*) = \frac{\partial \psi}{\partial \theta} \Big|_{\theta}$  is a  $(i * z * l \times k)$  matrix, which contains the first order derivatives of  $\psi$  w.r.t. each  $k^{\text{th}}$  parameter.

Substituting the Taylor series expansion into the non-linear equation yields the linear pseudomodel:

$$\bar{y}(\theta^*) = Z(\theta^*) * \theta + e, \text{ where } \bar{y}(\theta^*) = y_j - \psi(\theta^*) + Z(\theta^*) * \theta^*.$$

The variance covariance matrix for the parameter vector  $\hat{\theta}$  is then defined as:  $\hat{\Sigma}_{\theta} = \hat{\sigma}^2 * [Z(\hat{\theta}) * Z(\hat{\theta})]$ , where  $\hat{\sigma}^2 = e' * e / (T - K)$  and  $T = i * z * l$  the number of observations and  $K = k$  the number of parameters.

The first derivatives are computed as:

$$\frac{\delta \psi}{\delta \theta_k} = \frac{\psi(\hat{\theta}_k + 0,001) - \psi(\hat{\theta})}{0,001}$$

## 6 The MATLAB Program

In this section I will give a brief introduction to the program, a kind of manual, an overview of the building blocks of the program and point at possible problems that can arise.

### 6.1 A Manual

Before the program can be executed, the user has to make some preparations. First of all he has to order the time series in a data matrix:

$$\begin{bmatrix} y_{1,1} & y_{2,1} & \cdots & y_{m,1} \\ y_{1,2} & y_{2,2} & \cdots & y_{m,2} \\ \vdots & \vdots & & \vdots \\ y_{1,T} & y_{2,T} & & y_{m,T} \end{bmatrix}$$

The matrix has to be saved as  $D$ .

Furthermore, an initial guess as a starting point for the parameters of the model has to be inserted and saved as  $b$ . If the user wants to restrict the interval for the parameter, the upper and the lower bound has to be entered and saved as  $bu$  and  $bl$  respectively.

After these preparations, the program can be executed by entering "program" in the MATLAB prompt.

In the following part I will explain the questions the user will be asked. The first question is about the characteristics of the VAR process. The number of lags can be determined. This gives the user the opportunity to vary the number of lags for different experiments. Another choice offered concerns the prior distribution. The user can choose between the flat prior version and the Minnesota prior version. Another prior could be added later.

Afterwards he will be asked about the Monte Carlo procedure to compute the impulse response function of the data. The number of draws from the posterior distribution and the number of periods can be specified. The program computes impulse response functions for every variable in the VAR. In case the user wants to use only some of the functions for the estimation procedure, he is asked about the number of impulse response functions and their

position within the VAR. If the user, for example, only wants to use the second variable impulse response function, he has to enter 1 and 2.

Subsequent to the choices above, the user can decide on the estimation criterion he wants to employ and whether the intervals for possible solutions should be restricted or not. It is possible to try different settings, i.e. to repeat the estimation of the parameters with a different estimation criterion or with/without restrictions on the intervals.

## ***6.2 The building blocks of the program***

The program consists of different functions as building blocks. I want to shortly describe their characteristics in order to give an orientation for the MATLAB code. I find that useful because all functions can be used independent of each other. A more detailed description can be found in the MATLAB code in the appendix.

Function “*order*”

The function “*order*” constructs the VAR matrices  $Y$  and  $X$  as described above from the matrix  $D$ , which contains the time series for the variables.

Function “*prior*”

The function “*prior*” specifies the prior distribution for the coefficient matrix  $B$  and the VCV-matrix  $\Sigma$ .

Function “*posterior*”

The function “*posterior*” calculates the posterior distribution for the coefficient matrix  $B$  and the VCV-matrix  $\Sigma$ .

Function “*impulldata*”

The function “*impulldata*” contains a routine to compute the impulse response function for the data and the standard deviation for each period.

Function “*kriterium*”

The function “*kriterium*” contains the equation that should be minimized.

Function “*estimat*”

The function “*estimat*” includes the choice of the different estimation criteria and the choice between a restricted and unrestricted intervals for the parameter. It computes the VCV matrix, which is used in the estimation procedure, and executes the minimization routine.

Function “*standardb*”

The function “*standardb*” incorporate the routine to calculate the standard deviation of the parameter.

Function “*ziehen*”

The function “*ziehen*” is part of the function “*impulsedata*”. It embodies an algorithm to draw from a Wishart and a Normal probability distribution given the posterior distribution of  $B$  and  $\Sigma$ .

Function “*impulseth*”

The function “*impulseth*” contains the solution of Hansen’s RBC model in dependence of the parameter. This function has to be rewritten for another model.

### **6.3 Problems**

There are some problems that can arise when executing the program. I want to describe them and give an overview about possible solutions.

One problem that emerges easily is related to the savings routine in MATLAB. The command “*save y*” should normally save only the variable  $y$ . In the version I used, the whole workspace was saved. Because some vectors are loaded every time the minimization routine is executed, this problem can lead to a long calculation time especially with a high number of draws. Furthermore, it can happen that some variables are overwritten.

I experienced another problem with the function “*standardb*”. An inference on the parameters can generate very inappropriate results. It can be possible that the function fails to compute standard deviations for these values. This kind of problem did not occur in relation with reasonable estimates.

A limitation of the program can lead to another problem. The program can be applied for one shock. The extension of the program to several shocks would be beneficial, but is left out due to time constraints.

## 7 Results

In this section the results are presented. I will also give a description of the dataset. As the different experiments show, the results are not only dependent on different estimation criteria but also on the prior specification and the number of draws from the posterior distribution. Since some parameters only have an economic meaning, it can be necessary to put restrictions on the estimated parameters when they are outside a certain interval. This will influence the inference as well.

To show these effects, the section is organized in the following way: The first case is an unconstrained estimation procedure employing the flat prior and the Minnesota prior specification for 1000 draws from the posterior distribution. Afterwards I will restrict the interval for each parameter. While I accomplished the experiments above, I determined a high uncertainty in the Monte Carlo simulation. I, therefore, repeated the experiments 1000 times and computed mean and standard deviation. The results showed that the first case is not a representative one. In order to reduce the uncertainty in the Monte Carlo procedure 100,000 draws from the posterior distribution are generated. This is the second case. The parameter vector is estimated employing the flat prior and the Minnesota prior specification. Again this is an unconstrained and a constrained procedure considered.

I will judge the criteria by their ability to generate reasonable estimates with a low standard deviation and to produce a graph inside the band of mean plus/minus one time the standard deviation of the VAR impulse response function. The obtained estimates can give an idea how the procedure works and the relationship between the model's impulse response function and the VAR impulse response function. I will point out this as well.

### ***7.1 The Dataset***

The dataset consists of two time series for multi factor productivity and average weekly working hours in the U.S.A. from 1949-01-01 to 2000-01-01. The multi factor productivity index is an annual index (with 1996=100) for the manufacturing sector and was provided by the Bureau of Labour Statistics, a U.S. governmental department. I downloaded the time series for average weekly working hours from freelunch.com. As a source, the Bureau of

labour statistics is named. It is a quarterly index for the non-farm business sector (1992=100). I only used the annual observations that fit to the productivity index.

## 7.2 *The First Case*

### 7.2.1 Unrestricted Estimation

The parameters were estimated for each criterion employing a flat prior specification and for  $n=1000$  draws from the posterior distribution. Standard deviations are reported in brackets (where available, for a detailed explanation see section 6.3)

**Table 7.1 Results for Flat Prior and  $n=1000$**

Parameter/Criterion	1	2	3
$\theta$	14.6695 (0.6582)	15.2765 n.a.	16.0868 n.a.
$\delta$	-1.0727 (0.2198)	-0.3707 n.a.	-0.1966 n.a.
$\rho$	0.6788 (0.0293)	-0.0059 n.a.	0.8766 n.a.
$\beta$	0.4618 (0.0648)	6.8959 n.a.	44.1746 n.a.
$\gamma$	388.8450	-2.2016	-7.3148
$\bar{n}$	16.7319	-1.4940	-8.3340
Figure	10.1/10.2	10.3/10.4	10.5/10.6

All parameter estimates lie outside a reasonable interval. But they illustrate the procedure and show important differences between the estimation criteria. I, therefore, want to interpret some values.

The response of the VAR is much lower than the response predicted by the calibrated model. In order to fit the model responses, the procedure estimates parameters, which decrease the response of the model. It results in a very high share of capital, negative depreciation, low persistence of the shock and a very low discount rate.

The higher the share of capital in the production function, the lower is the share of labour. A technology shock leads to a lower impulse of labour the lower the share is in the production.

The lower the discount factor is, the higher are future periods discounted and present utility becomes more important to the agent. Since working lowers the utility, the agent is not willing to work much more after a shock in technology. This lowers the response to the shock. The low impulse of the VAR is also approaching the steady state fast. This is replicated by a low persistence of the shock in the model. While these results are possible to interpret, the very odd results for  $\delta$  (criterion 1+2),  $\beta$  (criterion 2) and  $\rho$  (criterion 2) hardly give any insight.

When comparing the figures 10.1 and 10.3, the differences between the estimation criteria become apparent. The first estimation criterion takes every period separately into consideration. This causes the model’s impulse response function to lie within the impulse response function of the VAR. The second estimation criterion considers the correlations between different periods. It looks much smoother than the first one. Both lie within the band of the impulse response function plus/minus the standard deviation.

The results change when the Minnesota prior is applied. The impulse from the VAR is much stronger than it was before. This prevents the first estimation criterion from a number of odd results:

**Table 7.2 Results for Minnesota Prior and  $n=1000$**

<b>Parameter/Criterion</b>	<b>1</b>	<b>2</b>	<b>3</b>
$\theta$	0.8121 (0.0240)	6.1893 n.a.	1.1836 (0.0342)
$\delta$	-0.0046 (0.2039)	-0.0018 n.a.	-0.0037 (0.1152)
$\rho$	0.9964 (0.0389)	0.1250 n.a.	0.9940 (0.0481)
$\beta$	0.9985 (0.0193)	7.5927 n.a.	1.0016 (0.0177)
$\gamma$	0.3173	-23.8701	-2.2383
$\bar{n}$	0.5313	-5.0499	-0.3857
Figure	10.7/10.8	10.9/10.10	10.10/10.11

The figure for the first estimation criterion shows a strong response followed by a sharp downturn for the first two periods. Afterwards the curve approaches the steady state.

The estimates for  $\beta$  and  $\rho$  employing the first criterion are appropriate estimates. The long persistence of the VAR impulse response function induces a value for  $\rho$  near unity.  $\theta$  is still too high to be reasonable, but it does lie between  $[0,1]$  meaning that the labour share in production is not a negative one. The high value of  $\theta$  is probably the reason that the response of labour is falling so fast (in two periods from 1.2 to 0.2).

The second criterion produces estimates similar to the first case when the flat prior was employed, although they are not as high, because of a stronger response in the first period. Both graphs lie within the band of mean plus/minus one time the standard deviation.

From these results I can conclude that the procedure leads to dynamics of the model which are similar to the dynamics in the data. Despite of the exception of the first criterion combined with the Minnesota prior all criteria generate inappropriate parameter values. In order to receive reasonable estimates, I will restrict the intervals for possible solutions.

**7.2.2 Restricted Estimation**

To choose the right interval is clearly a difficult task. I tried to find appropriate intervals in such a way that would allow the upper and lower bounds to remain at reasonable values while not constraining the estimation too much.

**Table 7.3 Parameter Restrictions**

Parameter	Lower bound	Upper bound
$\theta$	0.25	0.44
$\delta$	0.003	0.1
$\rho$	0.5	0.999
$\beta$	0.9	0.99
$\gamma$	0.05	10
$\bar{n}$	0.05	10

The program is then rewritten, to minimize the different criteria for all values inside the intervals.

I start again by reporting the results for the flat prior version:

**Table 7.4 Results for Flat Prior and  $n=1000$**

Parameter/Criterion	1	2	3
$\theta$	0.44 0.039	0.44 0.039	0.44 0.039
$\delta$	0.1 0.0421	0.1 0.0396	0.1 0.0421
$\rho$	0.5 0.0455	0.5184 0.0429	0.5 0.0455
$\beta$	0.9 0.0389	0.9 0.0372	0.9 0.0389
$\gamma$	1	1	1
$\bar{n}$	0.33	0.33	0.33
Figure	10.25/10.26	10.27/10.28	See Criterion 1

All criteria generate similar results. The restrictions lead to appropriate estimates but also to an impulse response function of the model, which differs from the VAR response for the first four periods. All generated functions are above the mean of the VAR impulse response function plus one standard deviation.

$\beta$ ,  $\delta$  and  $\theta$  are at their bounds. As the unrestricted experiment showed will the estimation procedure induce high values for  $\theta$  and low values for  $\beta$ . It also produces the highest value for  $\delta$ . This is surprising because  $\delta$  has been estimated negative in the unrestricted experiment. A high depreciation rate causes a lower rate of return on capital and a lower steady state level for capital. A lower rate of return lead to less investment after a technology shock, and therefore to a lower increase in the marginal rate of productivity in labour. This lowers the response of labour to a shock in technology. The high value for  $\delta$  can be a substitute for the higher values of  $\theta$ , which are not possible anymore.

The standard deviations of the parameters are indicating a low rate of uncertainty. The standard deviations from the second criterion are slightly smaller than the standard deviations of the first one.

Employing the Minnesota prior version lead to the following results:

**Table 7.5 Results for Minnesota Prior and  $n=1000$**

Parameter/Criterion	1	2	3
$\theta$	0.44 0.0052	0.44 0.0052	0.44 0.0052
$\delta$	0.1 0.0386	0.1 0.0386	0.1 0.0386
$\rho$	0.999 0.0070	0.999 0.0070	0.999 0.0070
$\beta$	0.9 0.0565	0.9 0.0565	0.9 0.0565
$\gamma$	1	1	1
$\bar{n}$	0.33	0.33	0.33
Figure	10.33/10.34		

All criteria generate equivalent results. Except for  $\rho$ , the estimated values are also equivalent to the flat prior case. But the standard deviations of the parameters are smaller than the standard deviations before. This indicates that the prior choice matters for the estimation procedure.

The explanation for the values of the parameters was given above. Even though the graph looks similar to the graph in the figures 10.25 and 10.27, it does lies closer to the band of the VAR. But this is only because the standard deviation of the data is higher in the special case for the Minnesota prior than for the special case with the flat prior. This will not be the case in general.

### 7.3 The Second Case

Employing Bayesian inference on the VAR process, impulse response functions can vary over single computations. This generates different estimates for the parameters. To measure the uncertainty, I repeated the process n-times and compute first and second moments for each parameter estimate. The results showed a high uncertainty in the Monte Carlo procedure. In order to reduce it I set the number of draws equal to  $n=100,000$  and repeated the experiments from above.

Applying the flat prior lead to the following results:

**Table 7.6 Results for Flat Prior,  $n=100.000$  – Unrestricted Estimation**

Parameter/Criterion	1	2	3
$\theta$	0.5409 0.2751	0.2303 n.a.	0.9924 0.9182
$\delta$	-0.3408 0.8994	0.0147 n.a.	0.0451 55.7008
$\rho$	0.9784 0.1644	0.7678 n.a.	0.9793 0.9245
$\beta$	0.7351 0.4586	1.3186 n.a.	1.0006 0.6187
$\gamma$	60.6132	1.3031	0.2320
$\bar{n}$	-10.4431	0.4060	-0.4097
Figure	10.13/10.14	10.15/10.16	10.17/10.18

The first point to notice is the high standard deviation of the VAR impulse response function. This is probably due to the poor dataset. The estimated values differ from the first case, but the trend in the estimation criteria is the same (high  $\theta$ , low  $\beta$ , negative  $\delta$  for the first criterion). They are much more reasonable, but still not satisfying. The second criterion results in a low share of capital and a positive, reasonable  $\delta$ .

The difference in the parameters is caused by differences in the impulse response function. Corresponding to the first case is the flat and persistent response function estimated by the first criterion, while the response function estimated by the second criterion is a very strong immediate response, which only last for two periods. Both function are inside the band of the VAR, but this is not a surprising result given the very high standard deviation in the data. The standard deviations of the parameters are also higher than the standard deviation in the former cases.

The results for the restricted estimation procedure are:

**Table 7.7 Results for Flat Prior,  $n=100.000$  – Restricted Estimation**

<b>Parameter/Criterion</b>	<b>1</b>	<b>2</b>	<b>3</b>
$\theta$	0.44 0.1823	0.44 0.1823	0.44 0.1823
$\delta$	0.1 1.9912	0.1 1.8684	0.1 1.9912
$\rho$	0.5 2.1512	0.5184 2.0244	0.5 2.1512
$\beta$	0.9 1.8386	0.9 1.7584	0.9 1.8386
$\gamma$	1	1	1
$\bar{n}$	0.33	0.33	0.33
Figure	10.29/10.30	10.31/10.32	See Criterion 1

The results are the same as for the case with a lower  $n$ . The only difference is the standard deviations of the parameters, which are bigger than before. It is now possible to draw a conclusion about the estimation procedure applying a flat prior version. Firstly, the high uncertainty in the data leads to a big standard deviation of the mean of the VAR impulse response function. This prevents a valuation of the model's impulse response function relative to the standard deviation of the data. A sensitivity analysis may be necessary towards the number of draws in the Monte Carlo procedure. Secondly, the high standard deviation of the parameters makes the estimates useless in practice. Thirdly, if the flat prior is applied, an unrestricted estimation may generate inappropriate results.

The estimation of the parameters employing the Minnesota prior produced the following results:

**Table 7.8 Results for Minnesota Prior,  $n=100,000$  – Unrestricted Estimation**

Parameter/Criterion	1	2	3
$\theta$	3.7560 n.a.	4.5982 n.a.	2.9902 n.a.
$\delta$	-0.0317 n.a.	-0.0019 n.a.	-0.0132 n.a.
$\rho$	-0.2002 n.a.	0.0238 n.a.	-0.1766 n.a.
$\beta$	2.1315 n.a.	1.0402 n.a.	1.1232 n.a.
$\gamma$	-5.2042	-4.7380	1.8535
$\bar{n}$	-0.0453	-0.0597	-0.3985
Figure	10.19/10.20	10.21/10.22	10.23/10.24

All parameter values are inappropriate. The model's impulse response function is fitted very well to the impulse response function of the data relative to the standard deviation of data. The differences between the first and the second criteria are the same as described before.

The restriction of the intervals for the parameters lead to the following results:

**Table 7.9 Results for Minnesota Prior,  $n=100,000$  – Restricted Estimation**

Parameter/Criterion	1	2	3
$\theta$	0.44 0.0013	0.44 0.0013	0.44 0.0013
$\delta$	0.1 0.0095	0.1 0.0095	0.1 0.0095
$\rho$	0.999 0.0017	0.999 0.0017	0.999 0.0017
$\beta$	0.90 0.0139	0.90 0.0139	0.90 0.0139
$\gamma$	1	1	1
$\bar{n}$	0.33	0.33	0.33
Figure	10.35/10.36		

These results are satisfying. On the one hand, most of the parameters are either at their lower or upper bound, but on the other hand, the standard deviations of the parameters are small and the impulse response function of the model are close to the band of the VAR. These are workable estimates for the parameter, which could be used in Hansen's RBC model.

While the Minnesota prior outperforms the flat prior, it is hard to discriminate between the different estimation criteria. It is advisable to apply both criteria, the one with a diagonal VCV matrix and the one with included covariances. The third criterion can be left out, because of the lack of theoretical foundation.

## 8 Discussion

In this section I will discuss the results presented in the last section and possible extensions of the estimation procedure.

The results showed two essential weak points of the estimation procedure. Firstly, inference on parameters that embody an economic meaning can lead to unreasonable values for the parameters. This can be solved in two ways. One-way was suggested before, namely, to restrict the interval for possible solution. The weaknesses of this approach will be discussed later. Another way was provided by Christiano, Eichenbaum and Evans (2001) in their paper “Nominal rigidities and the dynamic effects of a shock to monetary policy”. They divided the parameters in two categories: parameters that do not enter the formal estimation criterion and parameters that do enter it. The first group of parameters was calibrated to avoid unreasonable results. The parameters, which belonged to the second group, were, for example, parameters entering Calvo price setting of wages and prices (fraction of households and firms that cannot renegotiate prices and wages), the steady state mark-up, the habit parameter and the parameter of investment adjustment costs. Scientists have a higher degree of freedom when choosing these parameters. The exclusion of some parameters is therefore a possible solution to the problem.

The second weakness was when the intervals for possible solutions were restricted; the outcome was likely to be at the upper or the lower bound. This makes the determination of the bounds look equivalent to the determination of the parameter values. This weakness can hardly be resolved. One way is an agreement among economists about the range of certain parameters. This would secure a unified framework. The estimated parameter value would then represent the best choice out of the interval and standard deviations for each choice could be reported.

It is also possible that the problems vanish, if the estimation procedure is extended to more variables. In this thesis I only looked at the relationship between a shock in technology and labour. This generates the effects described in the last section. Interest rate, consumption and output could be included in the VAR, impulse response functions could be computed and incorporated in the estimation procedure. It is possible that certain effects leading to odd estimates cancel out. An extension of the VAR is therefore recommendable.

As the discussion demonstrates it is possible to remove the weak points of the estimation procedure. The results showed the advantages of the estimation procedure as well. The dynamics of the model can be made to look like the dynamics of the data. The standard deviation of the impulse response function of the VAR can be used as a measure of distance between the model's impulse response function and the data ones. All parameters can be estimated within a unified framework. There is no lack of equation. The estimation procedure therefore avoids the problem of inconsistency between studies. The uncertainty in the choice of parameters can be quantified and used to evaluate the model. Furthermore, the framework can be applied easily. After all, the advantages outweigh the disadvantages.

The estimation procedure can be extended in several ways. One limitation is that only one shock can be considered. For an advanced use of the program code, the possibility to account for further shocks should be added. Another feature, which would improve the code, is to include more prior distribution especially the Sims-Zha prior (Sims and Zha, 1998). In connection with the calculation of the impulse response of the VAR process, an alternative to the Cholesky transformation could be built in. This would have the advantage that the user would not have to make the recursiveness assumption.

I planned to include all the features named above, but took a decision against it because I do not think they contribute further to the specific topic I want to discuss.

## 9 Conclusion

Because a large amount of research deals with the dynamic implications of a model, an estimation method developed by Christiano, Eichenbaum and Evans has been examined and extended. As the results showed, this method led to workable estimates for Hansen's RBC model. I therefore suggest the following framework for choosing parameters within a DSGE model:

1. Computation of the dynamics of a Bayesian VAR process for the variables of interest in the model
2. Derivation of the model's dynamics depending on the model's parameters (not all parameters have to be included)
3. Application of the estimation procedure
4. Improvement of the estimates by restricting the intervals for possible solution or by employing a different prior probability distribution
5. Report of estimated values and their standard deviations in connection with the graphs

This framework avoids problems of the methods presented in section two. It provides the researcher with a measurement of the uncertainty in the choice of parameters that can be used for a further evaluation of the model. Because it concentrates on the dynamics of the data, the method is mostly robust against measurement errors. All variables of interest are observable. It also avoids arbitrary choices of parameter and inconsistency between studies by offering a unified framework. There is no lack of equation for the estimation procedure.

Conclusively, I agree with Christiano, Eichenbaum and Evans, that research with DSGE models is about their dynamic implications. The estimation method presented in this thesis can then be helpful.

## 10 Appendix

### 10.1 MATLAB Code

#### 10.1.1 “Program”

```
%this m-file is the main program

%D is the matrix that contains the data
load D;

%collecting some information
m= size(D,2);
rho= input('number of lags');
T= size(D,1);

%builds the VAR process
[Y,X]= order (D, m, rho, T) ;

%decides the prior distribution
[N, S, v, B]=prior (D, m, rho, T);

%function that estimates the coefficients and computes
%the posterior distribution
[Bt, Nt, vt, St]=posterior (Y, X, B, N, v, S, T);

%collecting information about impulse responses for the VAR
dr=input ('number of draws?');
disp(' ');
im=input ('number of periods?');
save im;

%computation of the I-R-F
[IDA, VCV, IV, plt, t]=impulldata (Bt, St, vt, m, rho, Nt,
im, dr);

%IDA- contains the average for each period
save IDA
%VCV- contains the variance covariance matrix
save VCV
%IV- contains every variable, period for every draw
save IV
%plt- average +/- standard deviation of the data
save plt

loop=1;
while loop==1;
    %parameters are estimated and their standard deviation
    %is computed
    psi=estimat(VCV, t, im);
    stdb=standardb(IV, psi);
```

```

%models I-R-F for the estimated parameters
%are computed
ih=impulseth(psi);

disp ('your parameter estimate is: ');
disp(psi);
disp (' ');
disp ('press enter')
pause
disp ('the standard deviations for the parameters are: ')
disp (stdb);
disp ('press enter')
pause

%preparations for the plots
x=1:im;
for i=1:t
    %plot I-R-F data vs. I-R-F model
    r=[IDA(1,((i-1)*im)+1:(i*im));ih];
    plot(x,r)
    xlabel('periods after shock');
    ylabel('percent deviation from steady state')
    pause

    %plot that includes standard deviation
    w=[r; plt(:,((i-1)*im)+1:(i*im))];
    plot (x,w)
    xlabel('periods after shock');
    ylabel('percent deviation from steady state')
    pause

end

disp(' 1 - repeat the calculation for a different settings')
disp(' 2 - end')
disp(' ')
loop=input('What do you want');
end

```

### 10.1.2 Function “order”

```

function [Y, X]= order (D, m, rho, T)

%this function builds the VAR process from the data- matrix D
%it orders the data into the Y and X matrix

Y=ones (T-rho, m);
X=ones (T-rho, m*rho+2);
i=1;

```

```

%Y matrix
while i<=m
    Y(:,i)=D(rho+1:T, i);
    i=i+1;
end

%X-matrix
j=rho;
k=1;
for k=k:rho
    n=1;
    for n=n:m
        X(:,(j-1)*m+n)=D(k:(T-j),n);
    end
    j=j-1;
end

b=1:(T-rho);

%X matrix plus constant and time-trend
X(:,m*rho+2)=b';

```

### 10.1.3 Function “prior”

```

function [N, S, v, B]=prior (D, m, rho, T)
%function that chooses the prior distribution

disp('1 - flat prior')
disp('2 - Minnesota prior')
disp('          ')

pri=input('Prior?');

switch pri
%flat prior
case 1
    Bo=0;
    No=0;
    So=0;
    vo=0;
%minnesota prior
case 2
    %So
    So=zeros(m);
    for i=1:m
        Uy=D(1:(T-1), i);
        Ux=D(2:T, i);
        bs=( (Ux'*Ux)^(-1) ) *Ux'*Uy;
        e=Uy-Ux*bs;
        s=e'*e/T;
    end
end

```

```

        So(i,i)=s;
    end
    %No
    No=zeros(rho*m+2);
    %hyperparameters
    eta1=5; eta2=2; eta3=8;
    No(1,1)=eta3; No(1,2)=(-eta3^2)/2; No(2,1)=No(1,2);
    No(2,2)=(eta3^3)/3;
    Y0=D(1,:);
    for i=1:m
        for l=1:rho
            No(2+m*(l-1)+i,2+m*(l-1)+i)=(Y0(i))^2*eta1*l^(eta2);
        end
    end
    %Bo
    Bo=eye(m,m*rho+2)';
    %vo (I had annual data)
    vo=5;
end
N=No;
B=Bo;
S=So;
v=vo;

```

#### 10.1.4 Function “posterior”

```

function [Bt, Nt, vt, St]= posterior (Y, X, B, N, v, S, T)
%function that computes the posterior

%likelihood estimation

Bhat=inv((X'*X))*X'*Y;
shat=(1/T)*(Y-X*Bhat)'*(Y-X*Bhat);

%posterior computation

vt=v+T;
Nt=N+X'*X;
Bt=inv(Nt)*(N*B+X'*X*Bhat);
St=(v/vt)*S+(T/vt)*shat+(1/vt)*(Bhat-B)'*N*inv(Nt)*X'*X*(Bhat-
B);

```

#### 10.1.5 Function “impulsedata”

```

function[IDA, VCV, IV, plot, t]=impulsedata (Bt, St, vt, m,
rho, Nt, im, dr)

%this function computes for a given posterior distribution
%cholesky-impulse response function with horizon im
%it draws dr-times

```

```

%all draws are collected in I
I=zeros (m,dr,im);
%u is the shock - it is also possible to load a shock
u=[1;0];
%matrix J is a brick for the companion form of the VAR
J=eye(m,m*rho);

%the loop, that computes dr stable impulse response function
j=1;

while j<=dr

%first it draws from the posterior
[SZ, BZ]= ziehen (Bt, St, vt, m, rho, Nt);

%some preparations for the stability test
B=BZ';
B=B(:,1:m*rho);
C=zeros(m*rho,m*rho);

%building the companion form
if rho==1
    C=B;
else
    Ar=eye(rho-1);
    Ak=eye(m);
    Akm=kron(Ar,Ak);
    Ab=B;
    C(1:m, :)=Ab;
    C((m+1):m*rho,1:m*rho-m)=Akm;
end

%computing the roots
roots=abs(eig(C));
ro=max(roots);

%Test for stability
%if the VAR is stable the function computes the
%impulse response function
if ro<1
    A=chol(SZ);
    I(:,j,1)=A'*u;
    for i=2:im
        I(:,j,i)=J*C^(i-1)*J'*A*u;
    end
    j=j+1;
end

end
end

```

```

%after the impulse response function are drawn, they are
ordered from
%the 3-d matrix I into the 2-d matrix Ip
%this is necessary in order to compute first and second
moments
Ip=zeros(m*im,dr);
tal=1;
for k=1:im
    Ip(tal:tal+m-1,:)=I(:, :, k);
    tal=tal+m;
end

%in case the VAR process includes more variables than the user
wants to employ
%(for example a productivity index), he is asked about the
variables he is interested in and their position

t=input('For how many variables do you want to compute impulse
response functions?');
tb=zeros(1,t);
disp('please insert now the position of the variables in the
VAR. you will be asked once for each variable')

for i=1:t
    tb(i)=input('position:');
end

%generating matrices that should save the information

% matrix that saves the average impulse responses function for
each variable of interest
IDA=zeros(1,t*im);

%matrix that saves the variance covariance matrix for each
variable of interest
VCV=zeros(im,im,t);

%matrix that contains the information for the plot
plot=zeros(t*3,im);

% a matrix for the computation of the average
Du=ones(dr,1);
Du=Du/dr;

% the matrix for the computation of the standard deviation of
the parameters
IV=zeros(t*im,dr);

%loop that computes mean, variance for every variable of
interest
for i=1:t

```

```

ta=tb(i);

%matrix that contains every draw and period
%(needed to compute standard deviations of the parameters)
Iv=zeros(im,dr);

%loop to take the impulse response function for variable ta
for k=1:im
    Iv(k,:)=Ip(ta,:);
    ta=ta+m;
end

%computation of the average for each period
ID=Iv*Du;
M=repmat(ID,1,dr);
ID=reshape(ID,1,im);

%computation of the variance covariance matrix
Sg=Iv-M;
VC=(Sg*Sg')/dr;

%computation of the standard deviation for the VAR
v=(diag(VC)).^(0.5);
std=zeros(2,im);
std(1,:)=ID+v';
std(2,:)=ID-v';
pl=[ID;std];

%put the matrices into the matrices that save the values
for all variables of interest

IV(((i-1)*im)+1:i*im,:)=Iv;

IDA(1,((i-1)*im)+1:i*im)=ID;

VCV(:, :, i)=VC;

plot(((i-1)*3)+1:i*3,:)=pl;

end

IV=IV(:);

```

### 10.1.6 Function “`estimat`”

```
function [fit]= estimat (VCV, s, im)
```

```

%this function generates the matrix, which is needed for
different criteria and
%starts the minimization routine
%it hands the parameter estimate back (fit)

```

```

om=size (VCV,1);

%the user can choose, between the different criteria

disp(' ');
disp(' ');
disp('1 - only variances')
disp('2 - full variance covariance matrix')
disp('3 - weighted variance covariance matrix')

sce=input('which estimation criterion do you want to
employ?');

if s==1
switch sce
case 1
    v=diag(VCV);
    V=diag(v);
case 2
    V=VCV;
case 3
    V=zeros(size(VCV));
    for w=1:om
        for a=1:om
            if a==w
                V(a,w)=VCV(a,w);
            elseif a>w
                V(a,w)=VCV(a,w)*(1/a);
            else
                V(a,w)=VCV(a,w)*(1/w);
            end
        end
    end
end
end

else

switch sce
case 1
    v=zeros(s*im,1);
    for h=1:s
        v((h-1)*im+1:h*im,1)=diag(VCV(:, :, h));
    end
    V=diag(v);

case 2
    V=zeros(im*s);
    for h=1:s

```

```

        V((h-1)*im+1:h*im, (h-1)*im+1:h*im)=VCV(:, :, h);
    end

case 3
    V=zeros(im*s);
    for h=1:s
        v=zeros(im, im);
        for w=1:om
            for a=1:om
                if a==w
                    v(a,w)=VCV(a,w,h);
                elseif a>w
                    v(a,w)=VCV(a,w,h)*(1/a);
                else
                    v(a,w)=VCV(a,w,h)*(1/w);
                end
            end
        end
        V((h-1)*im+1:h*im, (h-1)*im+1:h*im)=v;
    end
end

end

save V
%the matrix V is saved and loaded again by the "kriterium"
function

load b
%b is the initial guess that has to be saved before

disp('1 - estimate the parameters without restrictions')
disp('2 - estimate the parameters with interval restrictions')
disp('          ')

cho=input('What kind of estimation do you prefer');

switch cho
    case 1
        fit=fminsearch('kriterium',b);
        % minimization routine with no restrictions
    case 2
        load bu
        load bl
        fit=fmincon('kriterium',b, [], [], [], [], bl, bu);
        %minimization routine with restrictions
end

```

### 10.1.7 Function “kriterium”

```
function [X]=kriterium (c);

%this function computes the sum that
%should be minimized

%the impulse response function of the data and the
%chosen variance covariance matrix(in "estimat") are loaded
load IDA
load V

%models impulse response are computed
IT=impulseth (c);

%minimization criterion
X=(IDA-IT)*V^(-1)*(IDA-IT)';
```

### 10.1.8 Function “standardb”

```
function [stdb]= standardb (IVS, bs)

%function that computes the standard deviations of
%the parameter

%number of parameter
k=4;%size(bs,1)
%number of observation
do=size(IVS,1);

%the models I-R-F
IO=impulseth(bs)';
%horizon
o=size (IO,1);

%computation of the error terms
f=repmat(IO, do/o ,1);
e=IVS-f;
%sum of the error terms
S=e'*e/(do-k);

%computation of the first derivates w.r.t. every
%parameter
bt=bs;
Zs=zeros(k,o);
for h=1:k
    bt(h)=bt(h)+0.01;
    Zs(h,:)=(impulseth(bt)-IO')/0.01;
    bt=bs;
end

%computation of the standard deviation
```

```

Z=zeros(k,do);
Z= repmat(Zs,1,do/o);
vcvm=S*((Z*Z')^(-1));

stdb=diag(vcv).^(0.5);

```

### 10.1.9 Function “ziehen”

```

function [SZ, BZ]= ziehen (Bt, St, vT, m, k, Nt)
%This function draws from the Wishart and the standard normal
distribution

```

```

%drawing one VCV matrix from the Wishart distribution
%inverse of sigma=precision matrix H
S=inv(St);
w=size (St);
%a matrix containing standard normal distributed entries
%P~N(0,1)
P=randn (w);
A=chol (S);
%after the transformation is P~(0,St)
P=A'*P;
%Si~W(St/vT,vT)
Si=P*P'/vT;
%inverting H to derive sigma
SZ=inv(Si);

```

```

%drawing a coefficient matrix B from a standard normal
distribution
%computation of E[B]
Ni=inv(Nt);
sigb=kron(SZ,Ni);
q=0;
beta=Bt(:);
l=size(beta);
%O~N(0,1)
O=randn (l);
%check if sigb is positive definite
[Q q]=chol (sigb);
if q~=0
    BZ=Bt;
else
    %O~N(0,sigma)
    O=Q'*O;

    %O~N(beta, sigb)
    O=O+beta;

    BZ=reshape (O, m*k+2, m);
end

```

### 10.1.10 Function “impulseth”

```
function [Itv]=impulseth (ve)

%this function computes for a given parameter vector the
impulse response
%function of the model

% the impulse response horizon
im=20;

%reading the parameters
teta=ve(1);
delta=ve(2);
rhot=ve(3);
beta=ve(4);
gamma=ve(5);
nbar=ve(6);

%steady state
Rbar=1/beta;
kappa=(Rbar-1+delta)/teta;
ybar=nbar*(gamma*kappa^(-teta))^(1/(1-teta));
kbar=ybar/kappa;
cbar=(kappa-delta)*kbar;
lambar=1/cbar;
A=(lambar*(1-teta)*ybar)/nbar;

%alpha coefficients
a1=kappa+1-delta;
a2=cbar/kbar+kappa*((1-teta)/teta);
a3=kappa/teta;
a5=1+((1-teta)*kappa)/Rbar;
a6=kappa/Rbar;

%comparing coefficients
etakk1=0.5*((a1+1/a5)+((a1+1/a5)^2-4*(a1/a5))^0.5);
etakk2=0.5*((a1+1/a5)-((a1+1/a5)^2-4*(a1/a5))^0.5);
if -1<etakk1<1
    etakk=etakk1;
else
    etakk=etakk2;
end

etalk=(etakk-a1)/a2;
etalz=(etalk*a5*a3+a6*rhot)/(1-etalk*a5*a2-a5*rhot);
etakz=a2*etalz+a3;
```

```

%generating the shock
z=eye(1,im);
for i=2:im
    z(i)=rhot*z(i-1);
end

%impulse response function

I=zeros(3,im);
I(1,:)=z;
LZ=[etakz;etalz];
LK=[etakk; etalk];

for i=2:im
    I(2:3,i)=LK*I(2,i-1)+LZ*I(1,i);
end

J=zeros(4,im);
L=[0          0          -1
   1/teta     0          (1-teta)/teta
   kappa/Rbar 0          (kappa/Rbar)*(1-teta)
   1/teta     0          1/teta];

O=[0          0          0
   0          1          0
   0          0          0
   0          1          0];
J(:,1)=L*I(:,1);
for i=2:im
    J(:,i)=L*I(:,i)+O*I(:,i-1);
end

It=[I;J];

%output
%the 7th variable is n for example
Itv=It(7,:);

```

10.2 Figures

Figure 10.1 First Criterion, Flat Prior, n=1000, unrestricted

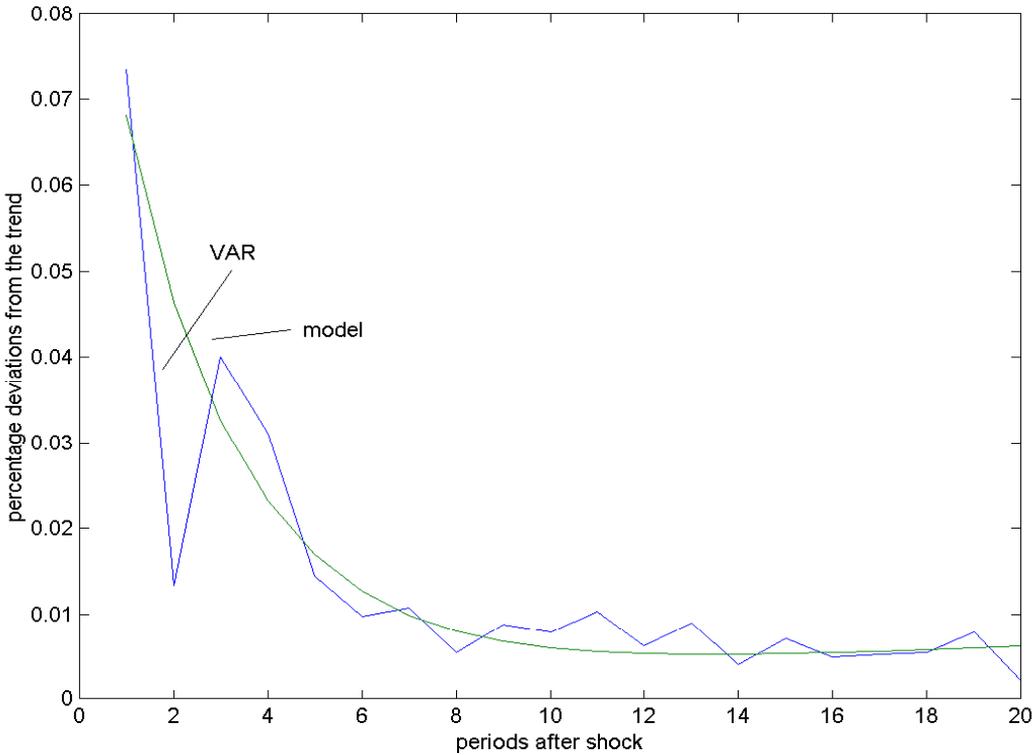
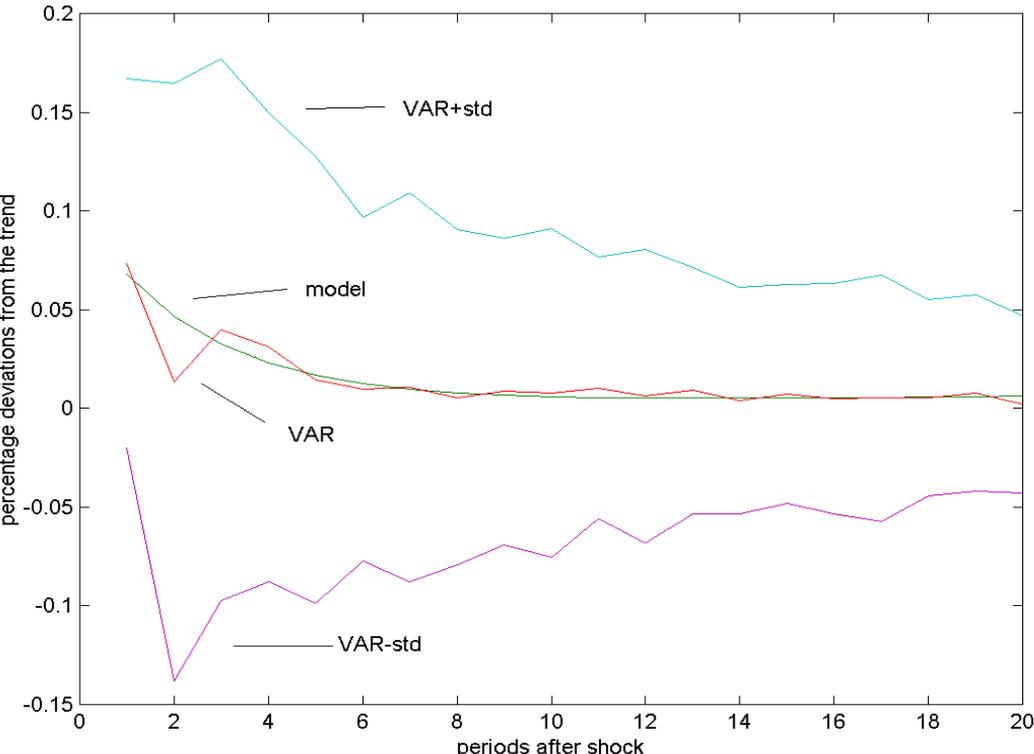
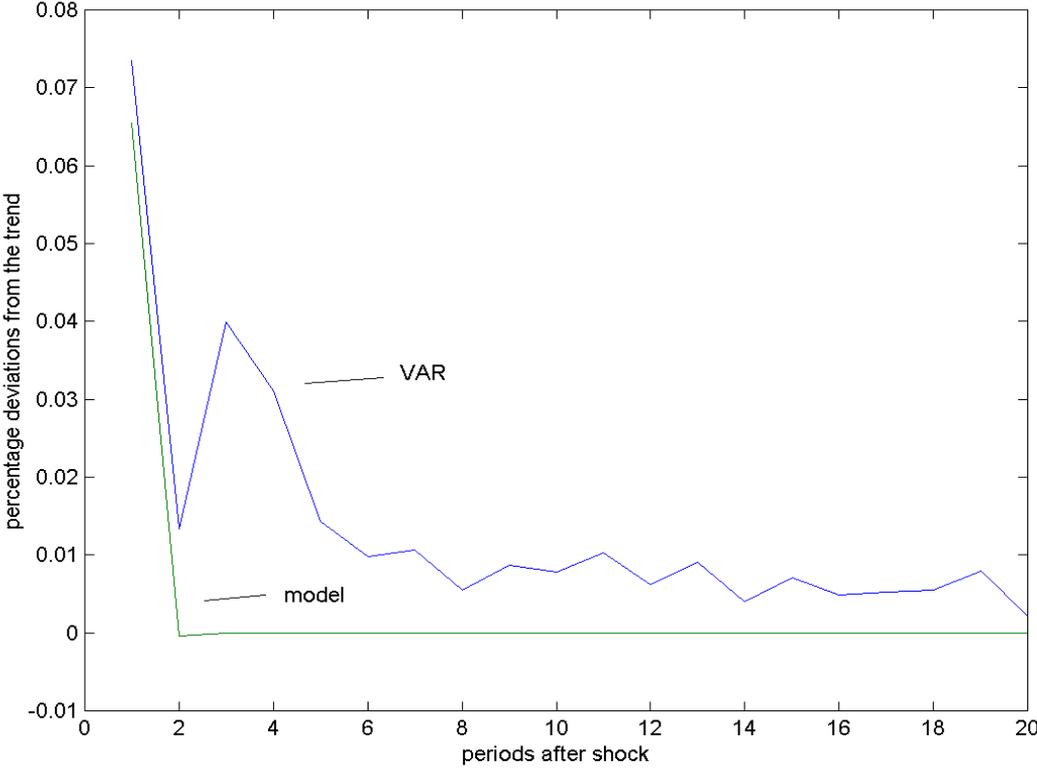


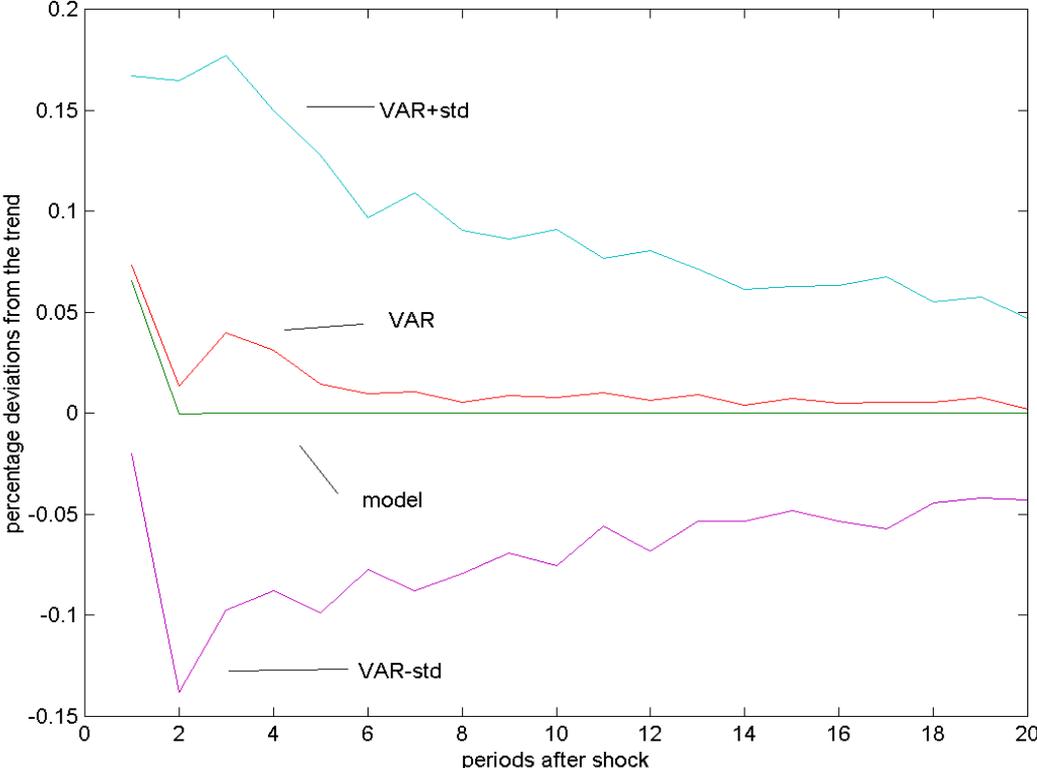
Figure 10.2 First Criterion, Flat Prior, n=1000, unrestricted



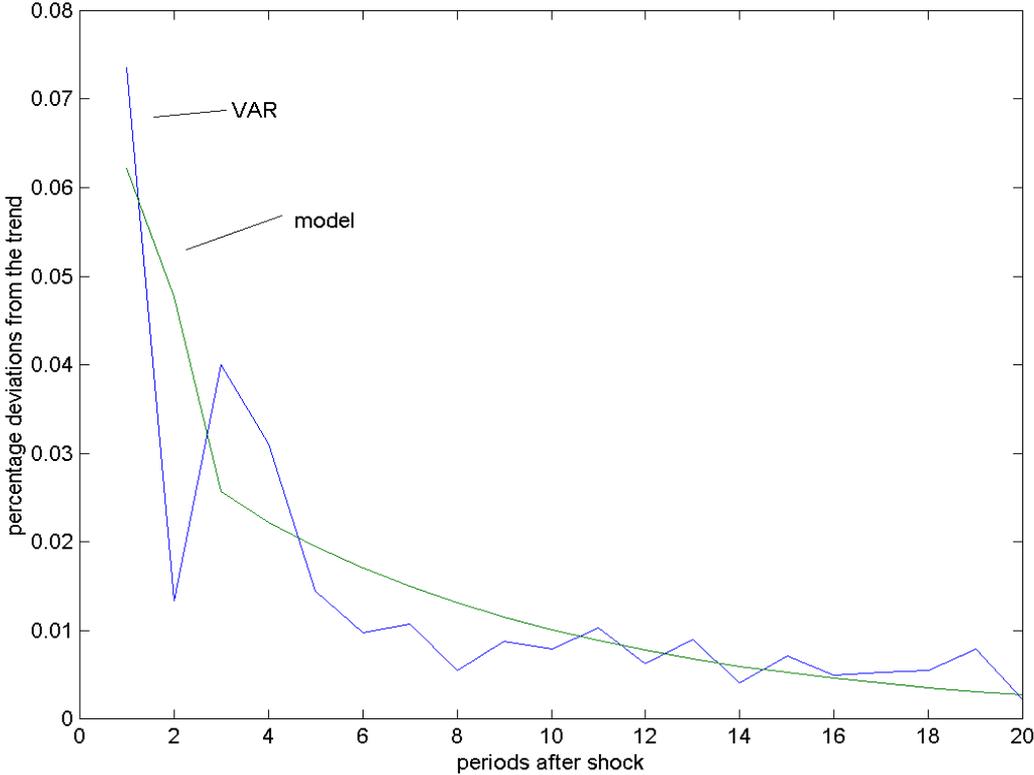
**Figure 10.3** *Second Criterion, Flat Prior, n=1000, unrestricted*



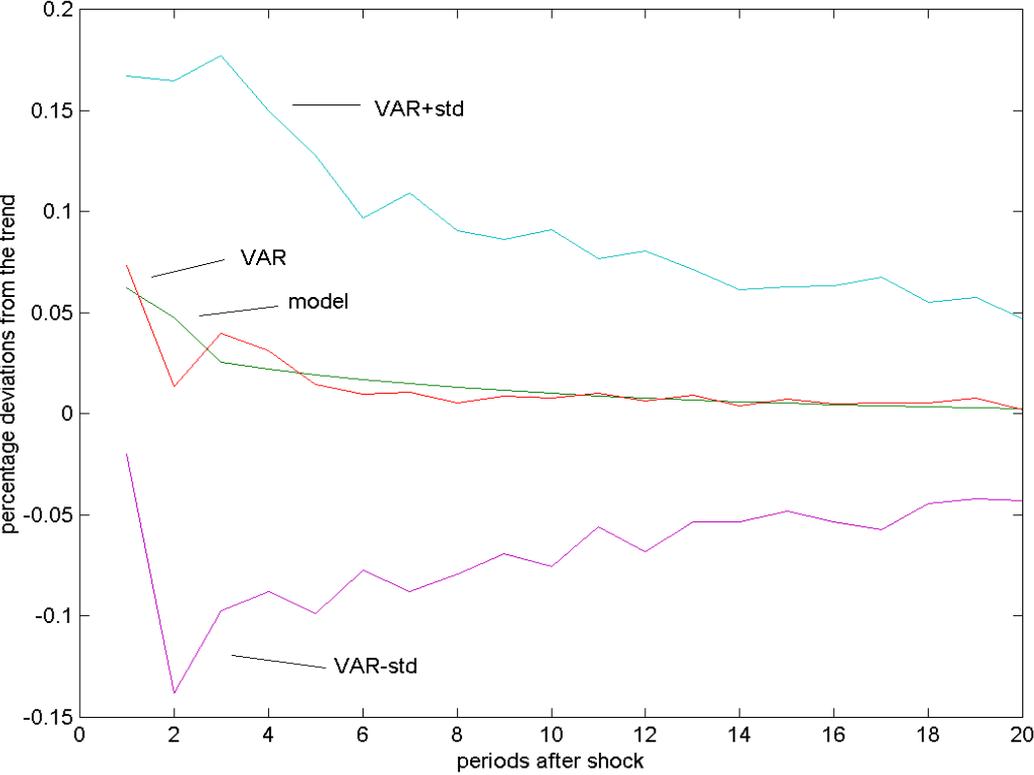
**Figure 10.4** *Second Criterion, Flat Prior, n=1000, unrestricted*



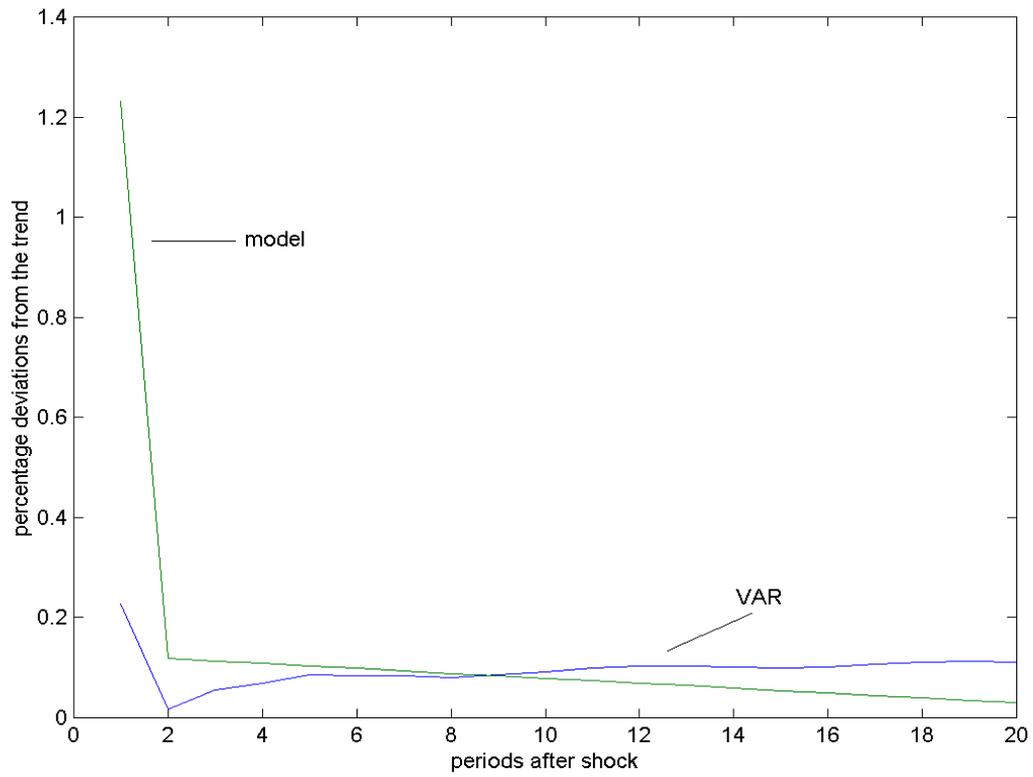
**Figure 10.5** *Third Criterion, Flat Prior, n=1000, unrestricted*



**Figure 10.6** *Third Criterion, Flat Prior, n=1000, unrestricted*



**Figure 10.7** *First Criterion, Minnesota Prior, n=1000, unrestricted*



**Figure 10.8** *First Criterion, Minnesota Prior, n=1000, unrestricted*

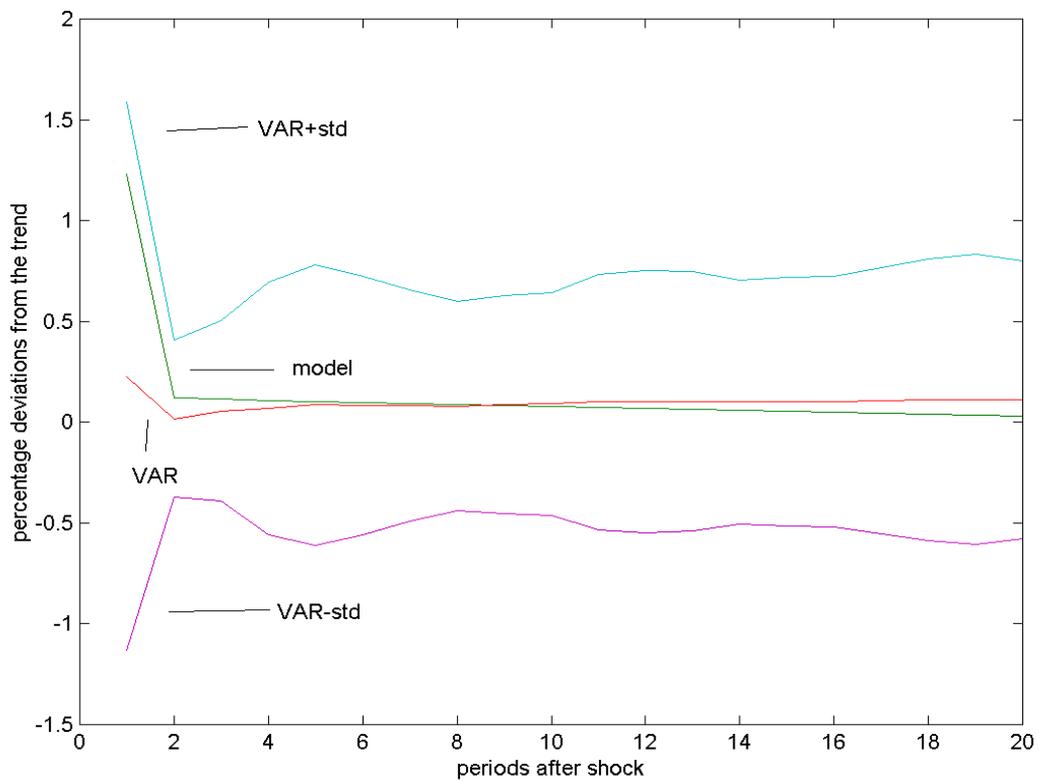


Figure 10.9 Second Criterion, Minnesota Prior, n=1000, unrestricted

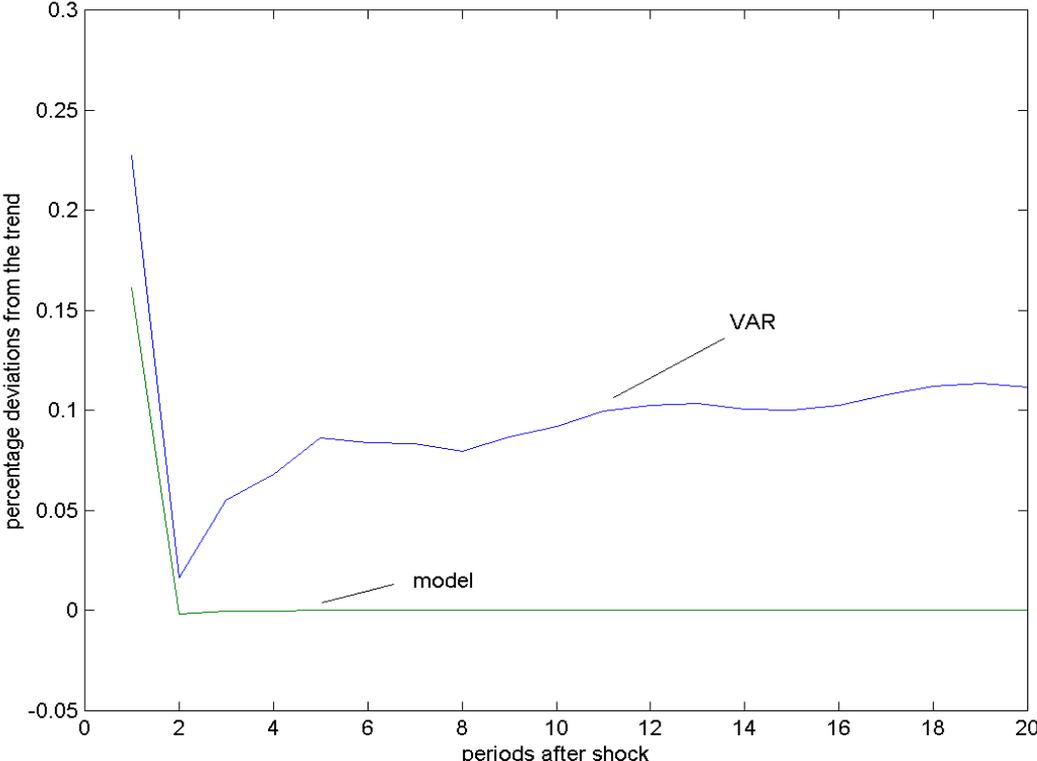
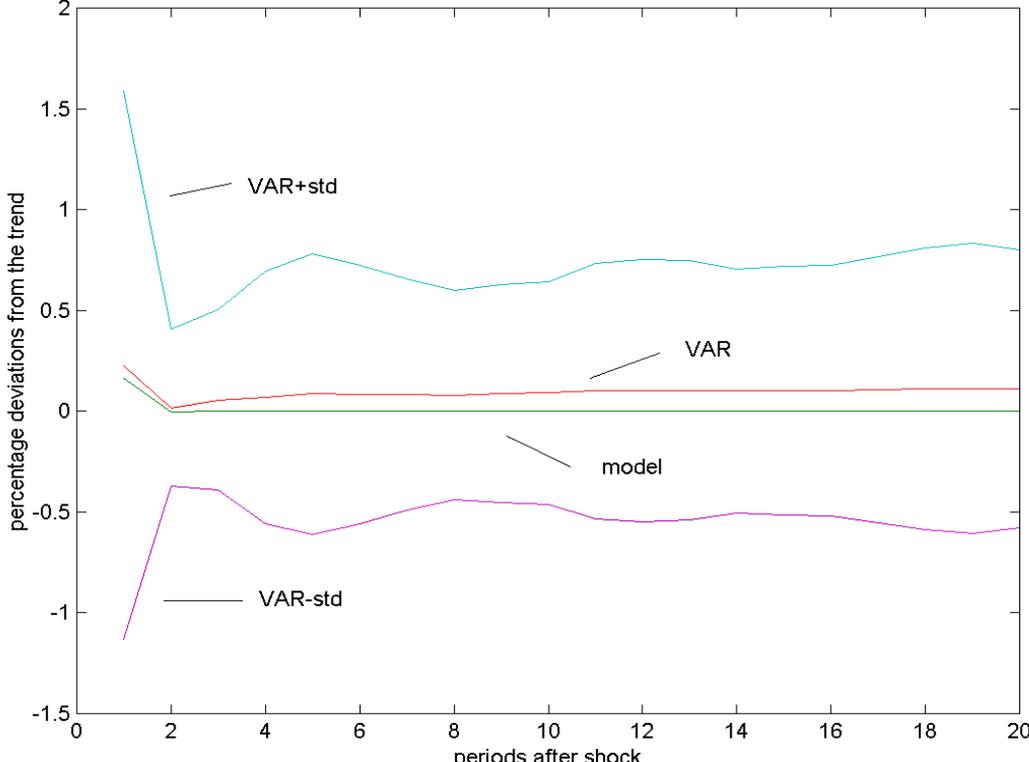
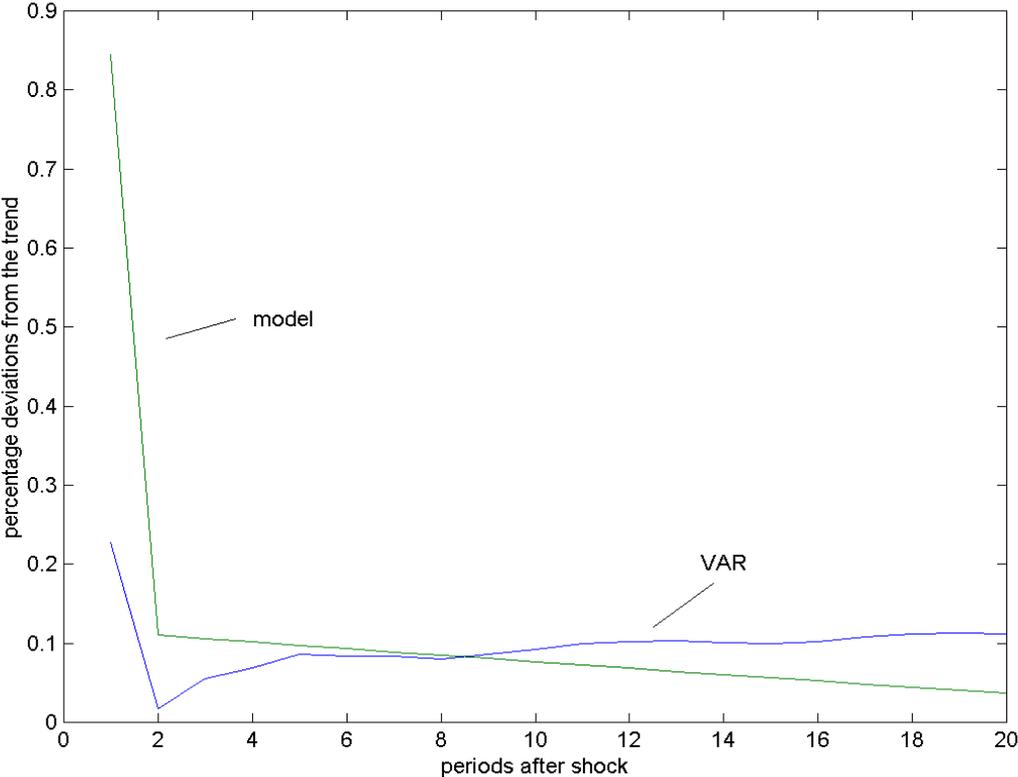


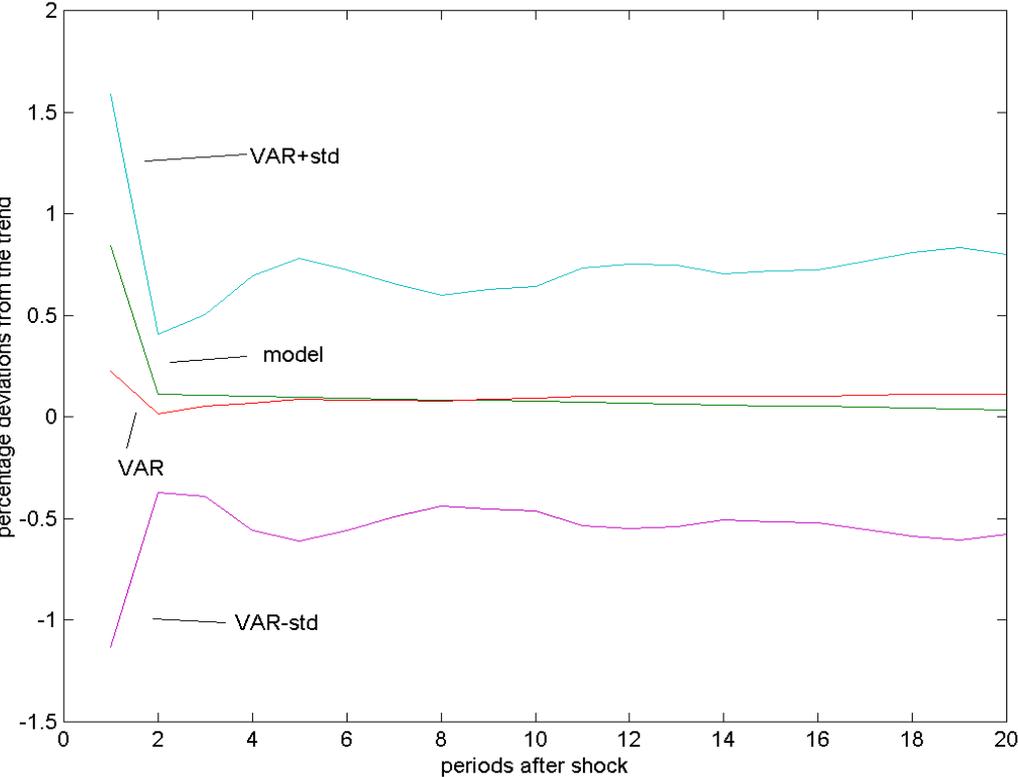
Figure 10.10 Second Criterion, Minnesota Prior, n=1000, unrestricted



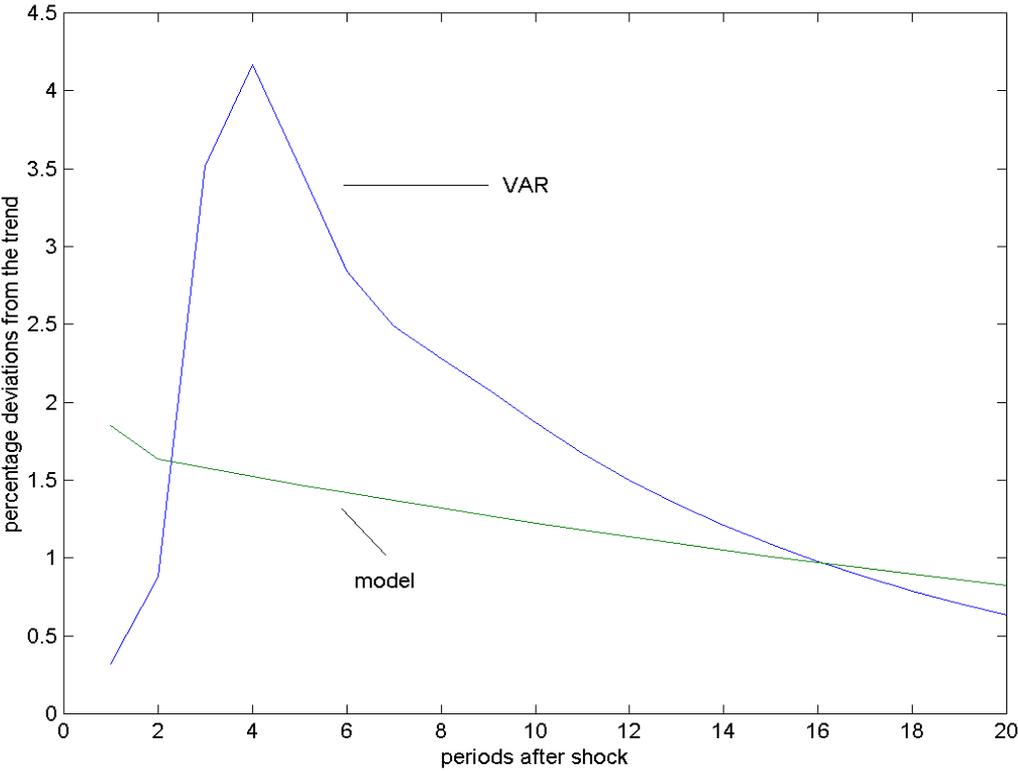
**Figure 10.11** *Third Criterion, Minnesota Prior, n=1000, unrestricted*



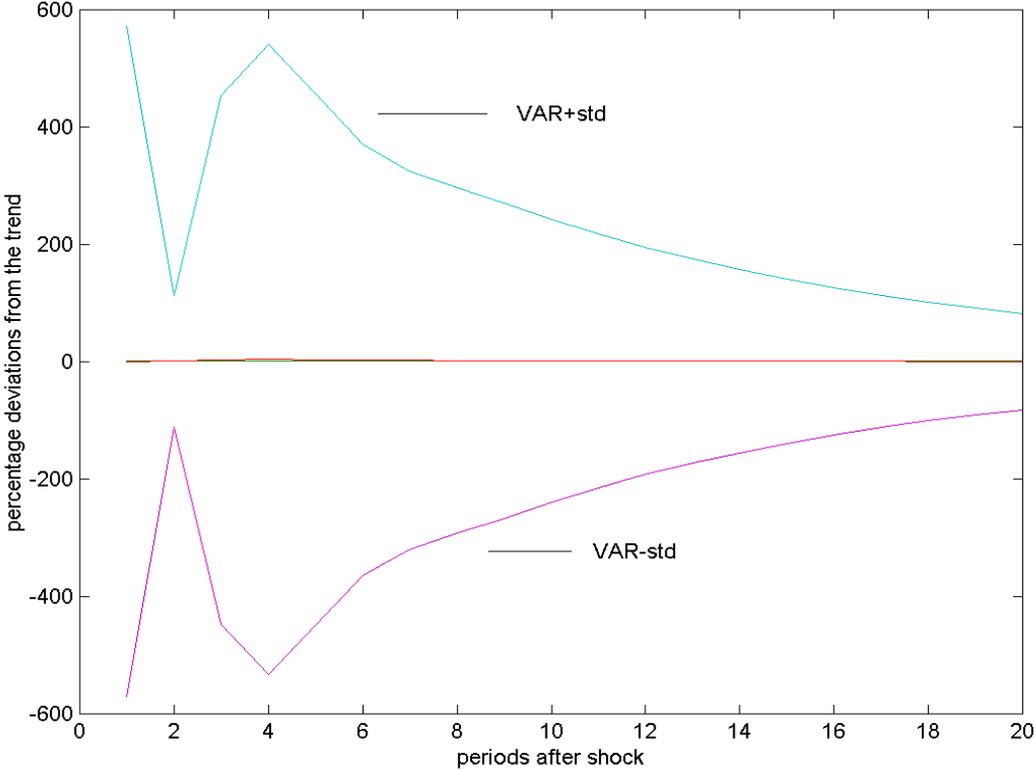
**Figure 10.12** *Third Criterion, Minnesota Prior, n=1000, unrestricted*



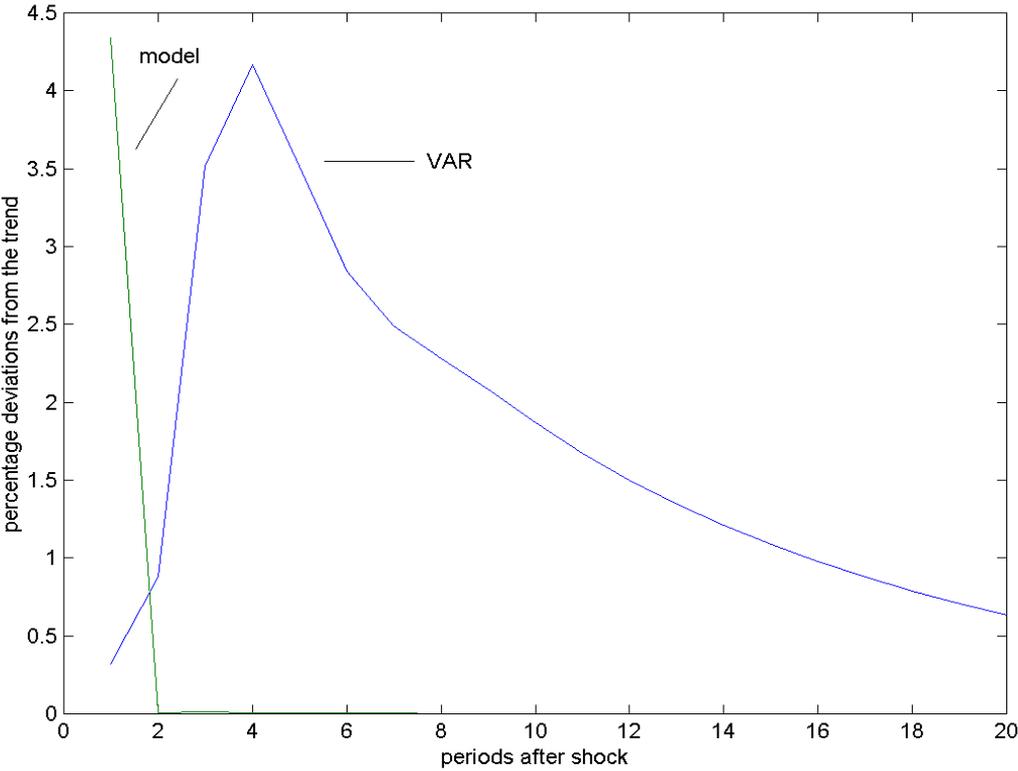
**Figure 10.13** *First Criterion, Flat Prior, n=100,000, unrestricted*



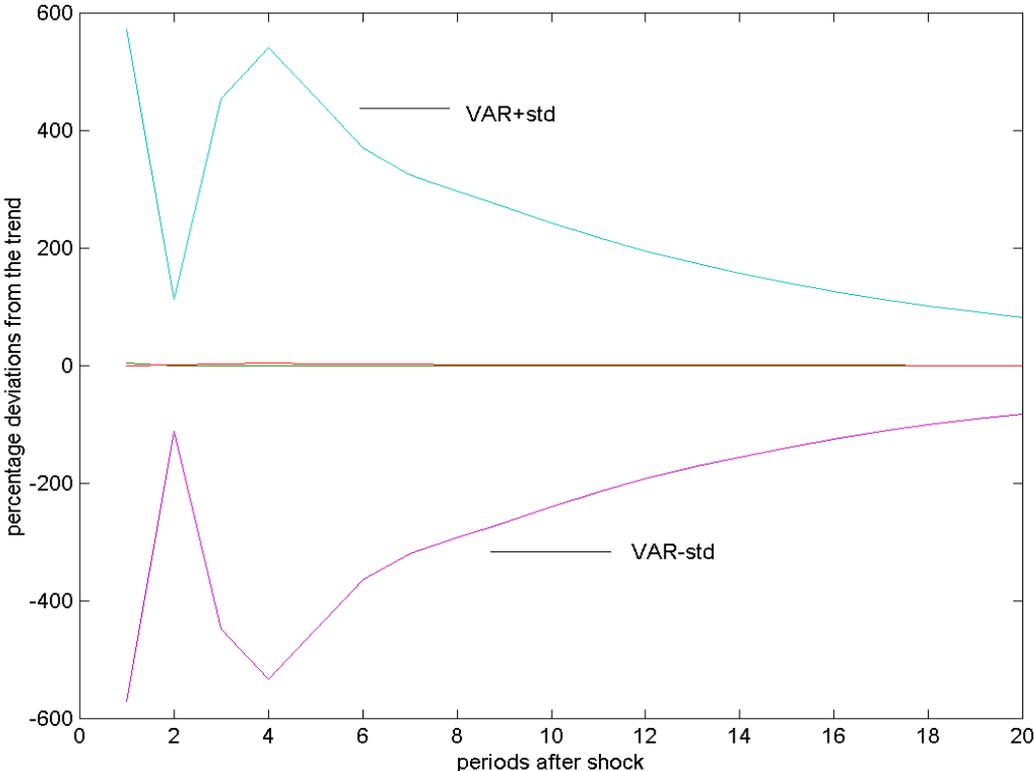
**Figure 10.14** *First Criterion, Flat Prior, n=100,000, unrestricted*



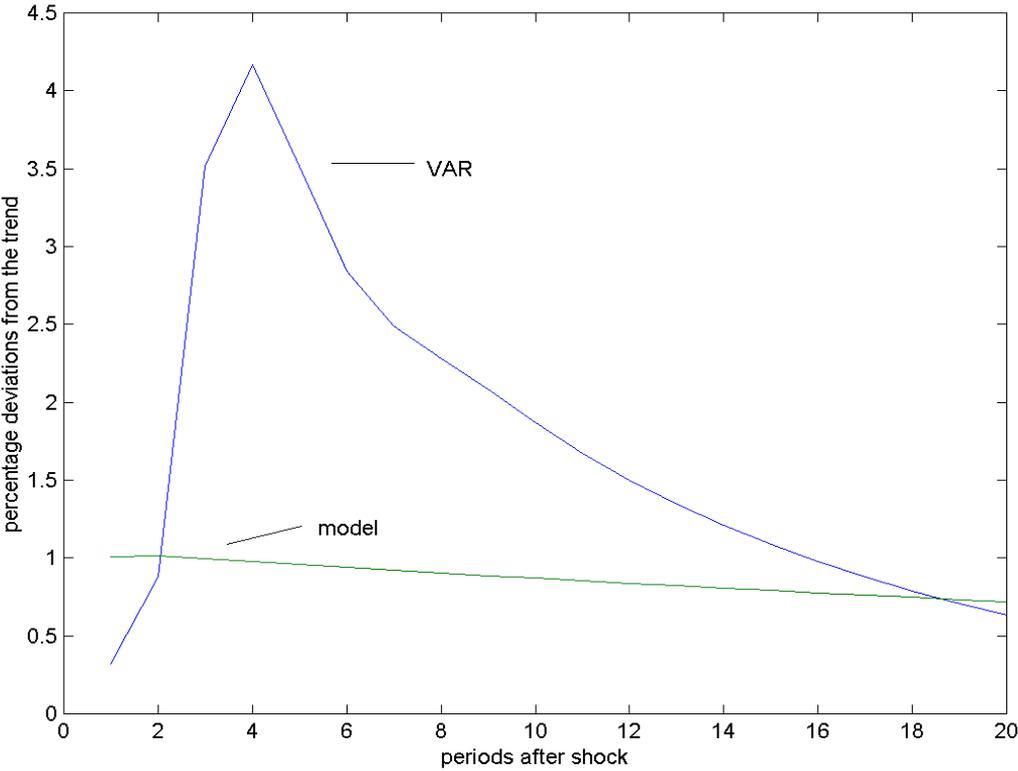
**Figure 10.15** *Second Criterion, Flat Prior, n=100,000, unrestricted*



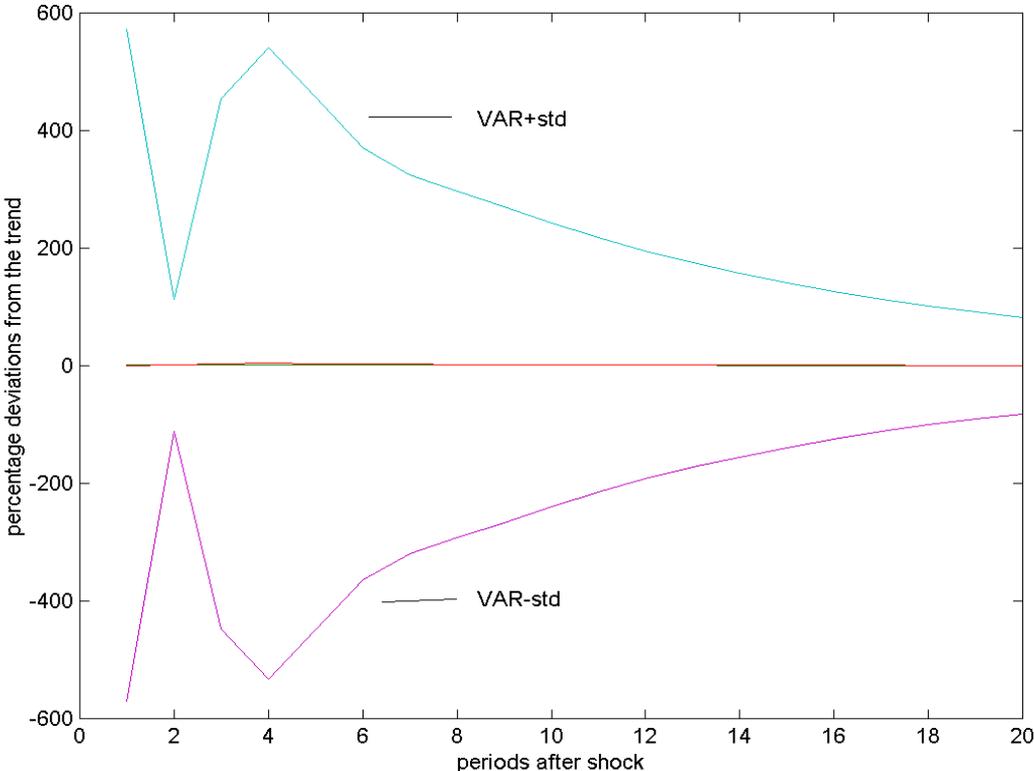
**Figure 10.16** *Second Criterion, Flat Prior, n=100,000, unrestricted*



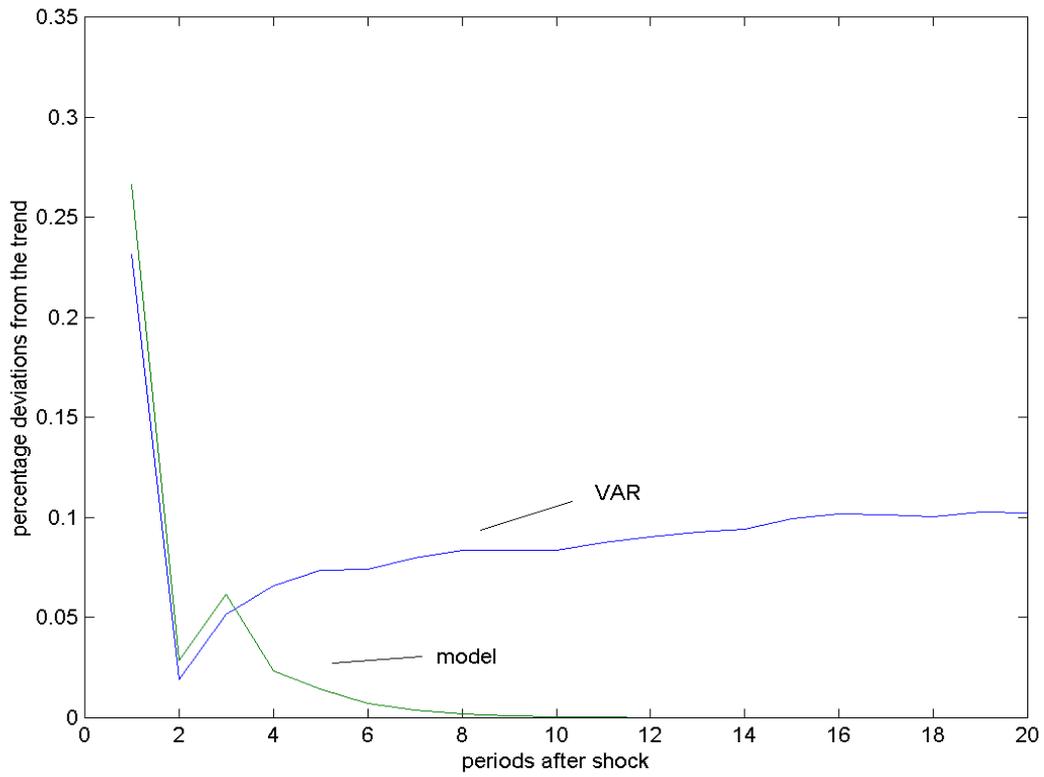
**Figure 10.17** *Third Criterion, Flat Prior, n=100,000, unrestricted*



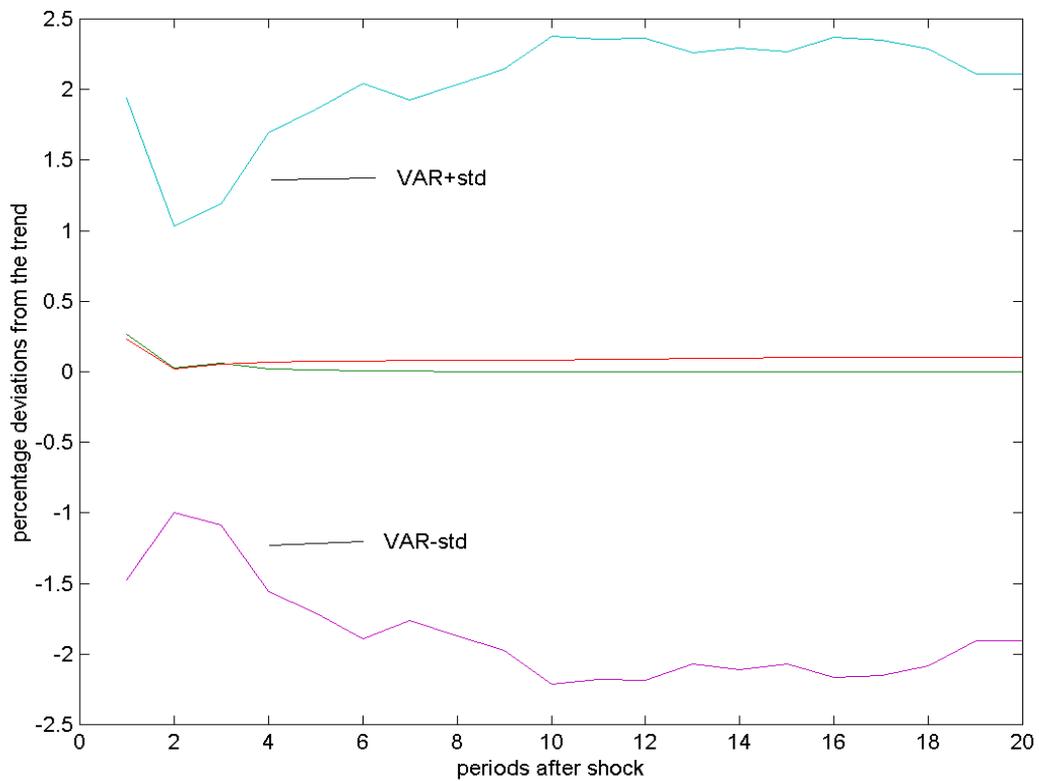
**Figure 10.18** *Third Criterion, Flat Prior, n=100,000, unrestricted*



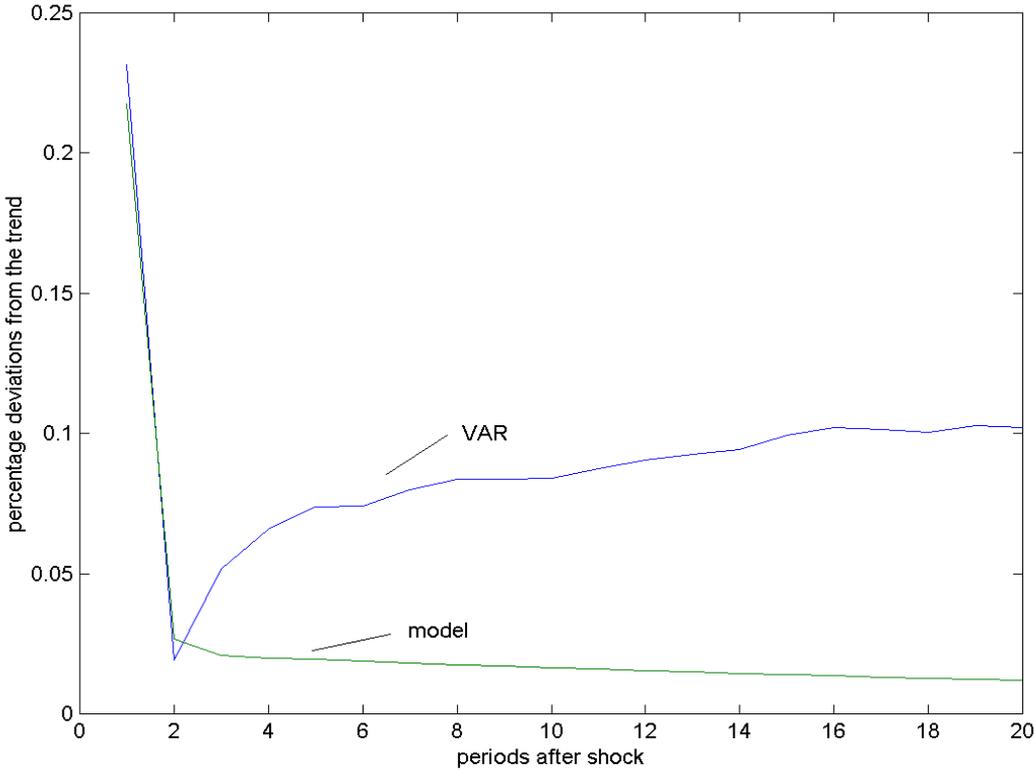
**Figure 10.19** *First Criterion, Minnesota Prior, n=100,000, unrestricted*



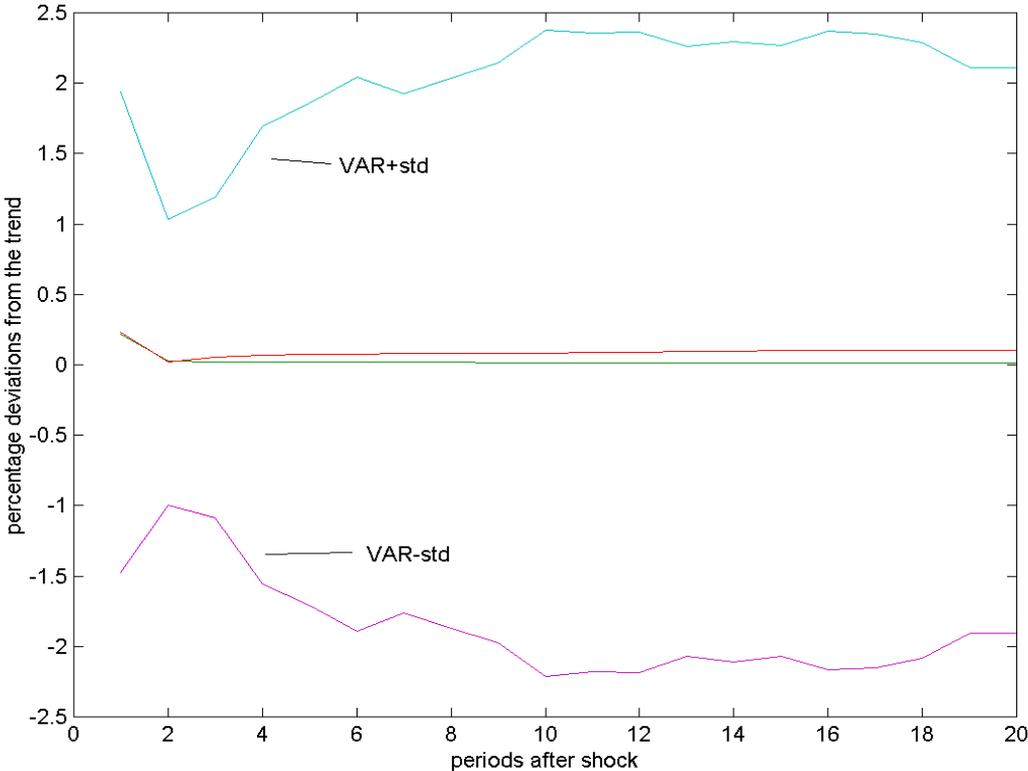
**Figure 10.20** *First Criterion, Minnesota Prior, n=100,000, unrestricted*



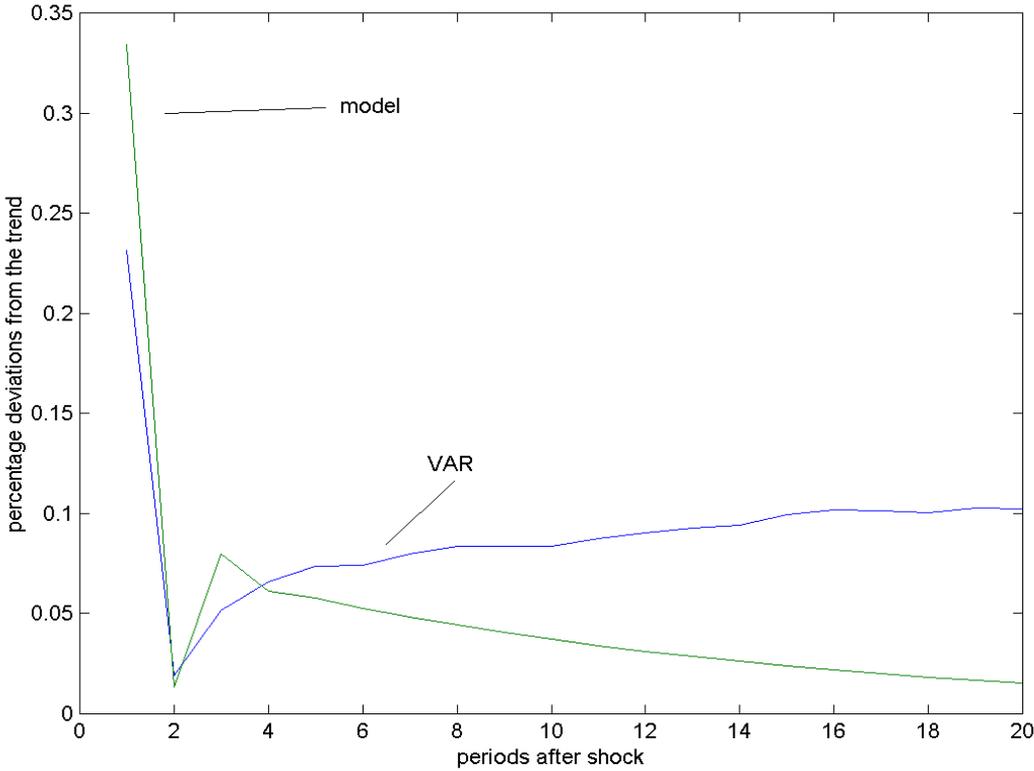
**Figure 10.21** *Second Criterion, Minnesota Prior, n=100,000, unrestricted*



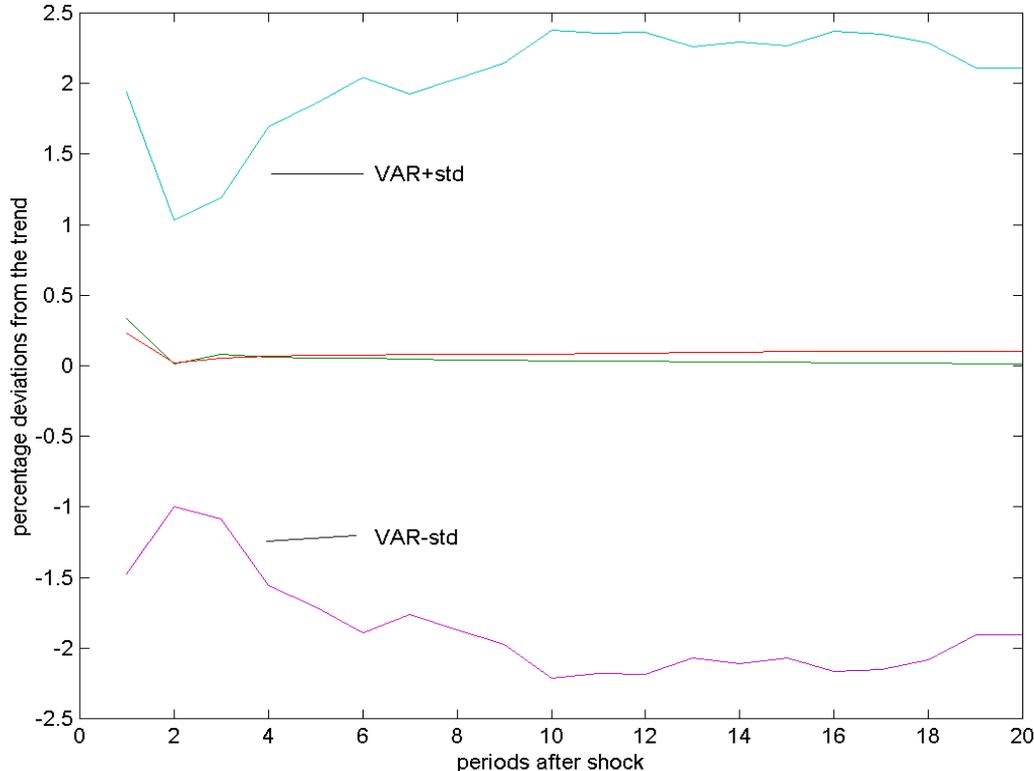
**Figure 10.22** *Second Criterion, Minnesota Prior, n=100,000, unrestricted*



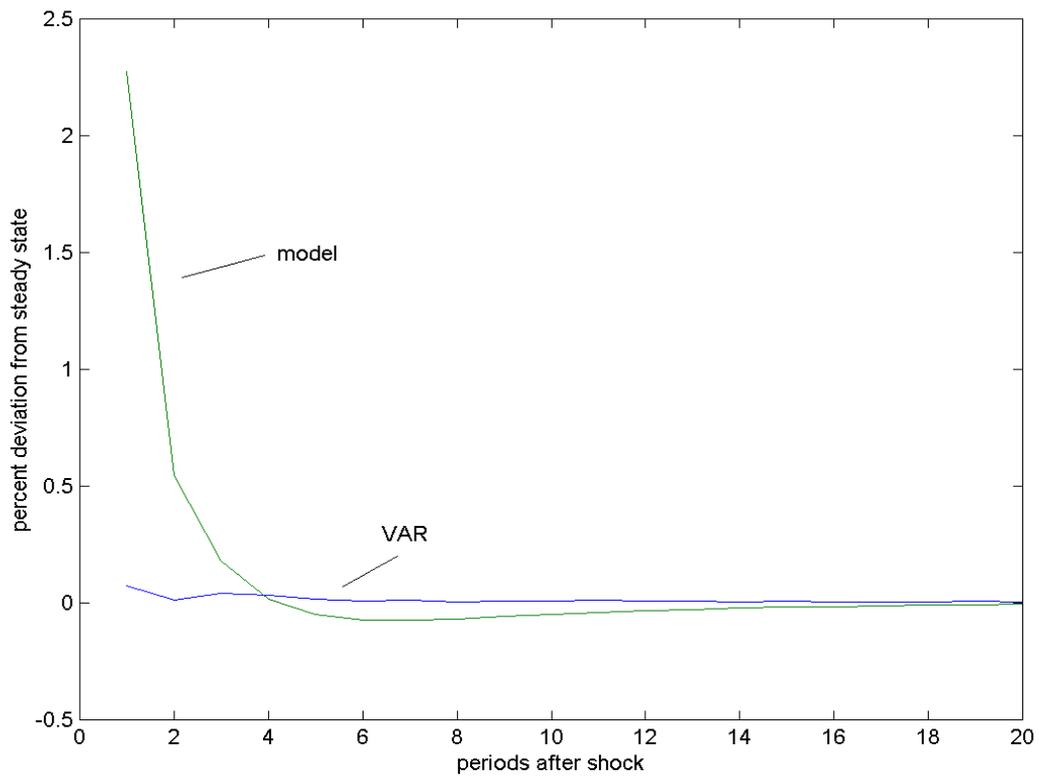
**Figure 10.23** *Third Criterion, Minnesota Prior, n=100,000, unrestricted*



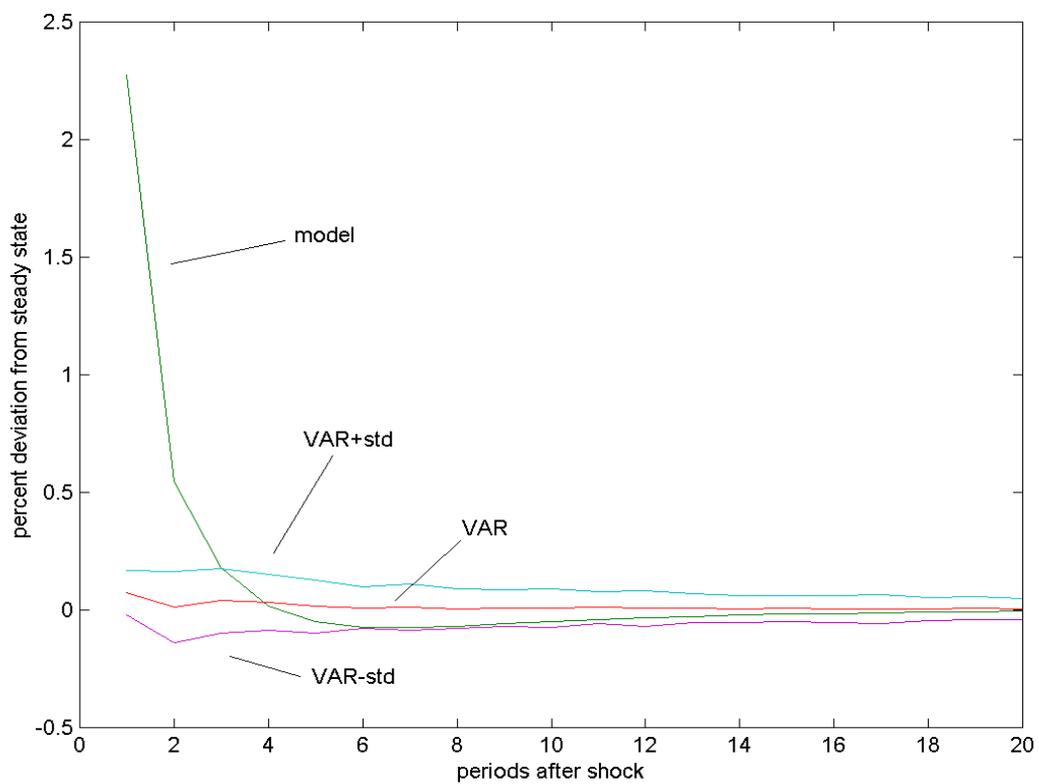
**Figure 10.24** *Third Criterion, Minnesota Prior, n=100,000, unrestricted*



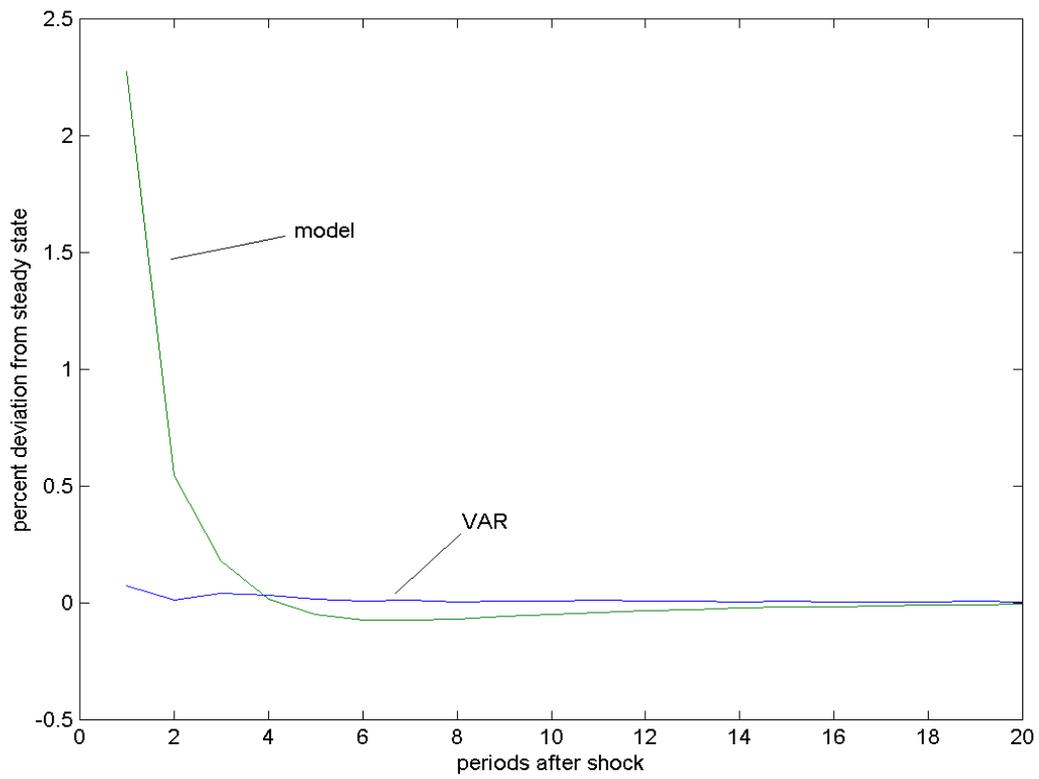
**Figure 10.25** *First Criterion, Flat Prior, n=1000, restricted*



**Figure 10.26** *First Criterion, Flat Prior, n=1000, restricted*



**Figure 10.27** *Second Criterion, Flat Prior, n=1000, restricted*



**Figure 10.28** *Second Criterion, Flat Prior, n=1000, restricted*

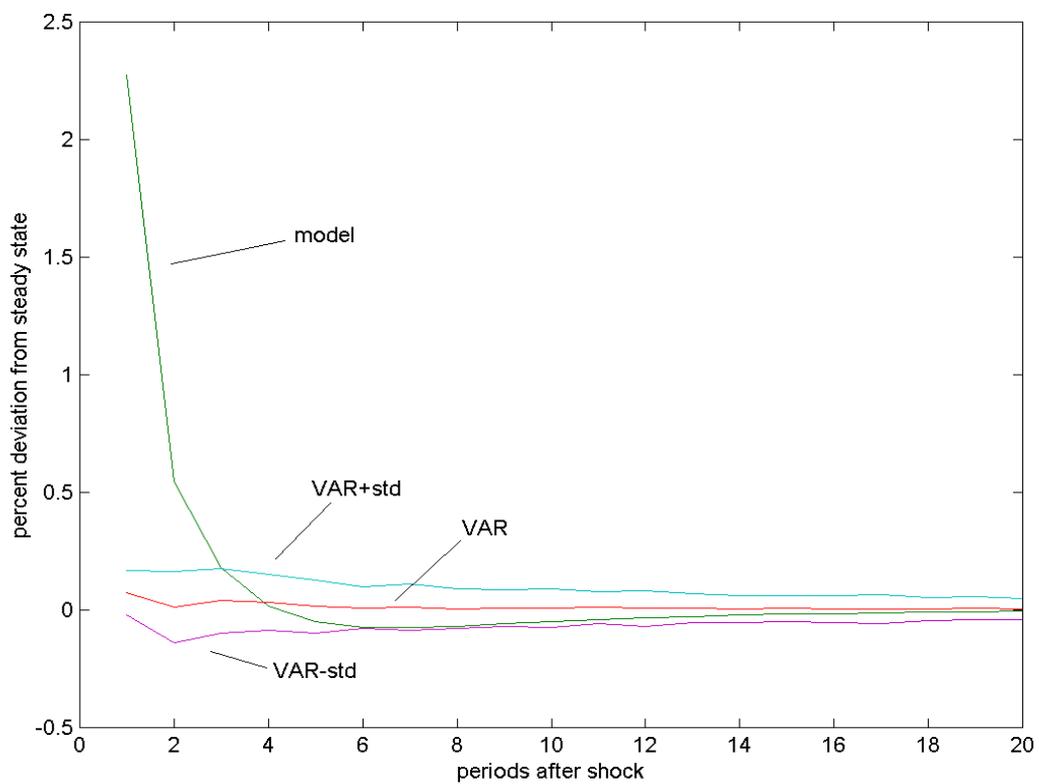


Figure 10.29 First Criterion, Flat Prior, n=100,000, restricted

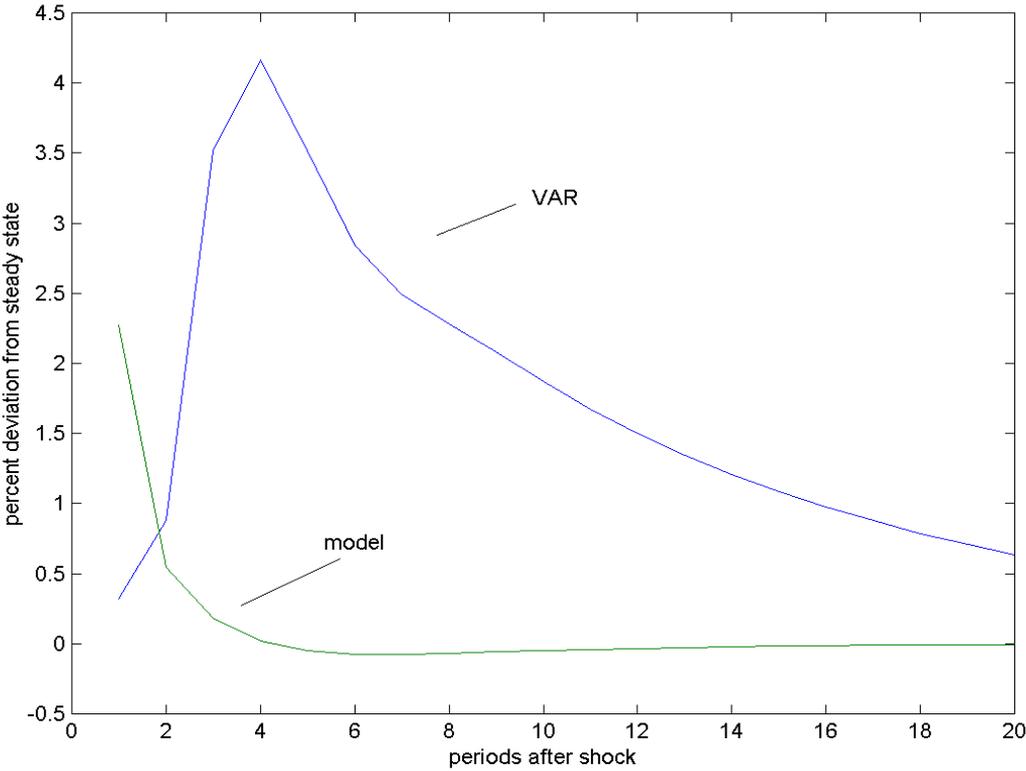
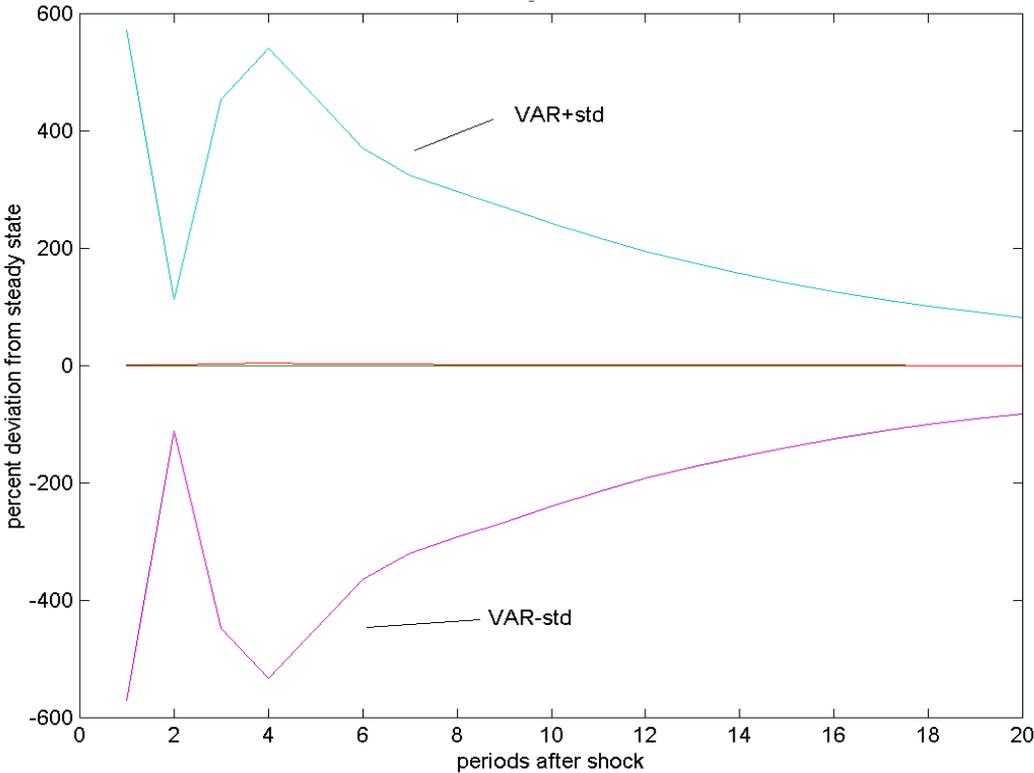
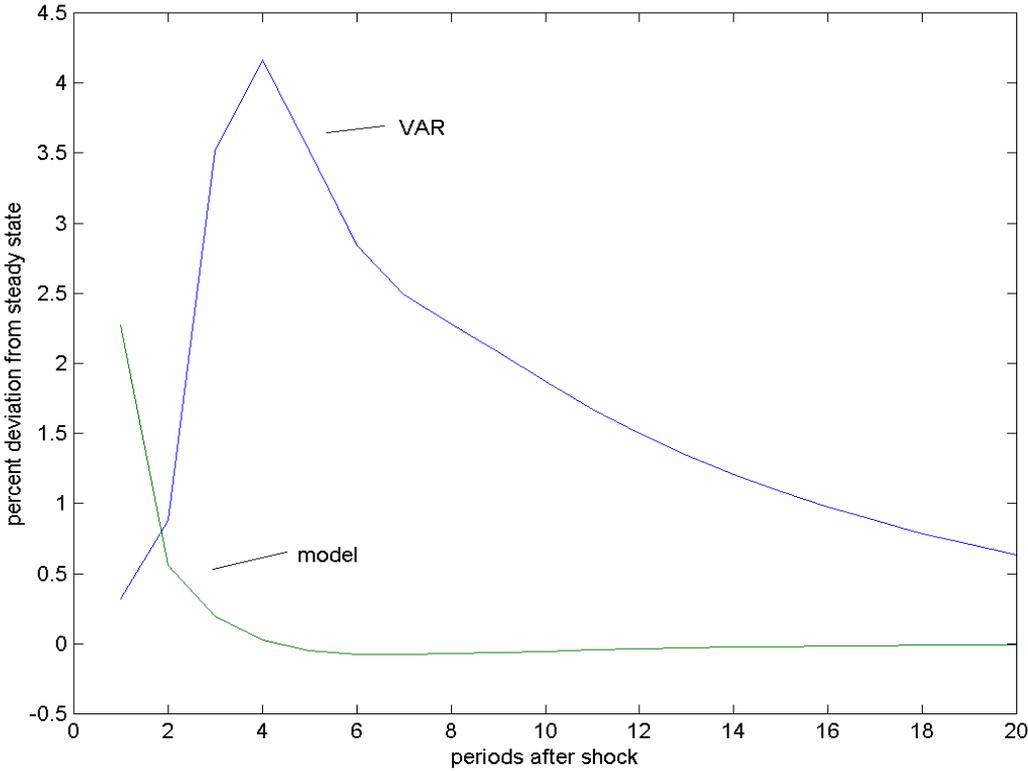


Figure 10.30 First Criterion, Flat Prior, n=100,000, restricted



**Figure 10.31** *Second Criterion, Flat Prior, n=100,000, restricted*



**Figure 10.32** *Second Criterion, Flat Prior, n=100,000, restricted*

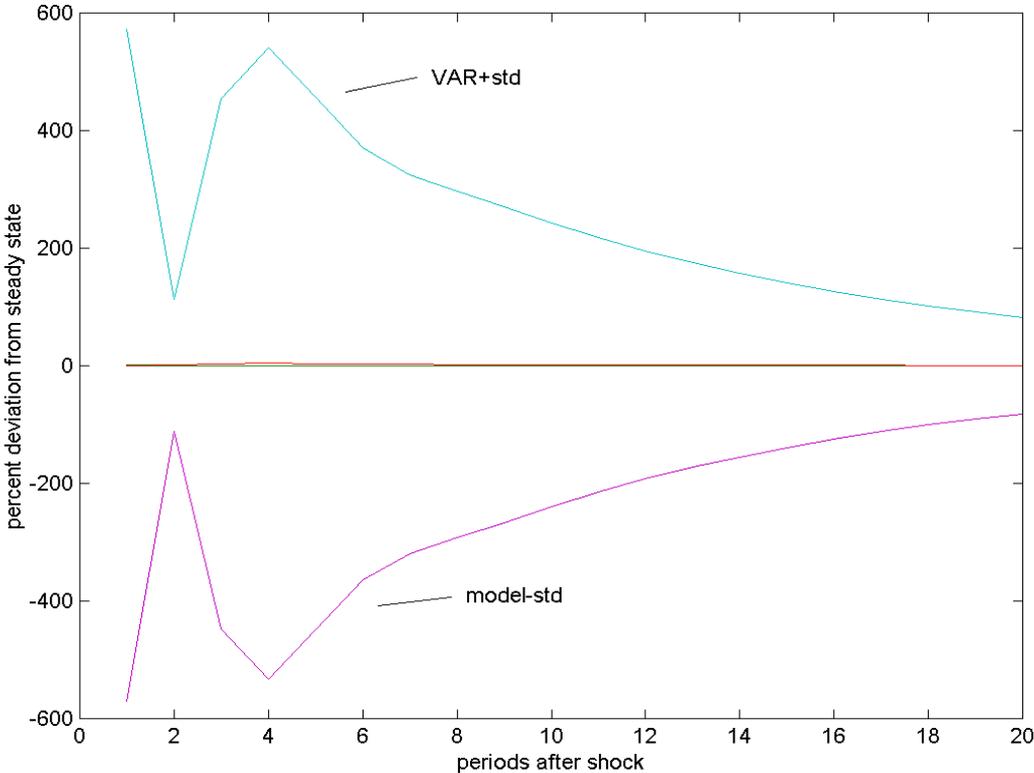


Figure 10.33 All Criteria, Minnesota Prior, n=1000, restricted

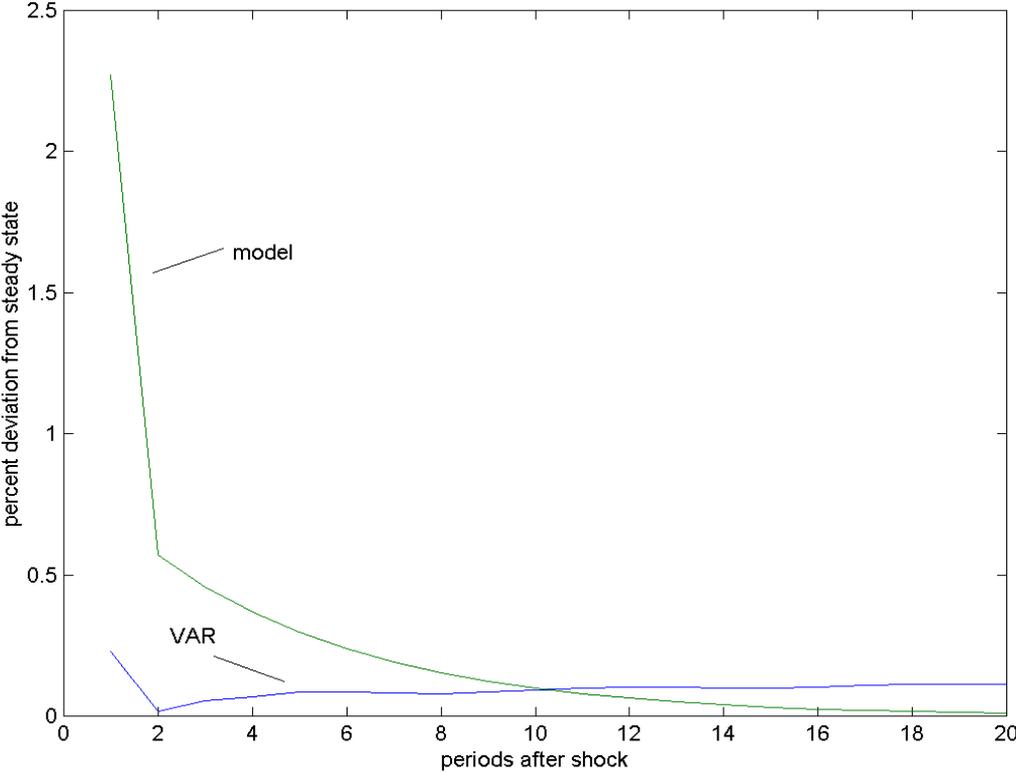
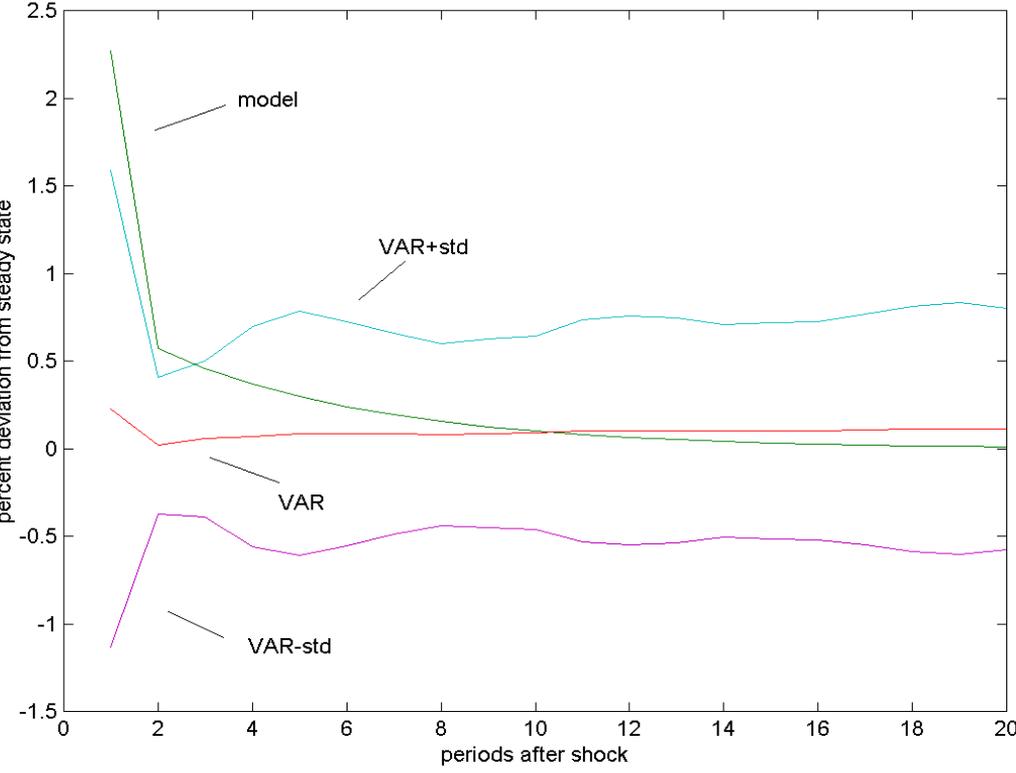
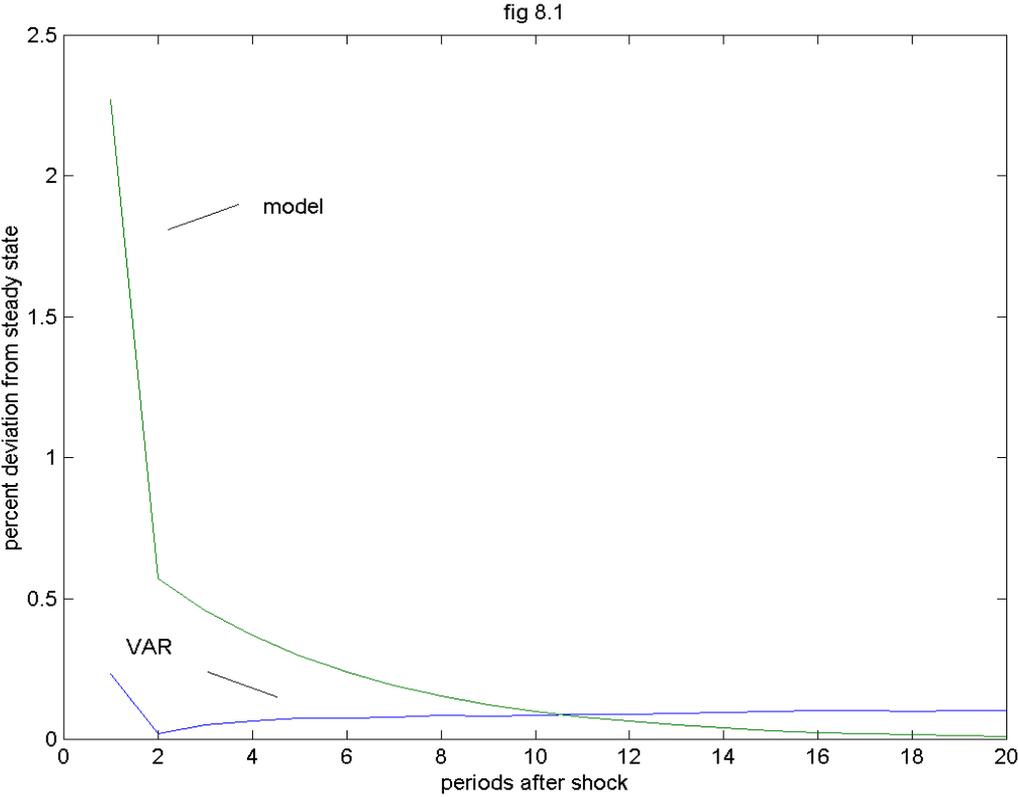


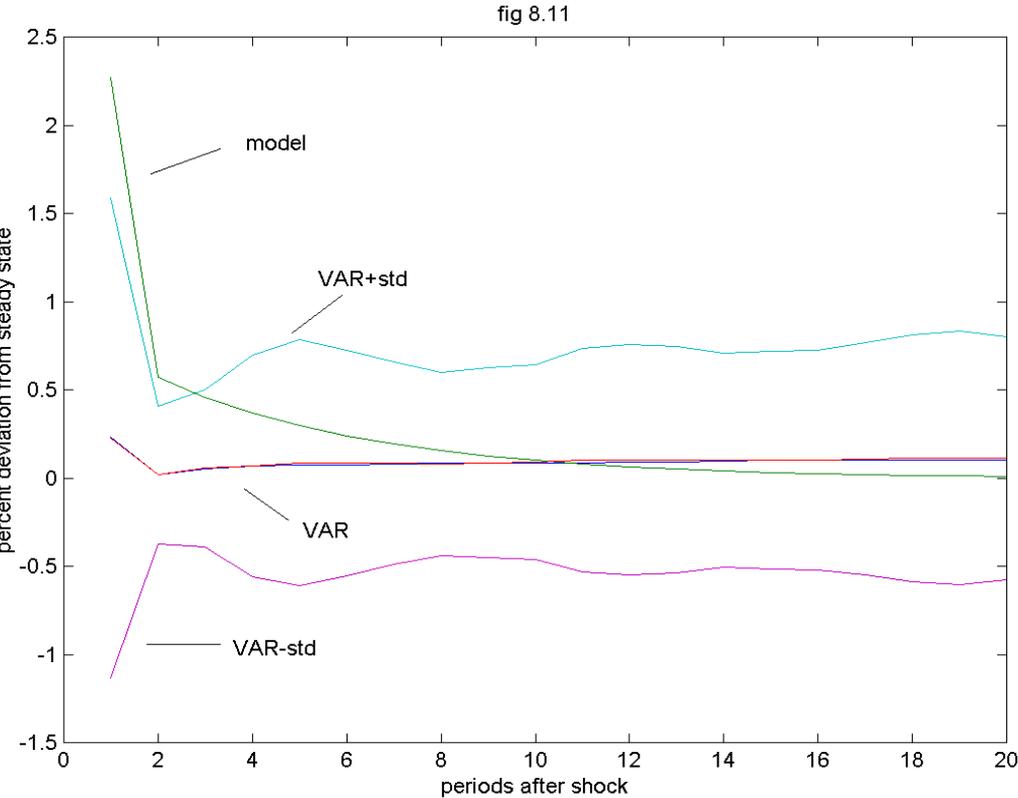
Figure 10.34 All Criteria, Minnesota Prior, n=1000, restricted



**Figure 10.35 All Criteria, Minnesota Prior, n=100,000, restricted**



**Figure 10.36 All Criteria, Minnesota Prior, n=100,000, restricted**



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## **Declaration of Authorship**

I hereby confirm that I have authored this master's thesis independently and without use of others than the indicated resources.

Alexander Kriwoluzky

Berlin, 26<sup>th</sup> December 2003